Electroweak theory (GWS theory)

1968 { Sheldon Glashow
Steven Weinberg
Abdus Salam

Nonabelian gauge theory w/ gauge group

$SU(2)_W \times U(1)$

contains 4 force-carrying fields

$\Rightarrow$ 4 spin-1 particles ("vector bosons")

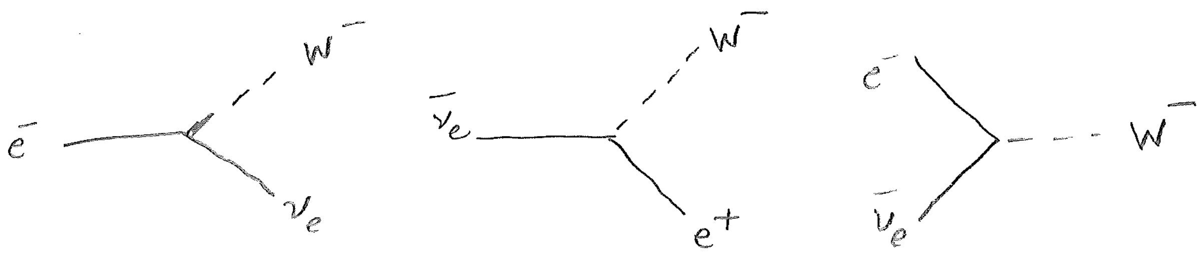
massless γ, plus 3 massive particles

antiparticles W^+, W^- $\quad m = 80.4$ GeV $\quad$ [N.B. 1st massive entry in PPB]

Z^0 $\quad m = 91.2$ GeV

These particles have mass because the gauge group symmetry is "spontaneously broken"

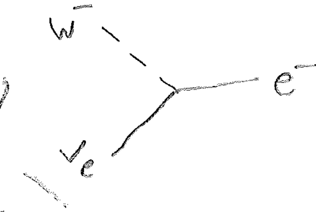
Leptonic weak vertices ["Theory of leptons"]



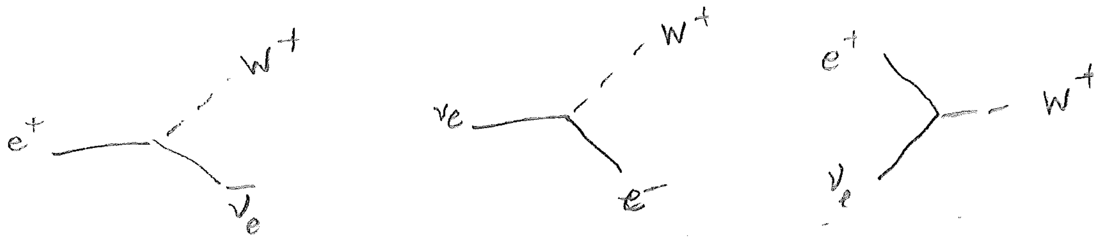
Unlike EM, each particle changes its identity when it emits W.

Charge and lepton number are conserved at each vertex.

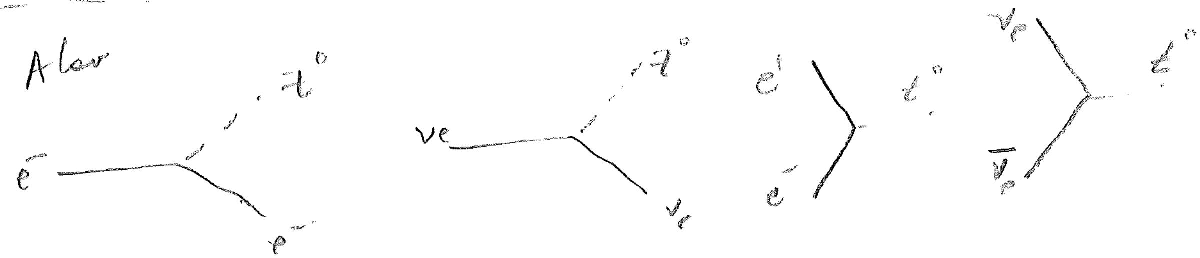
Also have time reversal of each of these, e.g.



Also, vertices w/ particles replaced by antiparticles



Also



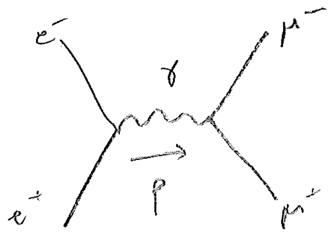
plus replace particles by antiparticles, and time reversal

[already saw  when discovered Z^0 peak in e^+e^-]

Finally, all of these vertices w/ e^-, ν_e

replaced by μ^-, ν_μ and τ^-, ν_τ

Reconsider $e^- e^+ \to \mu^- \mu^+$

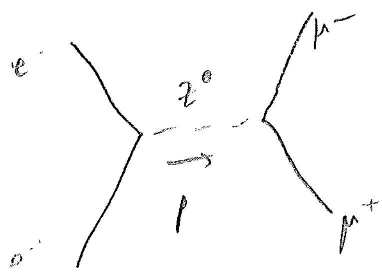


$$E_{cm} = E_{e^+} + E_{e^-}$$

$$p^2 = E_{cm}^2 - \cancel{\vec{p}_{cm}^{\,2}} \to 0$$

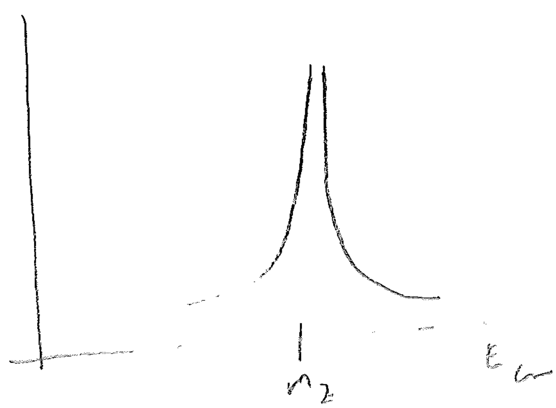
$$A \sim \frac{e^2}{p^2}(\text{stuff}) = \frac{e^2}{E_{cm}^2}(\text{stuff})$$

