(4-3-21)

Massive neutrinos

In SM, ν_e, ν_μ, ν_τ are massless

[Observations of oscillations of solar + atmospheric neutrinos + of neutrinos produced in reactors + accelerators]

Observations $\Rightarrow$ small but nonzero masses.

But not m_{ν_e}, m_{ν_μ}, m_{ν_τ}

Specific linear combinations of ν_e, ν_μ, ν_τ are mass eigenstates, ie states of well defined mass

Called ν_1, ν_2, ν_3 w/ m_1, m_2, m_3

Masses [are all probably] $\lesssim 1$ eV
(limits from tritium decay, + astrophysics)

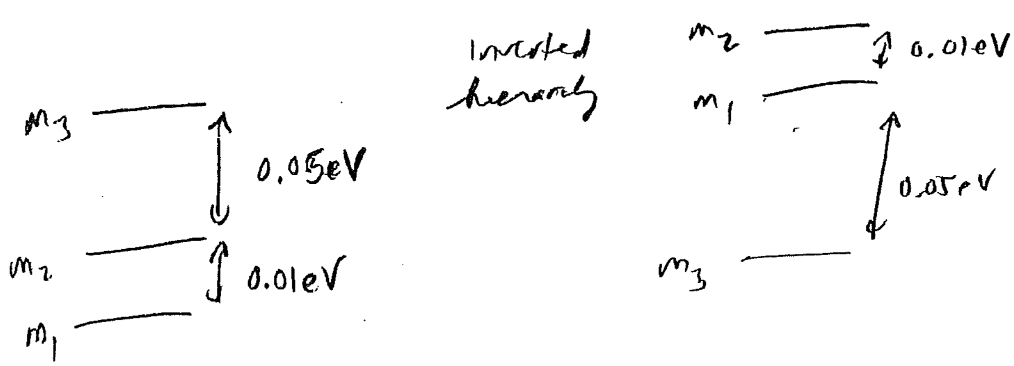
Oscillations only tell us mass differences

Two possibilities

defer until later

normal hierarchy

inverted hierarchy



Linear combinations

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

[like a rotation matrix but complex $\Rightarrow$ unitary]

U = PMNS matrix (Pontecorvo Maki Nakagawa Sakata)

(hand out) no need to write

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{bmatrix}$$

$c_{12} = \cos\theta_{12}$ etc

$s_{12} = \sin\theta_{12}$

current observations $\Rightarrow$

$$\theta_{12} \approx 34^\circ$$
$$\theta_{23} \approx 48^\circ$$
$$\theta_{13} \approx 9^\circ$$
$$\delta \approx 1.2\pi$$

e.g. $\nu_e = (\cos\theta_{12}\cos\theta_{13})\nu_1 + (\sin\theta_{12}\cos\theta_{13})\nu_2 + (\sin\theta_{13}e^{-i\delta})\nu_3$

$\approx 0.82\,\nu_1 + 0.55\,\nu_2 + 0.16\,e^{-1.2\pi i}\nu_3$

square to get probabilities: 67% 30% 3%

so ν_e is $\frac{2}{3}\nu_1$; $\frac{1}{3}\nu_2$, little ν_3

Parametrization of PMNS (for leptons) or CKM (for quarks) matrix in terms of 3 angles $\theta_{12}, \theta_{13}, \theta_{23}$ and a CP-violation phase δ_{CP}

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\mathrm{CP}}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & c_{23}c_{13} \end{bmatrix},$$

[θ_{13} is small, so to simplify matters we'll
[pretend it is zero. Then (letting $\theta = \theta_{12}$)

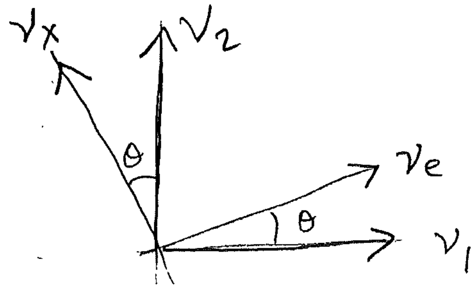
$$\nu_e = \cos\theta \, \nu_1 + \sin\theta \, \nu_2$$

The orthogonal linear combination we'll call

$$\nu_x = -\sin\theta \, \nu_1 + \cos\theta \, \nu_2 \, .$$

ν_x is neither ν_μ nor ν_τ but rather a
linear combination $\nu_x = \cos\theta_{23} \, \nu_\mu - \sin\theta_{23} \, \nu_\tau$.

We call ν_x orthogonal because if we represent
ν states as vectors, then ν_e and ν_x are $\perp$.



Use matrix notation

$$\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

rotation matrix for 2d vector

Invert this to write

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix}$$

or can just observe that rotate through opposite angle.

As we evolve in time, mass eigenstates (ie energy eigenstates) pick up a phase factor

$$\nu_i(t) = e^{-\frac{iE_i t}{\hbar}} \nu_i(0)$$

For now denote the phase as $\phi_i = -\frac{E_i t}{\hbar}$

$$\nu_i(t) = e^{i\phi_i} \nu_i(0)$$

Suppose a nuclear process ~~emits~~ a ν_e.
The initial ~~state~~ is

$$\psi(0) = \nu_e = \cos\theta\ \nu_1 + \sin\theta\ \nu_2$$

After time t, the state evolves into

$$\psi(t) = \cos\theta\ e^{i\phi_1}\ \nu_1 + \sin\theta\ e^{i\phi_2}\ \nu_2$$

$$= \cos\theta\, e^{i\phi_1}\left(\cos\theta\ \nu_e - \sin\theta\ \nu_x\right) + \sin\theta\, e^{i\phi_2}\left(\sin\theta\ \nu_e + \cos\theta\ \nu_x\right)$$

$$= \left(\cos^2\theta\, e^{i\phi_1} + \sin^2\theta\, e^{i\phi_2}\right)\nu_e - \sin\theta\cos\theta\left(e^{i\phi_1} - e^{i\phi_2}\right)\nu_x$$

Define ~~Let~~ average phase $\bar\phi = \frac{\phi_1+\phi_2}{2}$ and difference $\Delta\phi = \phi_1 - \phi_2$

$$\Rightarrow \phi_1 = \bar\phi + \tfrac{1}{2}\Delta\phi$$
$$\phi_2 = \bar\phi - \tfrac{1}{2}\Delta\phi$$

$$\psi(t) = e^{i\bar\phi}\left(\left[\cos^2\theta\ e^{+\frac{i\Delta\phi}{2}} + \sin^2\theta\ e^{-\frac{i\Delta\phi}{2}}\right]\nu_e - \sin\theta\cos\theta\left[e^{\frac{i\Delta\phi}{2}} - e^{-\frac{i\Delta\phi}{2}}\right]\nu_x\right)$$

Show that this can be simplified to [HW]

$$\psi(t) = e^{i\bar\phi}\left(\left[\cos\left(\tfrac{\Delta\phi}{2}\right) + i\cos(2\theta)\sin\left(\tfrac{\Delta\phi}{2}\right)\right]\nu_e + \left[-i\sin(2\theta)\sin\tfrac{\Delta\phi}{2}\right]\nu_x\right)$$

