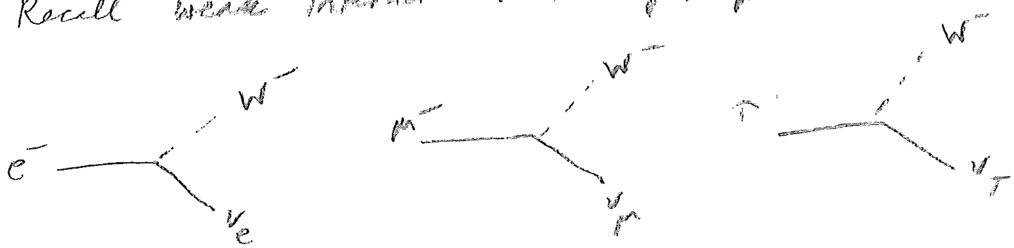
Weak interactions of quarks

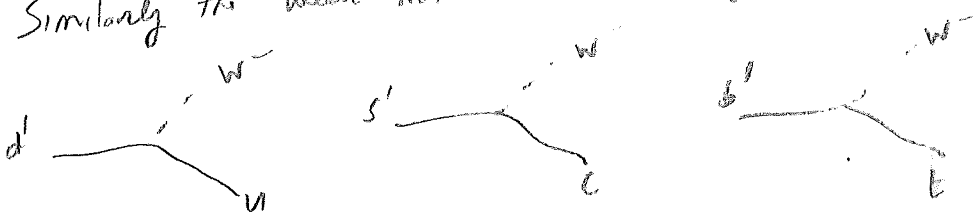
Recall weak interaction vertices of leptons



ν_e, ν_μ, ν_τ are not states of well defined mass. but are linear combinations of mass eigenstates ν_1, ν_2, ν_3

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta_{12}\cos\theta_{13} & \cdots \\ \vdots & \ddots \end{pmatrix}}_{\text{PMNS matrix (Pontecorvo-Maki-}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Similarly the weak interaction vertices of leptons are

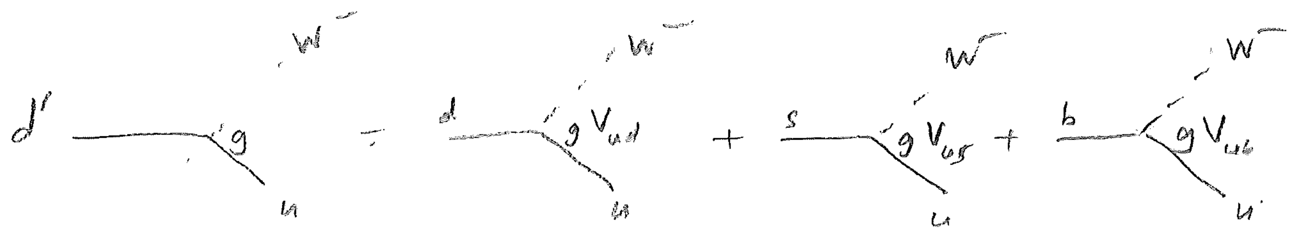


The weak interaction eigenstates d', s', b' are also not states of well defined mass, but are linear combinations of mass eigenstates d, c, s

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{\text{CKM (Cabibbo-Kobayashi-Maskawa) matrix}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

"mixing matrix"

e.g. $d' = V_{ud}\, d + V_{us}\, s + V_{us}\, b$



[which explains the labels on V]

experimentally. $V_{ud} \sim 0.97$

$V_{us} \sim 0.23$

$V_{ub} \sim 0.004$

These vertices allow intergenerational mixing of quarks

e.g. s & b quark can decay to u quark

If CKM matrix were unit matrix

strangeness etc would be absolutely conserved

The CKM matrix can be parametrized by 3 angle $\theta_{12}, \theta_{13}, \theta_{23}$ and one complex phase $e^{i\delta}$ (which allows CP violation)

$$V_{CKM} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

$c_{ij} = \cos\theta_{ij}$
$s_{ij} = \sin\theta_{ij}$

$\theta_{12} \approx 13^\circ \simeq 0.23$ rad
$\theta_{23} \approx 2.4^\circ \approx 0.042$ "
$\theta_{13} \approx 0.2^\circ \approx 0.0035$

θ_{ij} is roughly the decay amplitude between i & j generations.

Experimentally $\theta_{13} \ll \theta_{23} \ll \theta_{12} \ll 1$.