$$\sigma \sim |A|^2$$



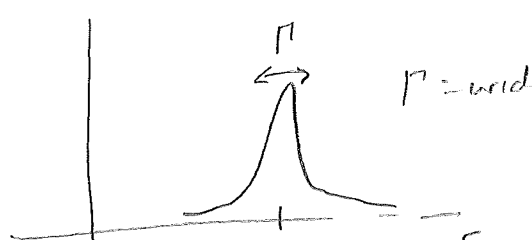
$$A \sim \frac{g^2}{p^2 - m_Z^2}(\text{stuff}) = \frac{g^2}{E_{cm}^2 - m_Z^2}(\text{stuff})$$

$$\sigma \sim |A|^2 \sim \frac{1}{(E_{cm}^2 - m_Z^2)^2}$$



[blows up at M_Z]
resonance

cf Griffiths 9-10



Γ = width of resonance

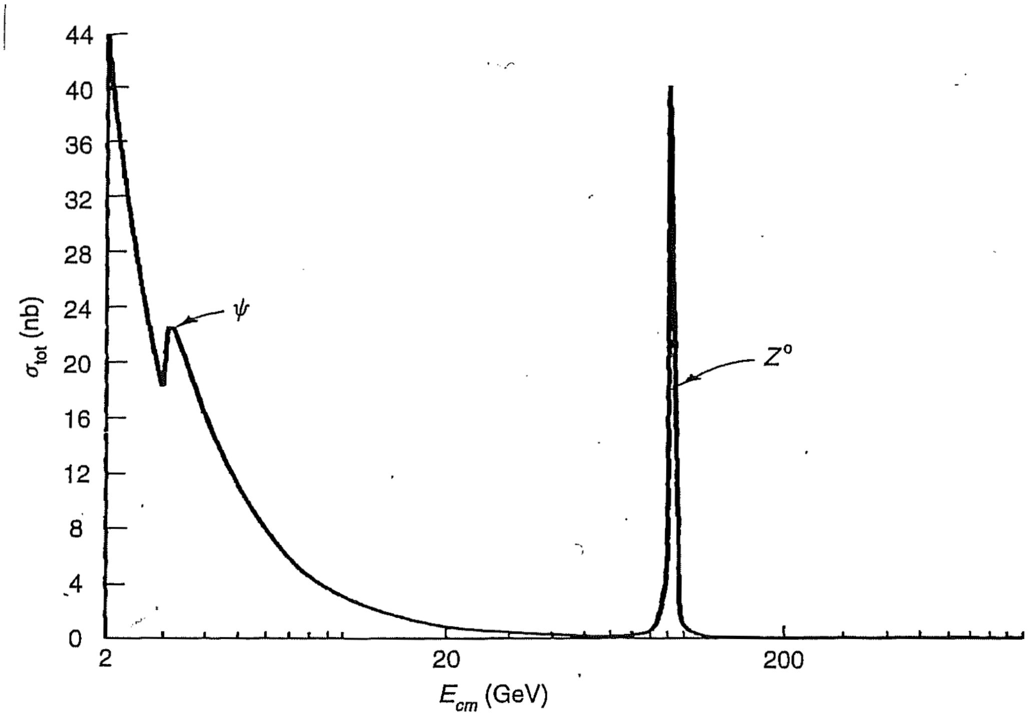


Fig. 9.10 Electron-positron scattering in the neighborhood of the Z^0 pole.

Griffiths → in "leptons"

Claim: Γ can be interpreted as imaginary part of mass.

[wow!]

Phys 2140: a stationary state (well defined energy) described by

$$\psi(x,t) = u(x)\, e^{-\frac{iEt}{\hbar}}$$

we'll only be interested in time dependence $e^{-\frac{iEt}{\hbar}}$

Particle at rest $E = m$ so $\psi = e^{-\frac{imt}{\hbar}}$

Prob density $|\psi|^2 =$ const in time, i.e. conserved

But if particle is unstable, expect

$$|\psi|^2 = e^{-\frac{t}{\tau}} = e^{-\frac{\Gamma t}{\hbar}}$$

~~where~~ define width $\Gamma = \frac{\hbar}{\tau}$

units of energy

$$|\psi| = e^{-\frac{\Gamma t}{2\hbar}}$$

$$\psi = e^{-\frac{\Gamma t}{2\hbar}}\, e^{-\frac{imt}{\hbar}}$$

$$= e^{-i\left(m - \frac{i\Gamma}{2}\right)\frac{t}{\hbar}}$$

$$= e^{-\frac{iMt}{\hbar}}$$

where $M = m - \frac{i\Gamma}{2}$

modify propagator w/ complex mass

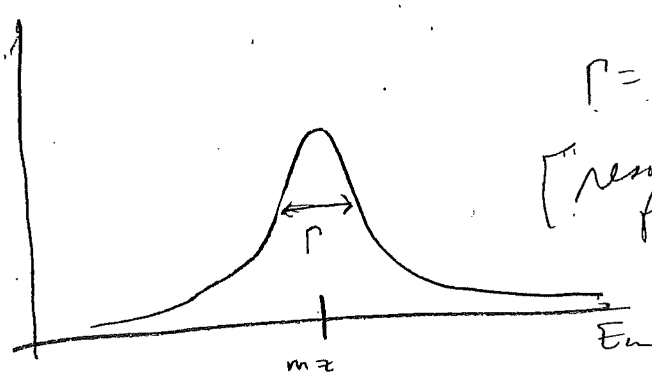
$$A \sim \frac{1}{E_{cm}^2 - m^2} \sim \frac{1}{E_{cm}^2 - \left(m_Z - \frac{i\Gamma}{2}\right)^2}$$

$$\sim \frac{1}{\left(E_{cm} + m_Z - \frac{i\Gamma}{2}\right)\left(E_{cm} - m_Z + \frac{i\Gamma}{2}\right)}$$