Probability that ν_e has evolved into ν_x is

$$P(\nu_e \to \nu_x) = \left| -i \sin(2\theta) \sin\left(\frac{\Delta\phi}{2}\right) \right|^2$$

$$= \sin^2(2\theta) \sin^2\left(\frac{\Delta\phi}{2}\right)$$

The probability that ν_e has remained a ν_e is

$$P(\nu_e \to \nu_e) = \left| \cos\left(\frac{\Delta\phi}{2}\right) + i \cos(2\theta) \sin\left(\frac{\Delta\phi}{2}\right) \right|^2$$

$$= \cos^2\left(\frac{\Delta\phi}{2}\right) + \cos^2(2\theta) \sin^2\left(\frac{\Delta\phi}{2}\right)$$

$$= 1 - \left(1 - \cos^2(2\theta)\right) \sin^2\left(\frac{\Delta\phi}{2}\right)$$

$$= 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta\phi}{2}\right)$$

As expected: $P(\nu_e \to \nu_e) + P(\nu_e \to \nu_x) = 1$

Prob that ν_e evolves into ν_x in time t

$$P(\nu_e \to \nu_x) = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta\phi}{2}\right)$$

Recall $\phi_i = -\dfrac{E_i t}{\hbar}$

$$\frac{\Delta\phi}{2} = \frac{\phi_1 - \phi_2}{2} = \frac{(E_2 - E_1)t}{2\hbar}$$

ν is nearly massless, travels distance $L = ct$ in time t

$$\frac{\Delta\phi}{2} = \frac{(E_2 - E_1)L}{2\hbar}$$

$\to \left[E_i = p_i + \dfrac{m_i^2}{2p_i} \right]$

$$E_i^2 = p_i^2 + m_i^2$$

Assume different mass eigenstates have common momenta p
(technically incorrect but gives correct answer)

$\hookrightarrow e^{ipx}$ same for all eigenstates so gives no phase difference

$$E_2^2 - E_1^2 = m_2^2 - m_1^2 \equiv \Delta m_{21}^2$$

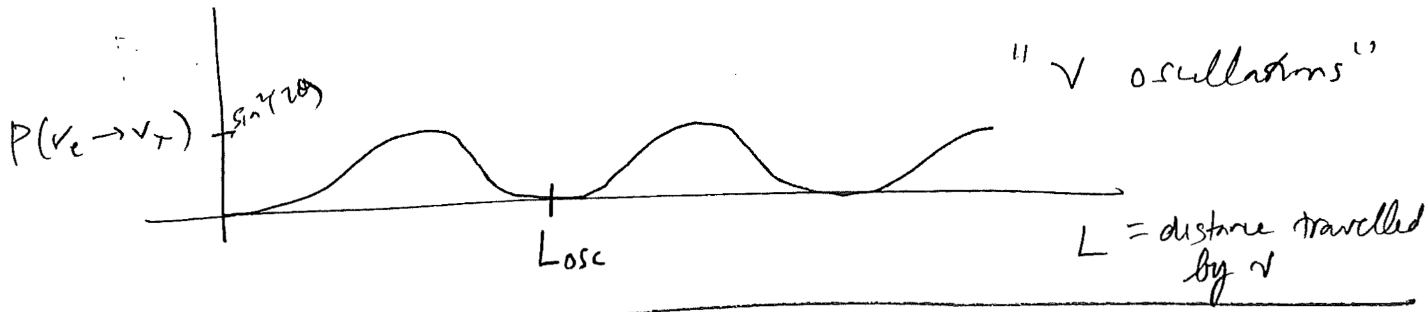
$$\underbrace{(E_2 - E_1)}\underbrace{(E_2 + E_1)}_{2E_\nu} = \Delta m_{21}^2$$

where E_ν = average energy

$$E_2 - E_1 = \frac{\Delta m_{21}^2}{2E_\nu}$$

$$\frac{\Delta\phi}{2} = \frac{\Delta m_{21}^2 L}{4\hbar E_\nu} \xrightarrow[\text{restore } c]{} \frac{(\Delta m_{21}^2)\, c^3 L}{4\hbar E_\nu}$$

$$P(\nu_e \to \nu_x) = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m_{21}^2\, c^3 L}{4\hbar E_\nu}\right)$$



Define oscillation length L_{osc} by $\dfrac{\Delta m_{21}^2 c^3 L_{osc}}{4\hbar E_\nu} = \pi$

$$\Rightarrow \boxed{L_{osc} = \frac{4\pi\hbar E_\nu}{(\Delta m_{21}^2)\, c^3}}$$

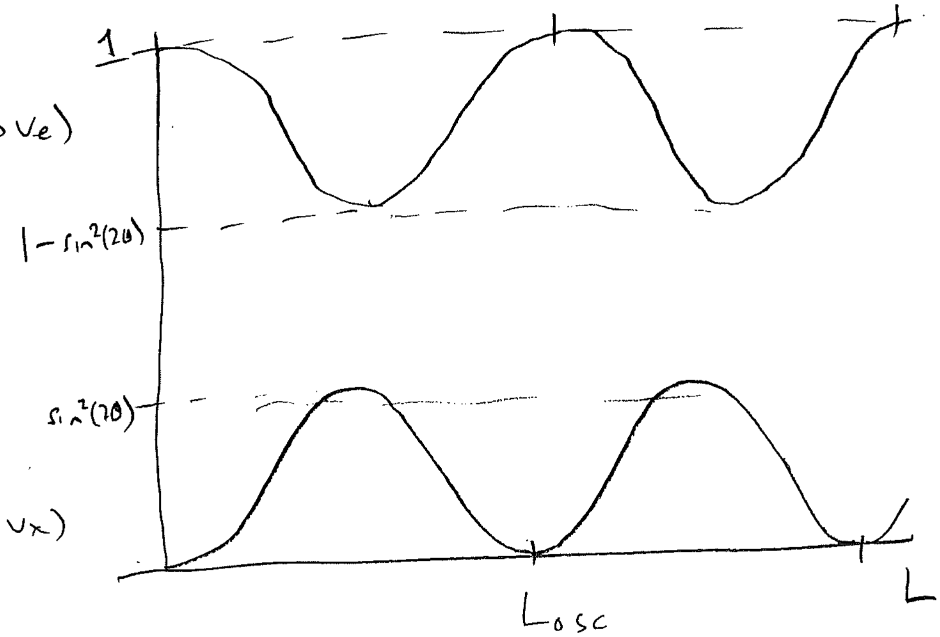
$$= 4\pi\hbar c\, \frac{E_\nu}{\Delta m_{21}^2 c^4}$$

$\hbar c = 197$ MeV·fm

$4\pi\hbar c = (2.48\,m)\, 10^{-12}$ MeV

$$L_{osc} = (2.48\,m)\left(\frac{E_\nu}{MeV}\right)\left(\frac{eV^2}{\Delta m_{21}^2 c^4}\right)$$

$$P(\nu_e \to \nu_x) = \sin^2(2\theta)\,\sin^2\left(\frac{\pi L}{L_{osc}}\right)$$



$$P(\nu_e \to \nu_x) = \sin^2(2\theta)\,\sin^2\left(\frac{\pi L}{L_{osc}}\right)$$

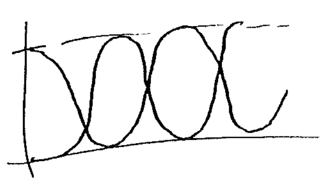
$$P(\nu_e \to \nu_e) = 1 - \sin^2(2\theta)\,\sin^2\left(\frac{\pi L}{L_{osc}}\right)$$