Let $\begin{cases} \theta_{12} \sim \epsilon \\ \theta_{23} \sim \epsilon^2 \\ \theta_{13} \sim \epsilon^3 \end{cases}$

Very roughly (ignoring CP violating phase) we can approximate

$$V_{CKM} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$$

where ϵ is small.

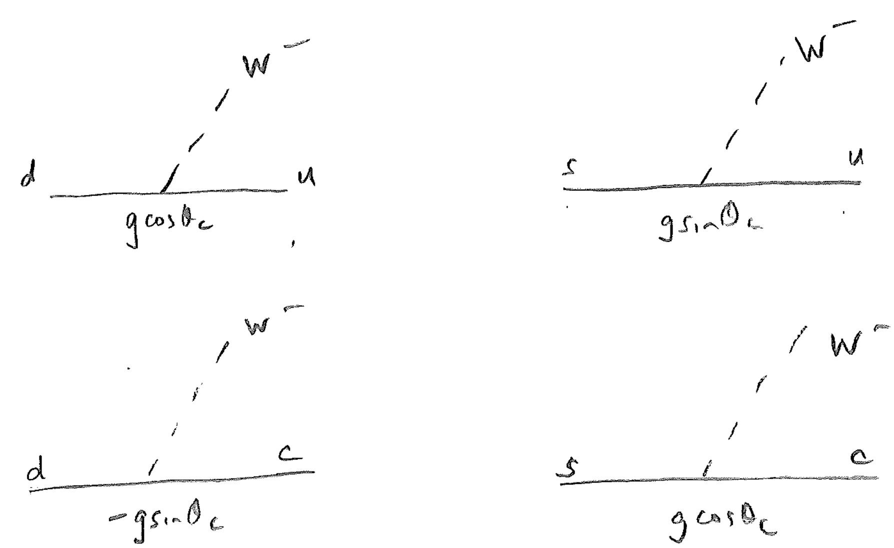
Hence top quark decays in stages:

$$t \xrightarrow{1} b \xrightarrow{\epsilon^2} c \xrightarrow{1} s \xrightarrow{\epsilon} u$$

If we set $\theta_{13} = \theta_{23} = 0$ and $\theta_{12} = \theta_c$ = Cabibbo angle

$$V_{ckm} = \begin{bmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{aligned} &\theta_c \approx 12^\circ \\ &\cos\theta_c \approx 0.98 \\ &\sin\theta_c \approx 0.23 \end{aligned}$$

we only have mixing between first two generations.



$\pi^+ = u\bar{d}$

$\pi^- = d\bar{u}$

$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$

$p = uud$

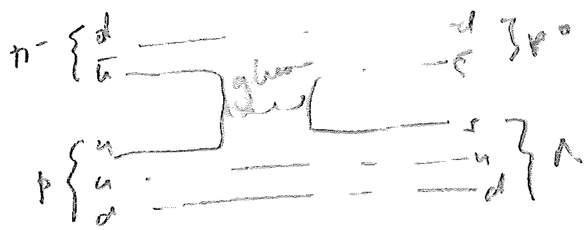
$n = udd$

strange baryon $\Lambda = uds$

strange meson $K^0 = d\bar{s}$

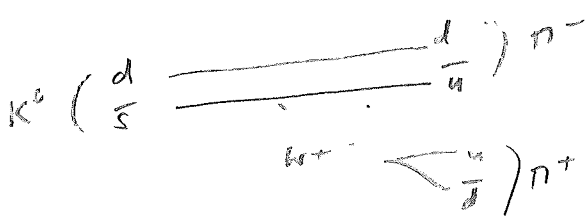
Strange particles are created in pairs by <u>strong</u> inter's

$\pi^- p \longrightarrow K^0 \Lambda$

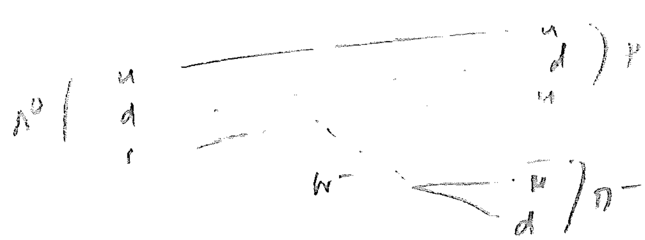


Strange particle decay individually by weak interaction (slowly)