For $E_{cm} \approx m_Z$, 1st term $= 2m_Z - \frac{i\Gamma}{2} \approx 2m_Z$

$$A \sim \frac{1}{2m_Z\left(E_{cm} - m_Z + \frac{i\Gamma}{2}\right)}$$

$$|A|^2 \sim \frac{1}{(2m_Z)^2\left[\left(E_{cm} - m_Z\right)^2 + \frac{1}{4}\Gamma^2\right]}$$



$\Gamma =$ FWHM

[resonance curve for damped driven hmc osc] Phys 3000

Breit-Wigner curve

[show some pictures of this; eg Higgs discovery?]

$$M = m - i\frac{\Gamma}{2}$$

For a fairly stable particle, $\Gamma \ll m$

Muon: $\tau \cdot 2.2\times10^{-6}$ sec

$$\Gamma = \frac{\hbar}{\tau} = \frac{6.6E-22 \text{ MeV}\cdot s}{2.2E-6 s} = 3E-16 \text{ MeV}$$

vs. $m = 105$ MeV [1 part in 10^{18}]

But for short lived particle such as Z

Z: $\tau \sim 2.6\times10^{-25}$ sec

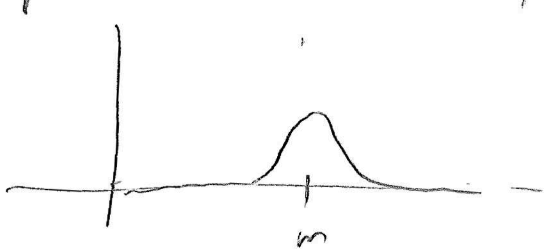
$$\Gamma = \frac{6.6E-25 \text{ GeV}\cdot s}{2.6E-25 s} \cdot 2.5 \text{ GeV}$$

vs $m = 90$ MeV about 3%

In fact, for some strongly decaying particles, Γ comparable to m

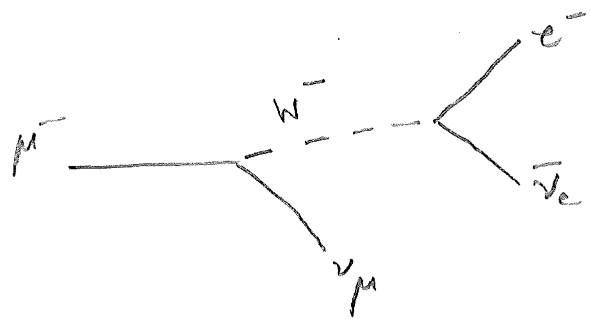
ρ: $\Gamma = 150$ MeV
$m = 770$ MeV about 20%

Such particles are sometimes referred to as resonances



[Higgs]

μ^- decay ($\mu^- \to e^- \bar{\nu}_e \nu_\mu$)



Each vertex involving W^- contributes a factor of the weak coupling constant g
("weak charge" analogous to electric charge e)

The internal W^- line contributes the propagator

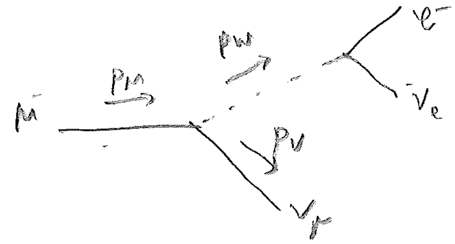
$$\frac{1}{p^2 - m_W^2}$$

[as we saw for γ]

So amplitude for μ^- decay is

$$A = \frac{g^2}{p_W^2 - m_W^2} \left(\begin{matrix}\text{spin} \\ \text{stuff}\end{matrix} \right)$$

[more complicated than QED
involves parity violation
"V−A"]



How determine p_W^2 ? Momentum conservation

Assume μ^- at rest decays to ν_μ in $+z$ direction

$$p_\mu = (m_\mu, 0, 0, 0)$$

$$p_\nu = (E, 0, 0, E) \qquad \text{where } E < \tfrac{1}{2} m_\mu$$

$$\Rightarrow p_W = p_\mu - p_\nu = (m_\mu - E, 0, 0, -E)$$

$$p_W^2 = (m_\mu - E)^2 - E^2 = m_\mu (m_\mu - 2E)$$

$$p_W^2 \le m_\mu^2$$

But $m_\mu^2 \ll M_W^2$ (100 MeV vs 80 GeV)

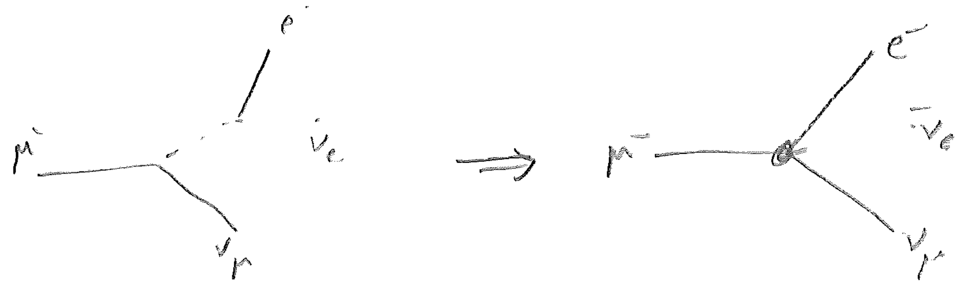
so W^- is far off-shell (virtual)

$$A = \frac{g^2}{p_W^2 - M_W^2} \begin{pmatrix} \text{spin} \\ \text{stuff} \end{pmatrix} \approx \frac{g^2}{-M_W^2} \begin{pmatrix} \text{spin} \\ \text{stuff} \end{pmatrix}$$