For oscillations need two things

① mass difference: $\Delta m^2_{21} \neq 0$

② mixing: $\theta \neq 0$

NB if $\theta = 45°$ we have maximal mixing



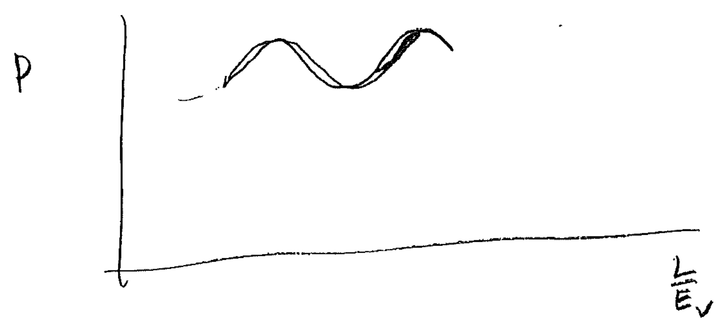
Kam LAND experiment (2011)

(Kamioka Liquid scintillator Anti Neutrino Detector)

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2(2\theta_{12}) \sin^2\left(\frac{\Delta m^2_{21}\, c^3 L}{4\hbar E_\nu}\right)$$

$\bar{\nu}_e$ from nuclear reactors at a distance $L \approx 180$ km

produced w/ E from 1.8 to 10 MeV



$\frac{L}{E_\nu}$ from 100 $\frac{km}{MeV}$ to 20 $\frac{km}{MeV}$

[Donelly, fig 18.6]
[see Peskin, fig 20.5]

[HW: use oscillations to compute Δm^2_{21}]

$\Delta m^2_{21} = 7.5 \times 10^{-5}$ eV

Recall $L_{osc} = (2.48\text{ m})\left(\frac{E_\nu}{MeV}\right)\left(\frac{eV^2}{\Delta m^2_{ij} c^4}\right)$

$L_{solar} = (33\text{ km})\left(\frac{E_\nu}{MeV}\right)$

$\left[\frac{(2.48\text{ m})}{7.5E-5} = 33.000\text{ m}\right]$

$\left[\sin^2\left(0.095 \left(\frac{L}{km}\right)\left(\frac{MeV}{E_\nu}\right)\right)\right]$

[previous expts in 80's to observe oscillation failed because $L << L_{solar}$]

V10

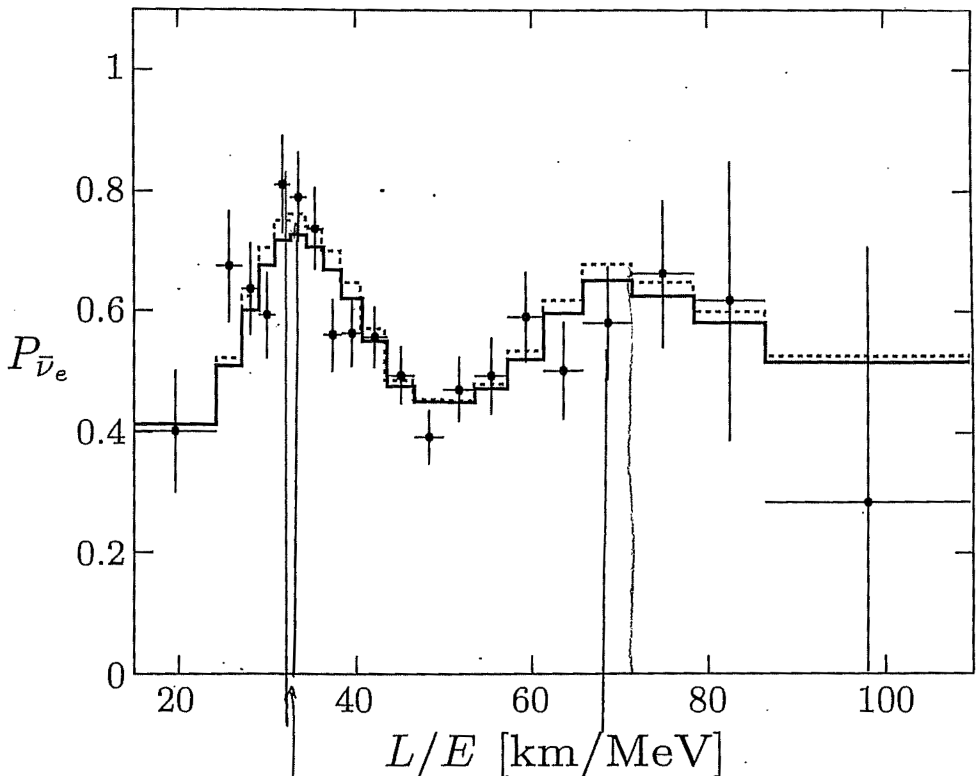


Fig. 18.6 The antineutrino survival probability versus L/E from the KamLAND experiment; figure adapted from [Gan11].

Donnelly

27.

70

70.

$$\overbrace{\frac{4\pi\hbar c}{\Delta m_{21}^2 c^4}}^{\text{one period}} = \frac{2475 \text{ meV}^2 \text{ fm}}{7.5\text{E-}17 \text{ meV}^2}$$

$\approx 3.3\text{E}19$ fm

$= 33 \frac{\text{km}}{\text{mev}}$

Solar neutrinos

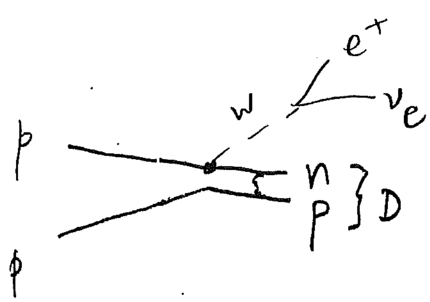
ν_e produced in sun w/ E from ~ 0.1 MeV to ~ 10 MeV

[Donnelly, fig 18.3]

most from $pp \to D\, e^+ \nu_e$ w/ $E < 0.42$ MeV

[recall HW]