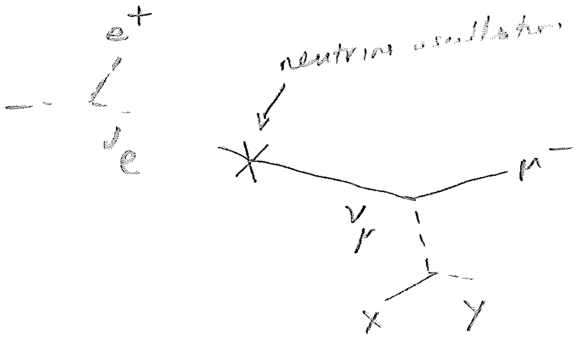
$K^0 \longrightarrow \pi^+\pi^-$



$\Lambda^0 \longrightarrow p\pi^-$



(5-7-25) [Added notes to WQ prompted by Peter Horvat's question abt why treat leptons differently from quarks] $\begin{pmatrix}\nu_e \\ e\end{pmatrix}$ $\begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}$



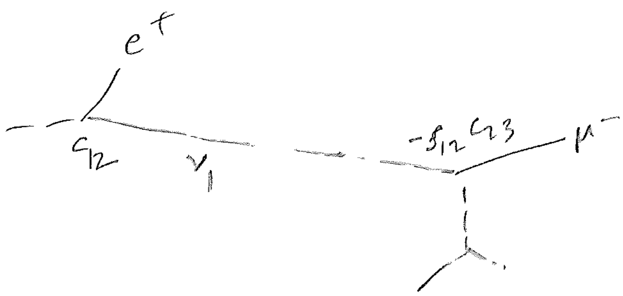
$$\nu_e = c_{12}\nu_1 + s_{12}\nu_2$$

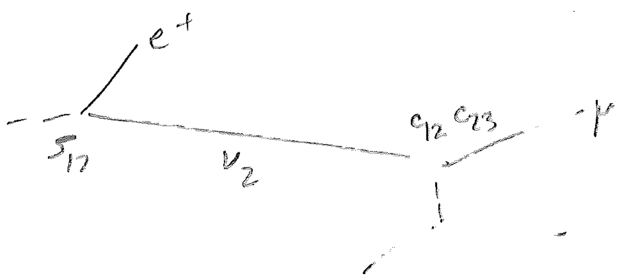
$$\nu_x = -s_{12}\nu_1 + c_{12}\nu_2$$

$$\nu_1 = c_{12}\nu_e - s_{12}\nu_x$$

$$\nu_2 = s_{12}\nu_e + c_{12}\nu_x$$

$$\nu_x = c_{23}\nu_\mu - s_{23}\nu_\tau$$

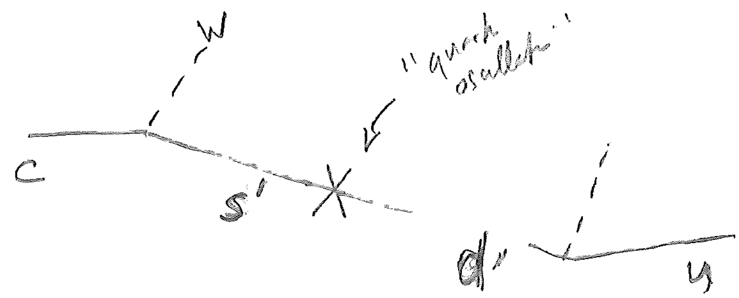




If $m_1 = m_2$ (or short distance, no phase developed)

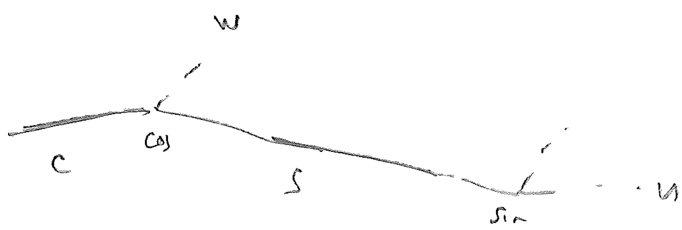
these cancel & get no μ^- production

If $m_1 \neq m_2$, then phase develops & these don't completely cancel



$$\begin{pmatrix}d'\\s'\end{pmatrix}=\begin{pmatrix}c & s\\ -s & c\end{pmatrix}\begin{pmatrix}d\\s\end{pmatrix}$$

$$\begin{pmatrix}d\\s\end{pmatrix}=\begin{pmatrix}c & -s\\ s & c\end{pmatrix}\begin{pmatrix}d'\\s'\end{pmatrix}$$