[Problem: work out max + min kinetic energy of e^-]

Since the 2 vertices plus W propagator reduce at low energies to a single constant $\frac{g^2}{M_W^2} = G_F$

we can replace them w/ a "four Fermi vertex"



$$A : \; G_F \left(\begin{matrix} \text{spin} \\ \text{stuff} \end{matrix} \right)$$

1932 Fermi theory of weak interactions

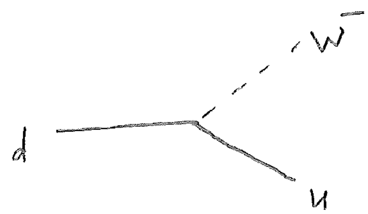
Fermi constant $G_F = \underbrace{1.1664 \times 10^{-5}}\,(\text{GeV})^{-2}$ $\left[\equiv \frac{G_1}{(\hbar c)^3} \right]$

Smallness of this # reflects weakness of weak int'n [cf PPB]

small not because g small but because M_W is big (compared to μ, p)
g about same as e

$$\left[\frac{g^2}{4\pi} = \frac{G_F M_W^2}{4\pi} \approx \frac{1}{167} \right]$$

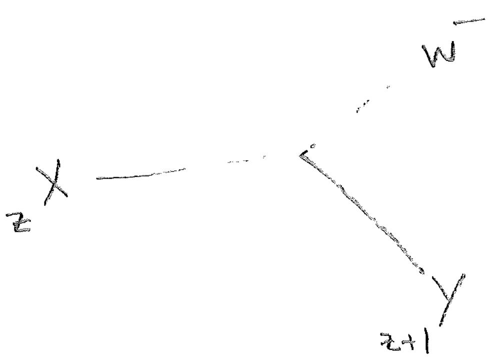
We'll return to μ decay later, but now consider nuclear β decay, Weak interaction also affects quarks, e.g.



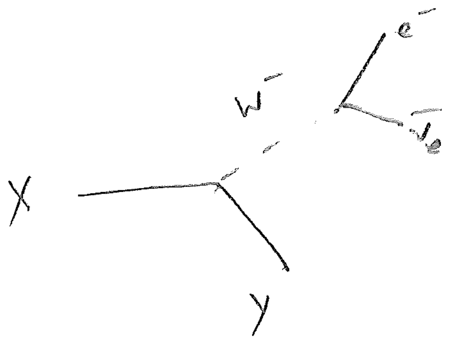
[charge conservation: $-\frac{1}{3} = -1 + \frac{2}{3}$]

but there are some complications (intergenerational mixing) and we haven't introduced quarks yet, so consider instead ($n = udd$, $p = uud$)

and for nuclei



Nuclear β-decay: ${}_{Z}X \longrightarrow {}_{Z+1}Y\, e^- \bar{\nu}_e$



$$A = \frac{g^2}{p_W^2 - m_W^2}\begin{pmatrix}\text{spin and} \\ \text{nuclear} \\ \text{stuff}\end{pmatrix}$$

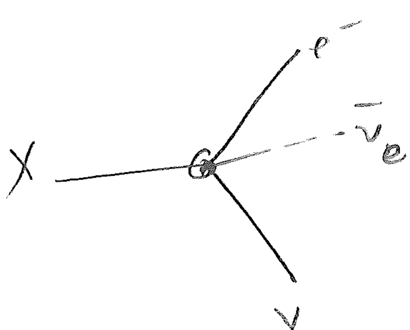
$$p_W = p_X - p_Y = (m_X - E_Y, 0, 0, p_Y)$$

but Y typically nonrelativistic, so $E_Y \approx m_Y$

$$p_W^2 \approx (m_X - m_Y)^2 \ll m_W^2$$

because $m_X - m_Y \approx$ few 10's of MeV

As before, simplify to



$$A = G_F\begin{pmatrix}\text{spin and} \\ \text{nuclear} \\ \text{stuff}\end{pmatrix}$$

Decay of particle X_0 into n_f particles

Energy-momentum conserved

$$p_0 = \sum_{j=1}^{n_f} p_j$$

$$\Delta p = \sum_{j=1}^{n_f} p_j - p_0$$

Decay rate $\quad R = \sum_{\substack{\text{final}\\\text{state}}} |A|^2$

" Golden rule for decay "

$$\boxed{R = \frac{1}{(2m_0)\hbar} \int (LIPS)_{n_f} |A|^2}$$

$$(LIPS)_{n_f} = \prod_{j=1}^{n_f} \frac{d^3 p_j}{(2\pi)^3 (2E_j)} (2\pi)^4 \delta^{(4)}(\Delta p)$$

[$\frac{1}{(2E_0)}$ for initial particle, but at rest become $\frac{1}{(2m_0)}$]

[restore $\frac{1}{\hbar}$ to make rate have dimensions of $\frac{1}{(\text{time})}$]

(Not for class)

Decay $X_0 \to 1 + 2 + \dots + n_f$

QFT → Decay rate $R = \frac{1}{2m_0\hbar} \int (LIPS)_{n_f} |A|^2$

$$(LIPS)_{n_f} = \prod_{j=1}^{n_f} \frac{d^3 p_j}{(2\pi)^3 (2E_j)} (2\pi)^4 \delta^{(4)}(\Delta p^\mu)$$

Dimensional analysis:

$(LIPS)_{n_f}$ has dimension $E^{2n_f} E^{-4}$

$R\, m_0 \hbar$ has dimension $t^{-1} \cdot E \cdot (E \cdot t) = E^2$

$\Rightarrow |A|^2$ has dimension $E^{-2n_f + 6}$

A has dimen. $E^{3 - n_f}$

[in general E^{4-n}, n = # of initial + final states]