many fewer from $^8B \to {}^8Be\; e^+ \nu_e$ but w/ $E \sim 10$ MeV



[Bahcall 10^{11} ν/s through thumbnail] ν/2

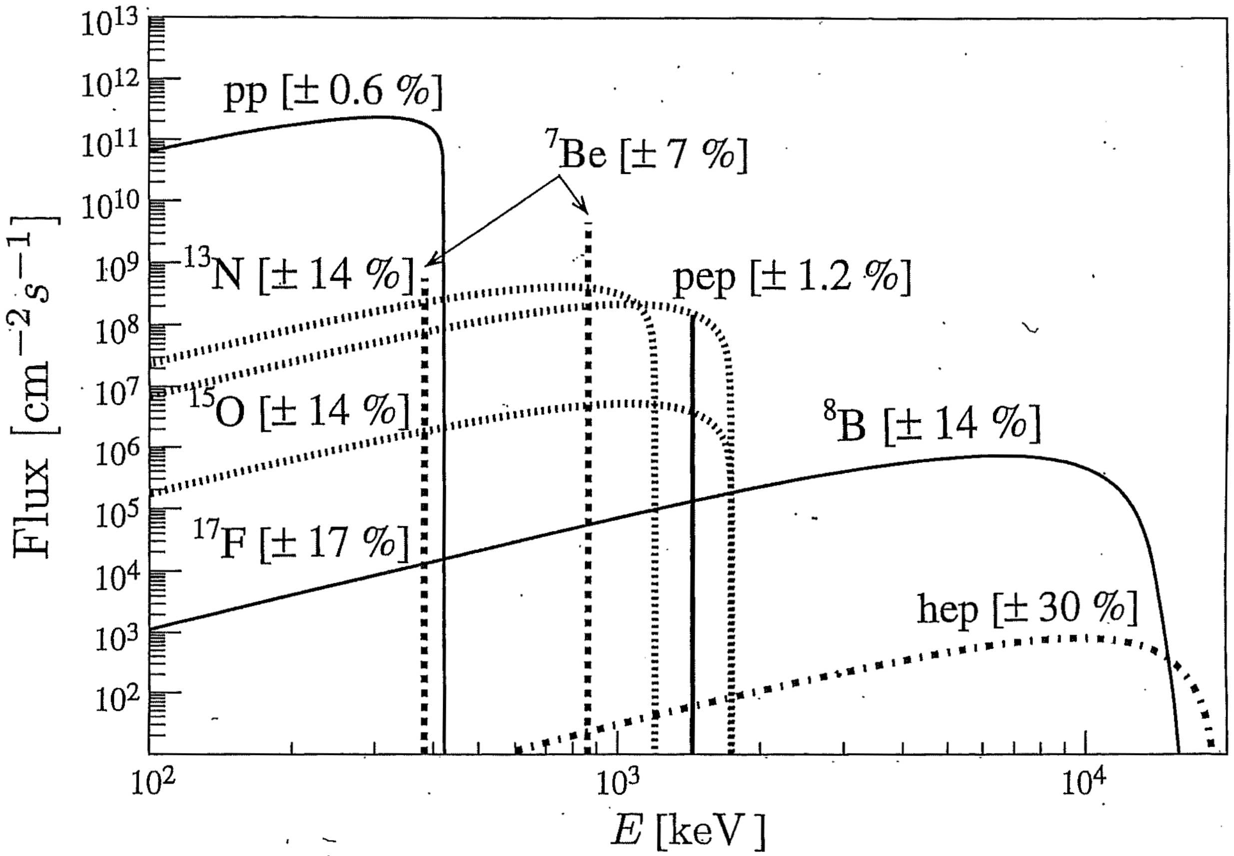


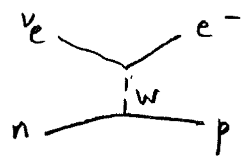
Fig. 18.3 The calculated solar neutrino energy spectrum from a variety of decay chain progenies for a particular solar model [Bel14].

Donnelly

How to detect ν_e?

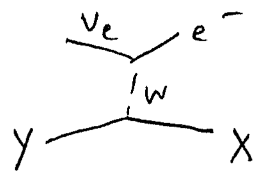
[Before had $\bar{\nu}_e$ from reactors: $\bar{\nu}_e p \to n e^+$ if $E_\nu > 1.8$ MeV]

$$\nu_e n \xrightarrow{?} p e^-$$



But no stable neutrons, except in nuclei

$$\nu_e \; {}^A_Z Y \longrightarrow {}^A_{Z+1} X e^-$$



$$Q = m(Y) - m(X) - m_e$$
$$= \Delta(Y) - \Delta(X)$$

$Q < 0$ for stable nuclei Y.

Reaction possible if $E_\nu > -Q$.

[a little bit higher since e^- carries off kinetic energy]

A practical process

$$\nu_e \; {}^{37}_{17}Cl \to {}^{37}_{18}Ar \; e^- \qquad Q = -0.814 \text{ MeV}$$

[calc in HW]

The byproduct ${}^{37}Ar$ is radioactive ~ $T_{\frac{1}{2}} = 35$ days

$$\underbrace{e^- \; {}^{37}_{18}Ar}_{\text{EC}} \to {}^{37}_{17}Cl \; \nu_e \qquad Q = 0.814 \text{ MeV}$$

↓

(Auger electrons)

[${}^{36,38,40}Ar$ are stable]

Solar neutrino detection

1968 Ray Davis [Nobel 2002]
Homestake mine
S. Dakota

615 tons of dry cleaning fluid
CCl_4 carbon tetrachloride (before 1970)
C_2Cl_4 PERC (perchloroethylene)
(aka tetrachloroethylene
tetrachloroethane)

• Flush out ^{37}Ar and concentrate about once a month

Sensitive only to ^{8}B ν_e because $|Q| > 0.42$ MeV

only detected $\frac{1}{3}$ expected #

"Solar neutrino problem"

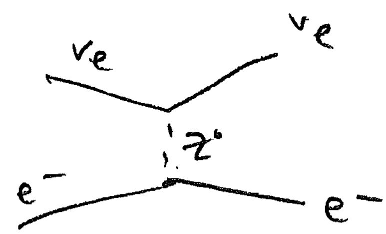
[previous expt by Davis + Harmer, 1959 [F + H, p. 187] w/ reactor $\bar{\nu}_e$ to demonstrate ν + $\bar{\nu}$ are distinct]

2001 super Kamiokande (Japan)
(orig built to detect proton decay)

50,000 tons of water

ν_e interact not w/ nuclei but w/ electrons

$$\nu_e e^- \rightarrow \nu_e e^-$$



If $E_\nu \gtrsim 4$ meV, outgoing e^- moves faster than the speed of light in water $\Rightarrow$ Čerenkov radiation

$\overrightarrow{\text{check}}$

Also detected a deficit of ν_e at earth.

[can also detect $\nu_\mu e^- \to \nu_\mu e^-$
but $\nu_e e^- \to \nu_e e^-$ is 6.5 X larger]

2002 Sudbury neutrino observatory (SNO), canada
heavy water (D_2O)

could also detect

$$\nu D \rightarrow n p \nu \qquad (E_\nu > 2.2 \text{ meV})$$
$$\nu_e D \rightarrow p p e^- \qquad (E_\nu > 1.4 \text{ meV})$$

What about lower energy neutrinos produced by pp rxn?

$\nu_e\ {}^{71}Ga \longrightarrow {}^{71}Ge\ e^-$ $\qquad Q = -0.233\ MeV$

SAGE (USSR)
GALLEX (Italy)

Also liquid scintillator expts $\qquad Q = -0.19\ MeV$

Borexino (Italy)

Also detected a deficit but not as much
High energy $\nu \Rightarrow P \approx 0.31$
Low energy $\nu \Rightarrow P \approx 0.55$

[see Donnelly, fig 1.8-4]

V17

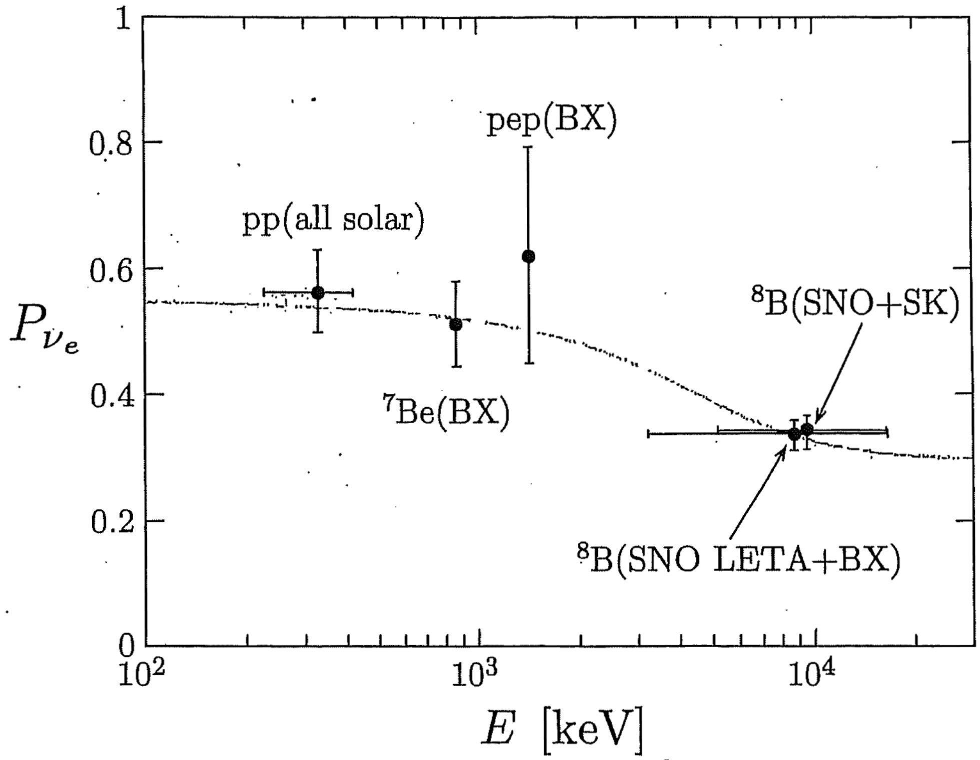


Fig. 18.4 The electron neutrino survival probability as a function of neutrino energy assuming matter-enhanced oscillations. The data points indicate measurements as made by the SNO and Super-Kamiokande experiments (for ^{8}B), Borexino, and radio-chemical experiments [Bel14].