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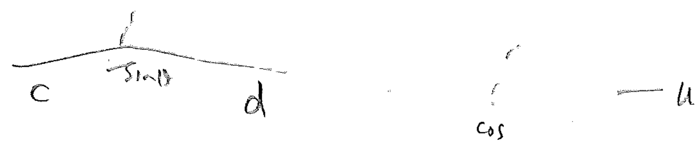
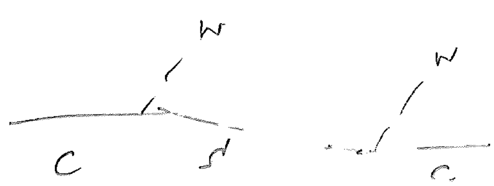
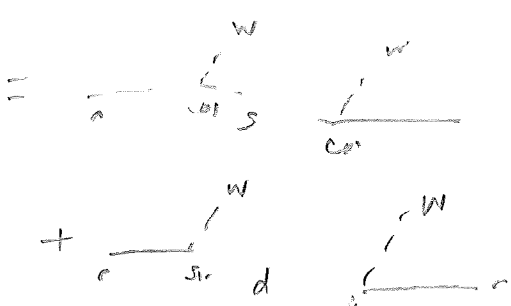


If $m_s = m_d$ then these cancel!
Thus no intergenerational mixing
But $m_s \neq m_d$,

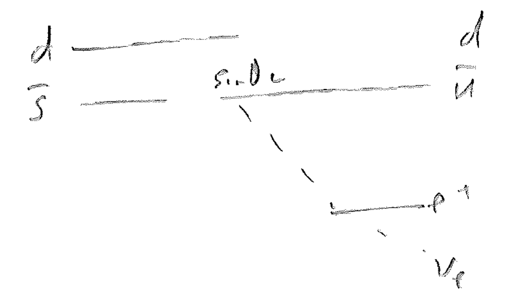




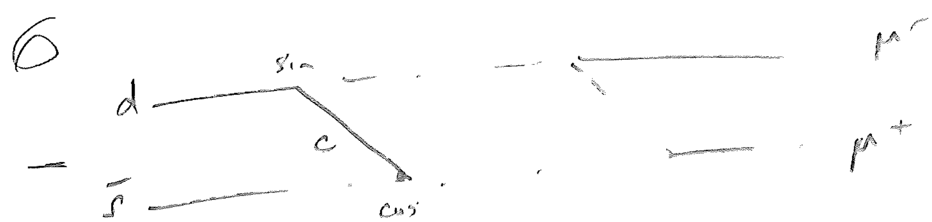
if $m_s = m_d$ add up to $\cos^2 + \sin^2 = 1$

$K^0 = \begin{cases} d \\ \bar{s} \end{cases}$

usual decay: $K^0 \to \pi^- e^+ \nu_e$ [40%]



Also $K^0 \to \mu^+ \mu^-$ [6.8×10⁻⁹]



If $m_u = m_c$ these cancel

Since $m_c >> m_u$, 2nd decay is suppressed.

(old note)

Electroweak theory (Glashow Weinberg Salam 1968)

gauge theory w/ gauge group $SU(2)_L \times U(1)_Y$ (weak isospin; weak hypercharge)

$SU(2)_W$ has 3 gauge bosons W^+, W^0, W^- (adjoint of $SU(2)_W$)
$U(1)_Y$ has 1 gauge boson B
} all spin 1

All 4 bosons are "initially" massless

perhaps do the "hot" one

The Higgs mechanism gives mass $M_W \approx 80.4$ GeV to W^+, W^-

and mass $M_Z \approx 91.2$ GeV to the linear combination

[$\sin^2\theta_W = 0.2312$ / $\overline{MS}$?]

$$Z^0 = W^0 \cos\theta_W - B \sin\theta_W \quad \text{where } \theta_W = 28.74^\circ$$

The $W^\pm$ & Z^0 were discovered 1983 at CERN [Nobel Dreams]

The other linear combination,

$$A = W^0 \sin\theta_W + B \cos\theta_W$$

remains massless, is identified as the photon

Eliminate B to obtain

$$Z^0 = W^0 \cos\theta_W - \left(\frac{A - W^0 \sin\theta}{\cos\theta}\right)\sin\theta$$

$$= \frac{W^0}{\cos\theta_W} - A^0 \tan\theta$$

This suggests $M_Z = \frac{M_W}{\cos\theta_W} = \frac{80.4}{0.877} = 91.6$ discrepancy due to quantum corrections