Nuclear β-decay: $X \longrightarrow e^- \bar{\nu}_e Y$

$$R = \frac{1}{(2m_X)\hbar} \int (LIPS)_3 \, |A|^2$$

$$= \frac{1}{(2m_X)\hbar} \int \frac{d^3p_e}{(2\pi)^3(2E_e)} \frac{d^3p_\nu}{(2\pi)^3(2E_\nu)} \frac{d^3p_Y}{(2\pi)^3(E_Y)} (2\pi)^4 \delta(p_e + p_\nu + p_Y - p_X) |A|^2$$

For 3-particle decay, $|A|$ is dimensionless

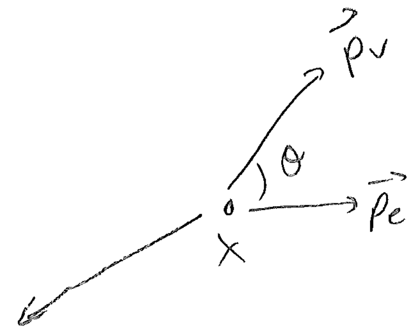
$$A = G_F \begin{pmatrix} \text{spin and nuclear} \\ \text{stuff} \end{pmatrix}$$

$G_F = 1.1664 \times 10^{-2}$ GeV^{-2} has units (energy)$^{-2}$

Let $\begin{pmatrix} \text{spin and nuclear} \\ \text{stuff} \end{pmatrix} = \sqrt{(2E_X)(2E_e)(2E_\nu)(2E_Y)} \; M$

"nuclear matrix element"
(dimensionless)

$$\boxed{R = \frac{G_F^2}{(2\pi)^5 \hbar} \int d^3p_e \, d^3p_\nu \, d^3p_Y \, \delta^{(3)}(\vec{p}_e + \vec{p}_\nu + \vec{p}_Y) \delta(E_e + E_\nu + E_Y - m_X) |M|^2}$$



$\vec{p}_Y = -\vec{p}_e - \vec{p}_\nu$

Do $d^3 p_Y$ using momentum δ-function

Energy δ-fn: $\delta(E_e + E_\nu + E_Y - m_X)$

$$= \delta(m_e + T_e + E_\nu + m_Y + T_Y - m_X)$$

<u>Define</u> $Q = m_X - m_Y - m_e$ (N.B we do <u>not</u> include m_ν)

$$\delta(T_e + E_\nu + T_Y - Q)$$

typically

$$\left[\; T_Y = \frac{(\vec{p}_e + \vec{p}_\nu)^2}{2 m_Y} \; , \ll T_e + E_\nu \le Q \ll m_Y \; \right]$$

which allows non rel. formula for T_Y

so ignore T_Y in this eqn:

$$\Rightarrow \boxed{R = \frac{G_F^2}{\hbar (2\pi)^5} \int d^3 p_e \, d^3 p_\nu \; \delta(T_e + E_\nu - Q) \, |M|^2}$$

$$d^3p_e\, d^3p_\nu = p_e^2\, dp_e\, d\Omega_e \;\; p_\nu^2\, dp_\nu\, d\Omega_\nu$$

$p_e = |\vec{p}_e|$
$p_\nu = |\vec{p}_\nu|$

Assume M is relatively insensitive to θ ($\measuredangle$ between $\vec{p}_e$ & $\vec{p}_\nu$)

[reasonably good assumption]

Integrate over angles $\int d\Omega_e = \int d\Omega_\nu = 4\pi$

$$\frac{(4\pi)^2}{(2\pi)^5} = \frac{1}{2\pi^3}$$

$$\Rightarrow R = \frac{G_F^2}{2\pi^3 \hbar} \int p_e^2\, dp_e \int p_\nu^2\, dp_\nu\; \delta(T_e + E_\nu - Q)\, |M|^2$$

[Segre 9-5.14
p. 352]

Next, assume the ν is strictly massless.

$$\Rightarrow E_\nu = p_\nu$$
$$dE_\nu = dp_\nu$$

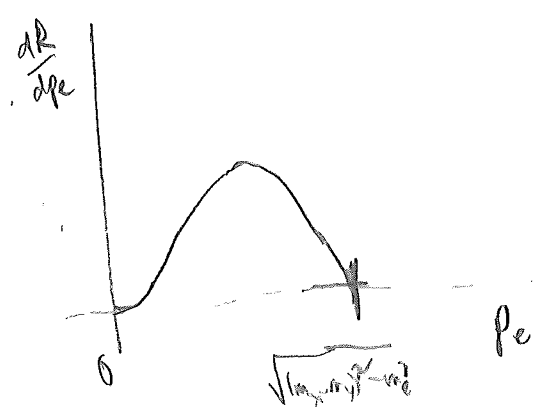
$$R = \frac{G_F^2}{2\pi^3 \hbar} \int p_e^2 \, dp_e \underbrace{\int E_\nu^2 \, dE_\nu \, \delta(T_e + E_\nu - Q) \, |M|^2}_{(Q-T_e)^2 |M|^2}$$

$(Q-T_e)^2 |M|^2$ evaluated at $E_\nu = Q - T_e$

$$= \int_0^{\sqrt{(m_x - m_y)^2 - m_e^2}} dp_e \underbrace{\frac{G_F^2}{2\pi^3 \hbar} p_e^2 (Q - T_e)^2 |M|^2}_{\text{call this } \frac{dR}{dp_e}}$$

from the

$dp_e \frac{dR}{dp_e}$ is the decay rate to electrons w/ momenta in the range p_e to $p_e + dp_e$ and can be experimentally measured.