Assuming solar models (Bahcall) and experiments are both accurate

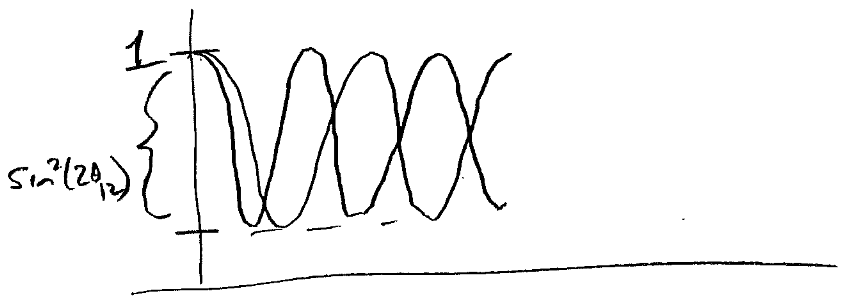
how explain solar neutrino deficit?

Neutrino oscillations

$$P(\nu_e \to \nu_e) = 1 - \sin^2(2\theta_{12}) \sin^2\left(\frac{\pi L}{L_{solar}}\right)$$

$$L_{solar} = (33\,km)\left(\frac{E_\nu}{MeV}\right)$$

so $L_{solar} \lesssim 500\,km \ll L_{sun-earth}$



Range of energies ⇒ oscillation average out at earth

$$P(\nu_e \to \nu_e) = 1 - \frac{1}{2}\sin^2(2\theta_{12}) \quad \text{(for low energy } \nu\text{)}$$

Note $\frac{1}{2} \le P \le 1$ so can explain low energy neutrino deficit but not high

$P_{avg} \approx 0.55 \Rightarrow \theta_{12} \approx 34^\circ$

[actually $\theta_{12} = 34^\circ \Rightarrow P = 0.57$ but θ_{13} correction $\Rightarrow P = 0.55$]

High energy neutrinos undergo
"matter enhanced oscillations" in the sun
(MSW effect) so
ν_e adiabatically evolve to ν_2
which then propagate to earth w/o
further oscillation

$$\nu_2 = (\sin\theta_{12})\,\nu_e + (\cos\theta_{12})\,\nu_x$$

so $P(\nu_2 \to \nu_e) = \sin^2\theta_{12}$ (high energy neutrinos)

$\theta_{12} = 34^\circ \Rightarrow P \approx 0.31$

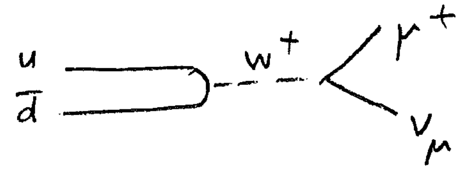
So $\theta_{12} = 34^\circ$ explains both low energy deficit
& high energy deficit

Atmospheric neutrinos

Cosmic rays (p, other nuclei)

produce $\pi^\pm$ in upper atmosphere

$\pi^+ (= u\bar{d}) \longrightarrow \mu^+ \nu_\mu$



[Peter Harris asked why not $\pi^+ \to e^+ \nu_e$
I explained abt chirality + how massless e^- + ν_e violate angular mom conservat because π has spin 0]

$\mu^+ \longrightarrow e^+ \nu_e \bar{\nu}_\mu$

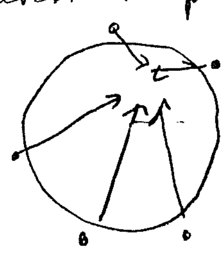
Similarly for π^-

$\therefore$ expect twice as many ν_μ as ν_e

E_ν typically $\sim$ GeV

$$L_{solar} = (33\,\text{km}) \left(\frac{E_\nu}{\text{MeV}} \right) > 30{,}000\,\text{km}$$

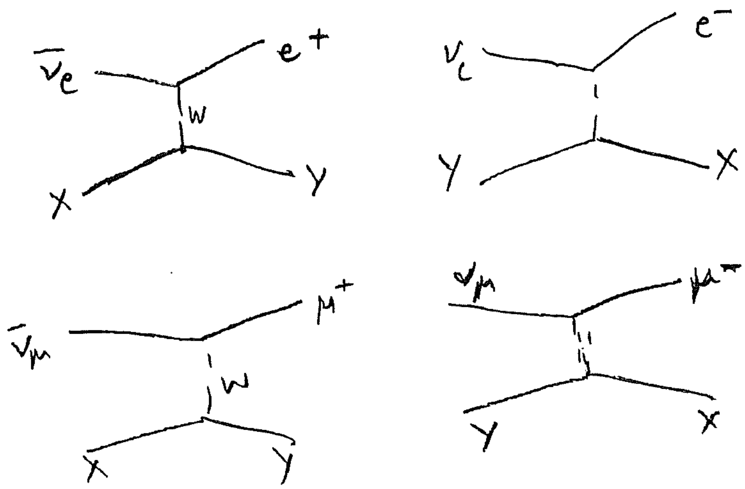
so expect no appreciable oscillations even if produced on other side of earth



$R_\oplus = 6400\,\text{km}$

[Atmos ν first observed in 1960's
cf Giunti + Kim: horizontal μ^-'s in scintillator
in gold mines produced by ν_μ interacting w/ rock]

Super K Čerenkov detector [50,000 tons water]



$e^\pm$ give more diffuse Čerenkov cone than $\mu^\pm$

[X, Y in rock around detector or inside]

[$\nu_\mu e^- \to \nu_e \mu^-$ only possible for $s \sim 2 m_e E_\nu > m_\mu^2$
ie $E_\nu \gtrsim 10$ GeV]

Late 1980's super K detected fewer ν_μ than expected relative to ν_e

Late 1990's distinguished directions:
- expected # of downward going ν_μ
- $\frac{1}{2}$ as many upward going ν_μ

Recall $$L_{osc} = \frac{4\pi\hbar E_\nu}{(\Delta m^2) c^3}$$

Perhaps ν_μ have <u>shorter</u> oscillation length than ν_e
due to a larger Δm^2 !

PMNS parametrization, with $\theta_{13}=0$, is

$$\nu_e = c_{12}\,\nu_1 + s_{12}\,\nu_2$$

$$\nu_\mu = (-s_{12}c_{23})\,\nu_1 + (c_{12}c_{23})\,\nu_2 + s_{23}\,\nu_3$$

$$\nu_\tau = (s_{12}\,s_{23})\,\nu_1 + (-c_{12}s_{23})\,\nu_2 + c_{23}\,\nu_3$$

Recall earlier we defined

$$\nu_x = -s_{12}\,\nu_1 + c_{12}\,\nu_2 \qquad \text{orthogonal to } \nu_e$$

(ν_x not quite a mass eigenstate but almost if $m_1 \approx m_2$)

$$\Rightarrow \quad \nu_\mu = c_{23}\,\nu_x + s_{23}\,\nu_3 \qquad \begin{pmatrix}\nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix} c_{23} & s_{23}\\ -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}\nu_x\\ \nu_3\end{pmatrix}$$

$$\nu_\tau = -s_{23}\,\nu_x + c_{23}\,\nu_3$$

We are assuming $m_1 \approx m_2 = m_x$ but $m_x \neq m_3$

the ν_e will not oscillate but

ν_μ will oscillate into ν_τ and vice versa.

We define the atmospheric oscillation length

$$L_{atm} = \frac{4\pi\hbar\, E_\nu}{\Delta m_{32}^2\, c^3}$$

$$P(\nu_\mu \to \nu_\mu) \cong 1 - \sin^2(2\theta_{23})\,\sin^2\left(\frac{\pi L}{L_{atm}}\right)$$

✓23

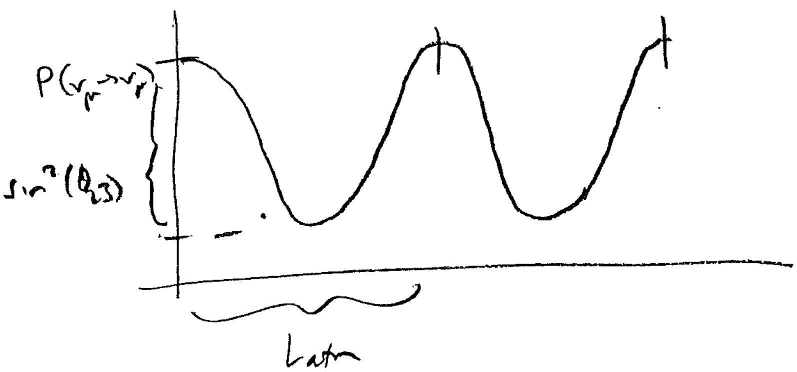
To measure Δm^2_{32}, use

accelerator neutrinos

$\pi^+ \to \mu^+ \nu_\mu$ w/ controllable energies $E_\nu \sim$ GeV

K2K (KEK to Kamiokande) $L = 250$ km

[Peskin] MINOS (Fermilab to Soudan, Minn) $L = 750$ km



"disappearance experiment"

Determined $\theta_{23} \approx 48^\circ$

$L_{atm} \cong (1.0\ \text{km})\left(\frac{E_\nu}{\text{MeV}}\right)$ (as opposed to 33 km for ν_e)

$$L_{atm} = \frac{4\pi\hbar E_\nu}{\Delta m^2_{32}\, c^2} = (2.48\,\text{m})\left(\frac{\text{eV}^2}{\Delta m^2_{32} c^4}\right)\left(\frac{E_\nu}{\text{MeV}}\right)$$

$$\Rightarrow \Delta m^2_{32} = 2.5 \times 10^{-3}\ \text{eV}^2$$

Reconsider atmospheric ν_μ

$E_\nu \sim 1\ \text{GeV} \Rightarrow L_{atm} = 1000\ \text{km} \ll R_\oplus$

oscillations average out in passing through earth

$$P(\nu_\mu \to \nu_\mu) = 1 - \frac{1}{2} \underbrace{\sin^2(2\theta_{23})}_{0.991} \approx 0.5$$

so expect only $\frac{1}{2}$ as many upward-going ν_μ as downward going ν_μ ✓

Pesheva?

$\nu_\mu \longrightarrow \nu_\tau$ appearance : OPERA

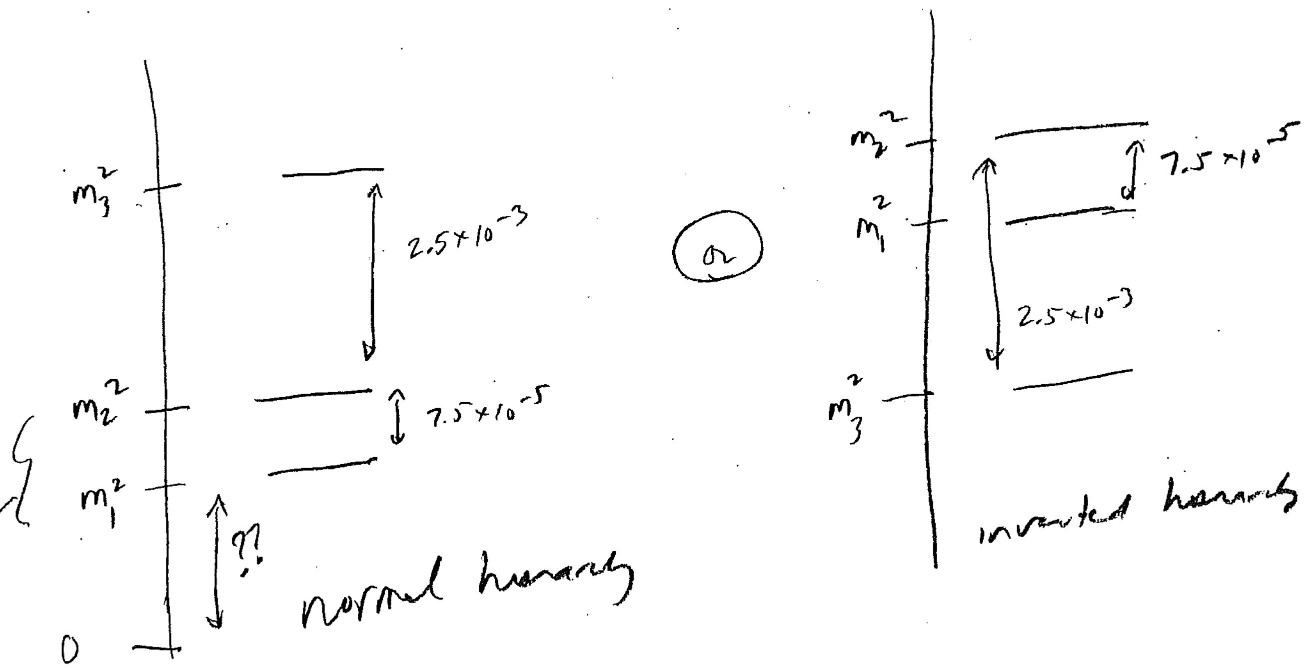
$\nu_\mu \longrightarrow \nu_e$ appearance : T2K
NOνA

Oscillations only measure mass differences

$|\Delta m^2_{21}| = |m_2^2 - m_1^2| = 7.5 \times 10^{-5}$ eV (KAMLand) reactor $\bar{\nu}_e$

$|\Delta m^2_{32}| = |m_3^2 - m_2^2| = 2.5 \times 10^{-3}$ eV (accelerator expts) accelerator ν_μ

What are the actual masses?