one measures $\frac{dR}{dp_e}$ and then plots

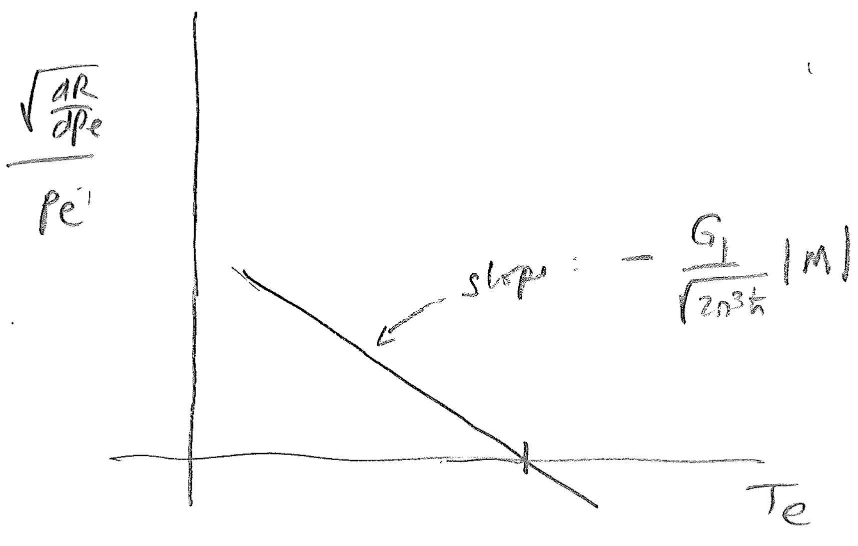
$$\frac{\sqrt{\frac{dR}{dp_e}}}{p_e} = \frac{G_F}{\sqrt{2\pi^3 \hbar}} (Q - T_e) \, |M|$$

as a function of T_e

(Kurie plot)

If $|M|$ is insensitive to T_e (as well as to $|\vec{q}|$)

this plot will be linear



(Indeed this seems to be the case)

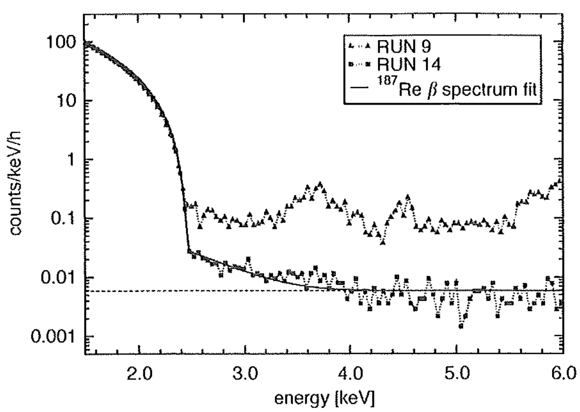


Fig. 4. Background in the 2000 and 2003 measurements.

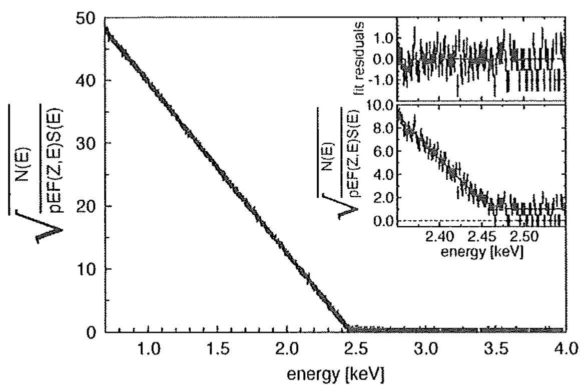


Fig. 5. Sum Kurie plot, where p and E are, respectively, the β momentum and kinetic energy, $F(Z,E)$ is the Coulomb factor and $S(E)$ is the shape factor.

and the flat residual background mainly caused by cosmic rays and environmental radioactivity.

The final Kurie plot resulting from the sum of the 8 detectors is shown in Fig. 5. It corresponds to $\sim 6.2 \times 10^6$ ^{187}Re-decays above the common energy threshold of 700 eV. The spectrum was fit in the energy interval 0.9–4 keV (see Section 2) and the χ^2/DOF of the fit is 0.905. The preliminary measured value for the end-point is $2465.3 \pm 0.5(stat.) \pm 1.6(syst.)$ eV. In the chosen fitting interval, the systematic error is determined by the uncertainties in the energy resolution, in the detector response function, and in the shape of the background below the β spectrum. By fitting the distribution of the time intervals between two successive β decays (see Section 2), we could precisely determine the ^{187}Re half-life, which was

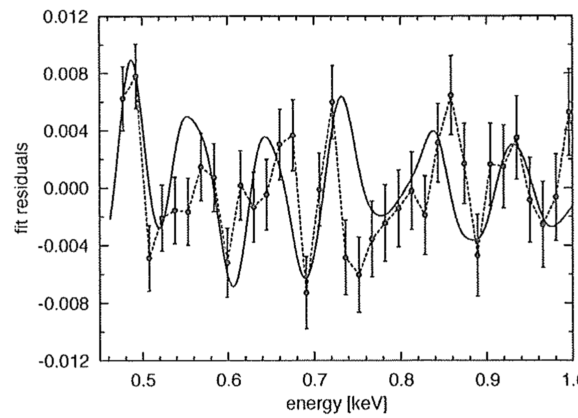


Fig. 6. Experimental BEFS superimposed to the theoretical prediction for $\Delta E_{\mathrm{FWHM}} = 30$ eV.

found to be $[43.2 \pm 0.2(stat.) \pm 0.1(syst.)] \times 10^9$ years. Here the statistical error is due to the uncertainties in the measurement of the mass of the absorbers and the systematic error is due to the uncertainties in the pile-up discrimination. The values for the end-point energy and for the half-life are the most precise existing in the literature. The latter has considerable impact in geochronology.

The squared electron antineutrino mass $m^2_{\bar{\nu}_e}$ is $-112 \pm 207(stat.) \pm 90(syst.)$ eV^2, where the systematic error has the same origin as for the end-point energy quoted above. The 90% C.L. upper limit to the electron antineutrino mass is 15 eV. This result is in agreement with the expected sensitivity deduced from a Monte Carlo simulation of an experiment with the same statistical significance as our data set [5].