Expts underway to determine which

or vice versa. Define $m_1 < m_2$

√26

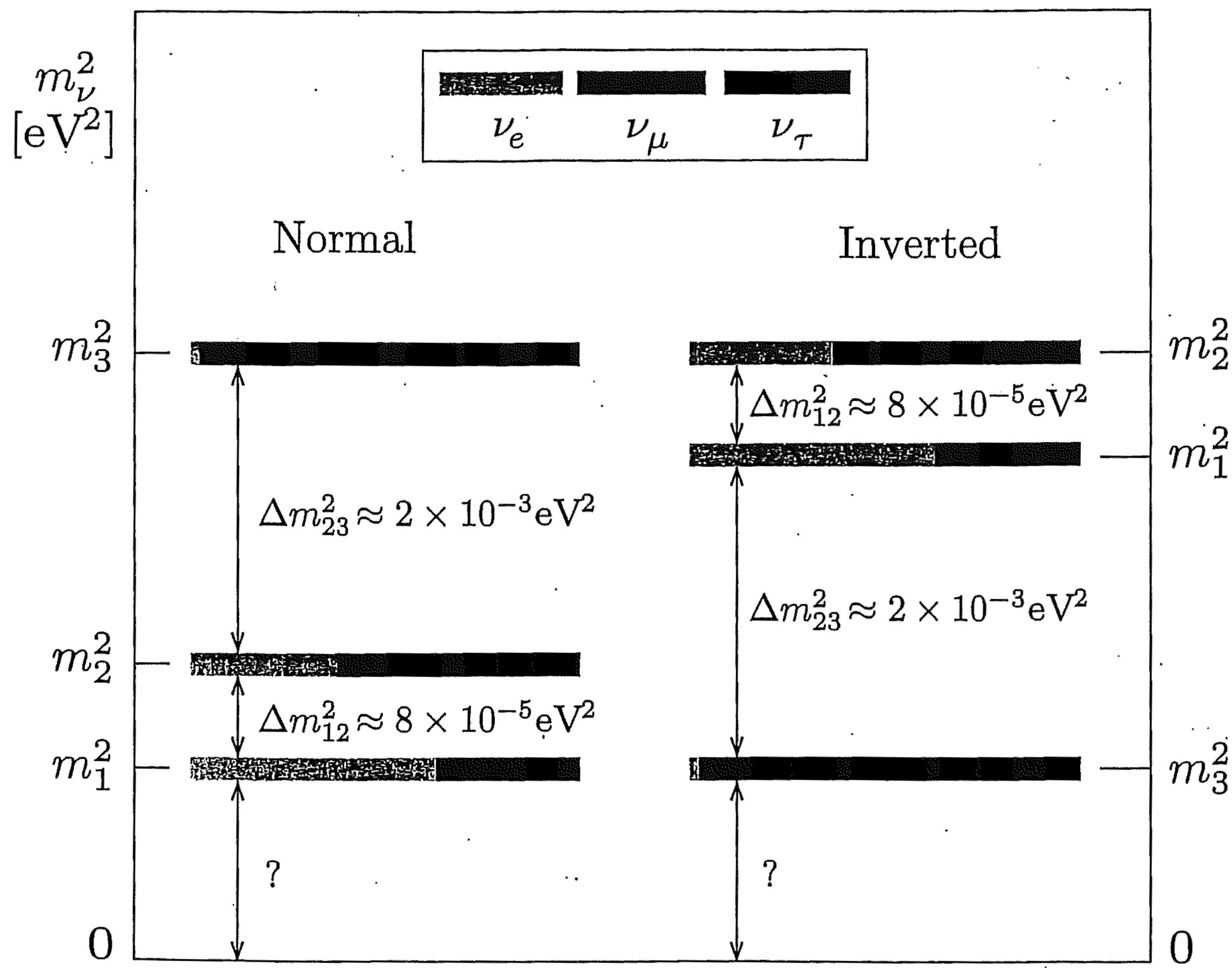


Fig. 18.7 The mass hierarchy problem, whereby the ordering and scale of neutrino masses remains unknown. The Δm_{ij}^2 values have been determined from experiments involving solar, reactor, and atmospheric neutrinos. The absolute vertical scale is constrained by the experimental limits on neutrino mass which are about 2 eV from tritium endpoint data and about 0.2 eV from interpretations of temperature fluctuations in the cosmic microwave background; see Section 21.3.

Actual masses:

$$m_i \lesssim 0.5\ eV$$

If assume normal hierarchy

and $m_1 \approx 0$

then $m_2 = \sqrt{7.5E-5} \approx 0.01\ eV$

$m_3 = \sqrt{2.5E-3} \approx 0.05\ eV$

(but no reason to assume $m_1 = 0$)

Summarize:

Assuming $\theta_{13} = 0$:

ν_e oscillate w/ $L_{solar} = (33\text{ km})\left(\frac{E_\nu}{\text{MeV}}\right)$

ν_μ oscillate w/ $L_{atm} = (1\text{ km})\left(\frac{E_\nu}{\text{MeV}}\right)$

solar neutrinos ν_e

ν_e from sun, (1 to 10 MeV) have $L \gg L_{solar}, L_{atm}$

so oscillations average out $\Rightarrow$ can measure θ_{12}

atmospheric neutrinos $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$

ν from atmosphere (~ GeV) have $L_{solar} \gg R_\oplus$

but $L_{atm} < R_\oplus$

so no ν_e oscillations observed ~~$\to$ measured~~

only ν_μ oscillations

accelerator neutrinos $\nu_\mu, \bar{\nu}_\mu$

ν_μ from accelerators used to measure L_{atm} & $\therefore$ Δm^2_{23} & θ_{23}

reactor antineutrinos $\bar{\nu}_e$

$\bar{\nu}_e$ from reactors (1 to 10 MeV)

KAMLAND ($L = 180$ km) observes $\bar{\nu}_e$ oscillation $\to$ measure L_{sol} & $\therefore$ ~~measure L_{sol}~~ $\to$ measure Δm^2_{12}

"Long baseline"

If $L \approx 1$ to 2 km "medium baseline" (China, Korea)

would expect to see ν_μ oscillations

but not $\bar{\nu}_e$ oscillations

However, if $\theta_{13} \neq 0$, then $\bar{\nu}_e$ can oscillate!

$$\nu_e = c_{12} c_{13} \nu_1 + s_{12} c_{13} \nu_2 + s_{13} e^{-i\delta} \nu_3$$

if $L \ll L_{solar}$, then observe no oscillations due to Δm^2_{21} i.e. Δm^2_{21} effectively vanishes

so ν_1 + ν_2 have same mass

Define $\nu_y = c_{12} \nu_1 + s_{12} \nu_2$

$$\nu_e = c_{13} \nu_y + s_{13} e^{-i\delta} \nu_3$$

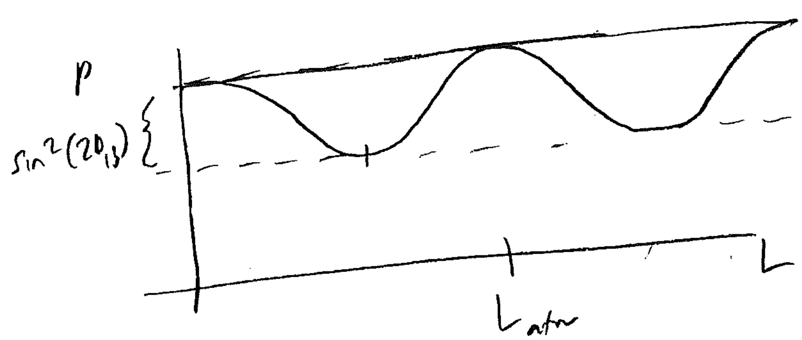
approximate mass eigenstate

$$\Delta m^2_{3y} \approx \Delta m^2_{32}$$

so $\bar{\nu}_e$ can also oscillate w/ $L_{atm} \approx (1\ \mathrm{km})\left(\frac{E_\nu}{\mathrm{MeV}}\right)$.

+ mixing angle θ_{13}.

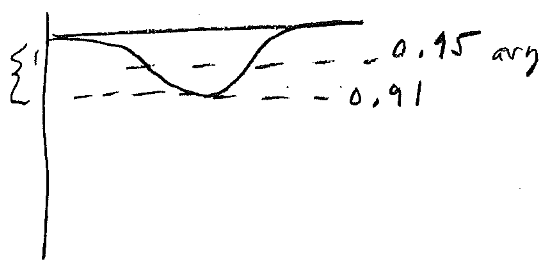
$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2(2\theta_{13}) \sin^2\left(\frac{\pi L}{L_{atm}}\right)$$



medium baseline expts

2012 { Double CHOOZ, Daya Bay, RENO } ← (cf Donnelly)

(see fig 3.1 Barger, fig 20.6 Peskin; An. et al)



$\sin^2(2\theta_{13}) \approx 0.09 \Rightarrow \theta_{13} \approx 9°$

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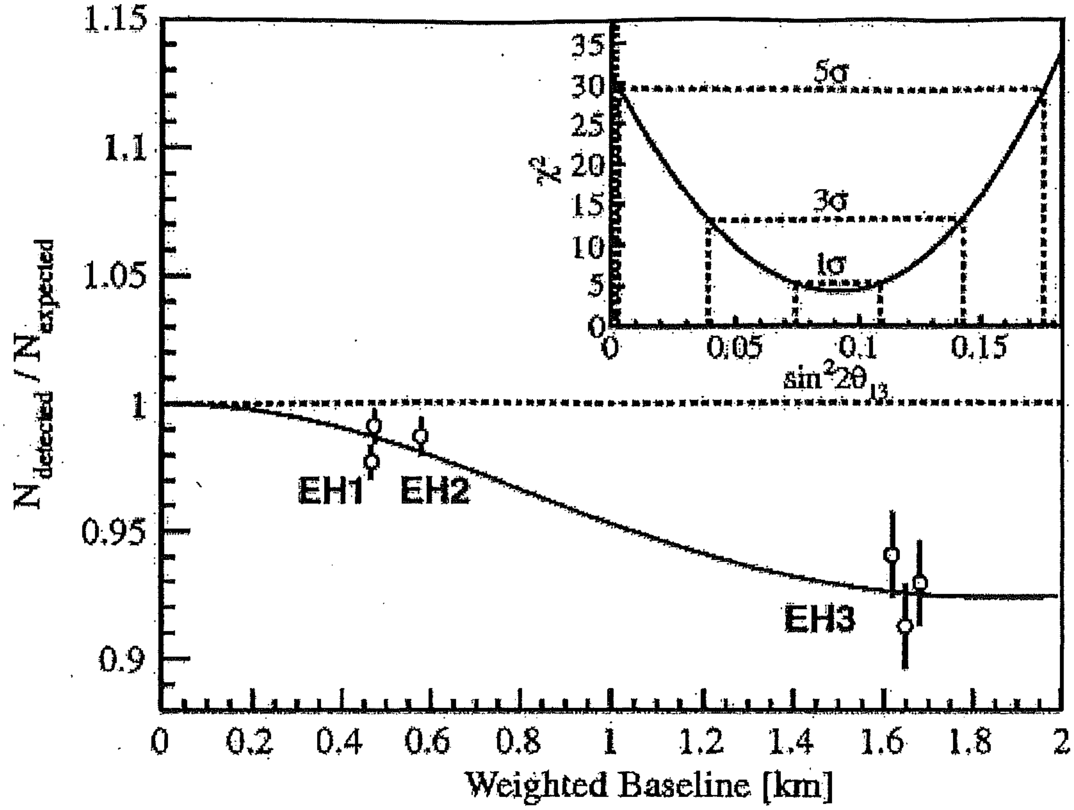
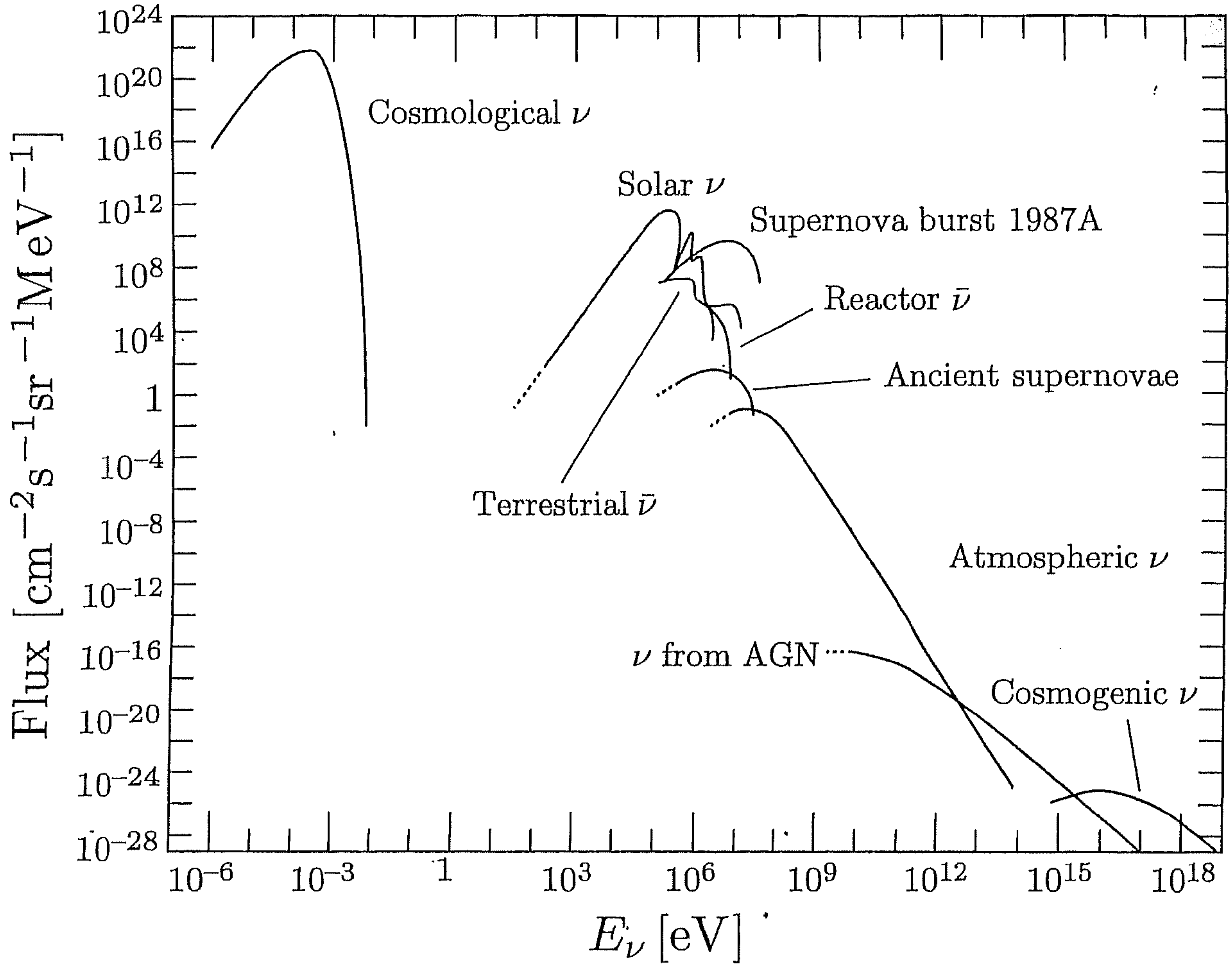
short baseline, for even larger mass difference

Los Alamos anomaly (LSND)



An et al PRL 108 (2012) 171803



Donnelly

Fig. 18.2 Flux of neutrinos by source versus neutrino energy [Kat12]. Note that the terrestrial antineutrinos arising from β-decay deep within the Earth were detected for the first time by the KamLAND experiment [Gan13] during the long-term shutdown of Japanese nuclear reactors following the March 2011 Fukushima nuclear accident.