The fit residuals in the energy interval between 470 eV (the common energy threshold for 7 of the 8 detectors) and 1.3 keV show a clear evidence of an oscillatory modulation of the data due to the Beta Environmental Fine Structure (BEFS) in $AgReO_4$ (Fig. 6). This important effect was first observed for metallic rhenium [4]. A quantitative analysis in terms of $AgReO_4$ lattice structure is presently on the way.

5. Conclusions

The limit on m_ν reported in this paper, even if not yet competitive to the limit of about 2 eV

If the neutrino has mass, then expect [HW]

$$\frac{\sqrt{\frac{dR}{dp_e}}}{p_e} \sim (Q-T_e)\left[1-\left(\frac{m_\nu}{Q-T_e}\right)^2\right]^{\frac{1}{4}} |M|$$

vanishes if $T_e = Q - m_\nu$

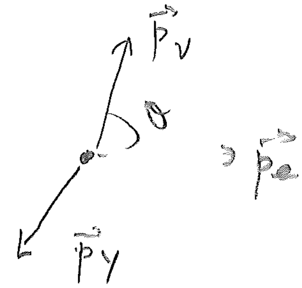
only important when $Q - T_e$ is small

~~(see plots)~~ not necessary

$m_\nu < 0.3$ eV

KATRIN 2025 $m_\nu < 0.45$ eV

$$X \longrightarrow e^- \bar{\nu}_e Y$$



Assumptions so far

1. $m_Y \gg m_e$ so can neglect T_Y

2. $m_{\bar{\nu}} = 0$

3. $|M|$ independent of θ

4. $|M|$ indep of energy of electron [borne out by Kurie]

$$\Rightarrow R = \frac{G_F^2 |M|^2}{2\pi^3 \hbar} \int_0^{(p_e)_{max}} dp_e \; p_e^2 \, (Q - T_e)^2$$

$Q = m_X - m_Y - m_e$ = kinetic energy released

change variable from p_e to E_e

$$E_e^2 = |\vec{p}_e|^2 + m_e^2$$

$$\Rightarrow E_e \, dE_e = |\vec{p}_e| \, dp_e$$

$$T_e = E_e - m_e$$

$$R = \frac{G_F^2 |M|^2}{2\pi^3 \hbar} \int_{m_e}^{m_e + Q} dE_e \; E_e \sqrt{E_e^2 - m_e^2} \, (Q + m_e - E_e)^2$$

If $Q \gg m_e$, we make further approximation

(5) $m_e = 0$ i.e. e^- + $\bar{\nu}$ both ultrarelativistic

$$R = \frac{G_F^2 |M|^2}{2\pi^3 \hbar} \int_0^Q dE_e \; E_e^2 (Q - E_e)^2$$

Let $x = \frac{E_e}{Q}$

$$= \frac{G_F^2 |M|^2}{2\pi^3 \hbar} Q^5 \int_0^1 dx \; \underbrace{x^2 (1-x)^2}_{x^2 - 2x^3 + x^4}$$

$$\left(\frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5} \right) \Big|_0^1 = \frac{1}{3} - \frac{1}{2} + \frac{1}{5} = \frac{1}{30}$$

$$= \frac{1}{60\pi^3} \frac{G_F^2}{\hbar} \cdot Q^5 |M|^5$$

Sargent's rule: $R \propto Q^5$

Larger Q ⇒ more phase space available

⇒ faster decay

[can use exptal data for R to measure $|M|^2$ for various nuclear transitions]

$$\left[\begin{aligned} G_F &: 1.166379\text{E}-11 \text{ MeV}^{-2} \\ \hbar &= 6.58212\text{E}-22 \text{ MeV} \cdot \text{s} \\ \frac{G_F^2}{\hbar} &= 0.206687 \text{ MeV}^{-5} \cdot \text{s}^{-1} \end{aligned} \right]$$

Neutron decay $\quad n \to p e^- \bar{\nu}_e$

experiment: $\tau = 878\ s$

$\Rightarrow R = \frac{1}{\tau} = 1.14 \times 10^{-3}\ s^{-1}$

$Q = m_n - m_p - m_e \doteq 0.782\ MeV$

Can't apply Sargent rule because $m_e \not\ll Q$!

Go back to previous expression

$$R = \frac{G_F^2 |M|^2}{2\pi^3 \hbar} \int_{m_e}^{Q+m_e} dE_e\ E_e \sqrt{E_e^2 - m_e^2}\ (Q + m_e - E_e)^2$$

Let $E_e = Q x$

$m_e = Q \mu$

$$R = \underbrace{\frac{G_F^2}{2\pi^3 \hbar}}_{3.333\times 10^{-3}\ s^{-1}\ MeV^{-5}} |M|^2 Q^5 \underbrace{\int_{\mu}^{1+\mu} dx\ \ x \sqrt{x^2 - \mu^2}\ (1 + \mu - x)^2}_{I(\mu)}$$

$I(0) = \frac{1}{30} = 0.033 \quad$ (Sargent rule)

neutron decay: $\mu = \frac{m_e}{Q} = 0.653, \quad I(\mu) = 0.1946 \quad$ [about 5.84 $\uparrow$ from $m_e \to 0$ limit, which is $I(0) = 1/30$]

$$R = \underbrace{(3.333E-3\ s^{-1})(0.782)^5 (0.1946)}_{1.90\ E-4\ s^{-1}} |M|^2 \quad \Rightarrow \quad |M|^2 = 6.0$$

$|M| = 2.45$

[for n = fund spin $\frac{1}{2}$ particle, $|M| = ?$]

```
In[ ]:= me = 0.511;
        mn = 939.565;
        mp = 938.272;
        mpiplus = 139.570;
        mpinought = 134.977;

In[ ]:  f[x_, mu_] :=  x Sqrt[x^2 - mu^2] (1 + mu - x)^2

In[ ]:= intf[mu_] := Integrate[f[x, mu], {x, mu, 1 + mu}]

In[ ]:= intf[0]

         1
Out[ ]= ——
        30

In[ ]:= Q = mn - mp - me;

In[ ]   intf[me / Q]

Out[ ]= 0.194601

In[ ].

In[ ]:= Q = mpiplus - mpinought - me

Out[ ]= 4.082

In[ ]:= intf[me / Q]

Out[ ]= 0.0565786

In[ ]:=
```

$$\pi^+ \rightarrow \pi^0 e^+ \nu_e$$

$\tau = 2.6033E-8$ s

$R = 3.8413E7 \ s^{-1}$

$BR = 1.036E-8$

partial rate $= 0.398 \ s^{-1}$ [cf Perkins, Particle Astrophysics, prob 1.6 (c)]

$Q = m_{\pi^+} - m_{\pi^0} - m_e = 4.082$

$$R = \frac{G_F^2}{7\pi^3\hbar} |M|^2 Q^5 I\left(\frac{m_e}{Q}\right)$$

1.7 times larger than $m_e \to 0$ limit which is 1/30

$$= \underbrace{(3.333E-3 \ s^{-1})(4.082)^5 (0.05658)}_{0.214 \ s^{-1}} |M|^2$$

mathematica

$\Rightarrow |M|^2 = 1.86$, $|M| = 1.36$

Alternatively, neutron decay rate

$$R = \frac{G_F^2}{2\pi^3\hbar} |M|^2 \int_{m_e}^{m_e+Q} dE_e \, E_e \sqrt{E_e^2 - m_e^2} \, (Q + m_e - E_e)^2$$

let $\begin{cases} y = \dfrac{E_e}{m_e} \\ q = \dfrac{Q}{m_e} \end{cases}$

$$R = \frac{G_F^2}{2\pi^3\hbar} |M|^2 m_e^5 \underbrace{\int_1^{1+q} dy \, y \sqrt{y^2-1} \, (q+1-y)^2}$$

For $q = \frac{Q}{m_e} = 1.530$, integral gives 1.636 [mathematica]

[cf Dodelson (3.62) exercise 3-3]

$$\underbrace{R}_{1.14\times10^{-3}\,s^{-1}} = \underbrace{(3.333E-3\ s^{-1})\ (0.511)^5\ (1.636)}_{1.900E-4\ s^{-1}} |M|^2 \Rightarrow |M|^2 = 6.00$$

$|M| = 2.45$

FermiIntegral.nb

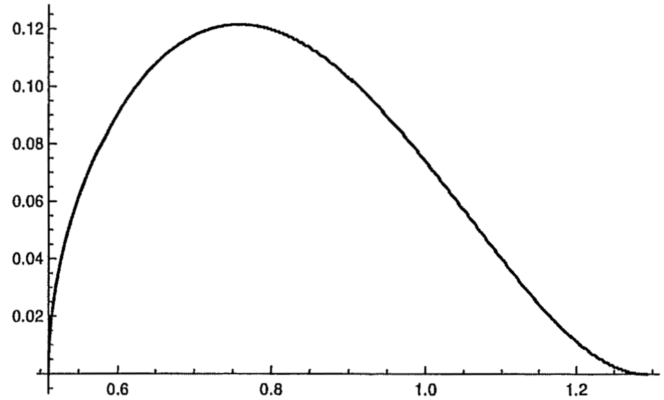
```
In[30]:= P[me_, del_] := Plot[e Sqrt[e^2 - me^2] (del - e)^2, {e, me, del}]

In[31]:= F[me_, del_] := Integrate[e Sqrt[e^2 - me^2] (del - e)^2, {e, me, del}]

In[32]:= me = 0.511; del = 939.565 - 938.272;

In[33]:= P[me, del]
```

Out[33]=

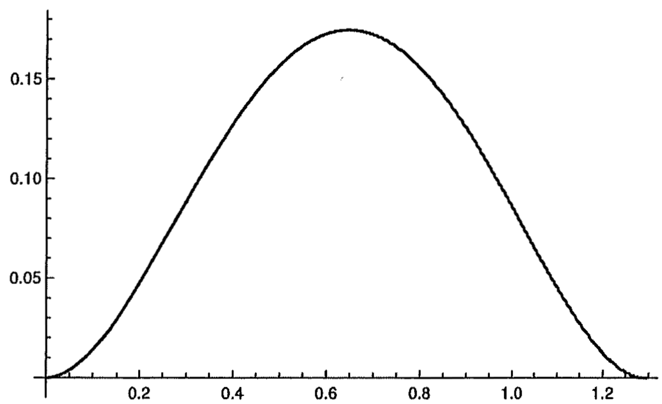



```
In[34]:= F[.511, del]
Out[34]= 0.0569086
```

$\longrightarrow = 1.636\ m_e^5$

```
In[35]:= P[0, del]
```

Out[35]=




```
In[36]:= F[0, del]
Out[36]= 0.120468
```

$= \frac{1}{30}(\text{del})^5$

neutron decay

```
In[10]:= a = (939.565 - 938.272) / (.511);
         (2 a^4 - 9 a^2 - 8) Sqrt[a^2 - 1] / 15 + a Log[a + Sqrt[a^2 - 1]]
Out[11]= 6.53332
```

We find

$$R = \frac{1}{2\pi^3} \frac{G_F^2}{\hbar} m_e^5 \,(1.636)\, |M|^2$$

Griffiths (1e) (10.63) and (10.68)

(2e) (9.60) and (9.65)

correction factor

$$R = \frac{1}{4\pi^3 \hbar} (2 G_F^2)\, m_e^5\, (6.533)\, \underbrace{\tfrac{1}{4}(c_V^2 + 3c_A^2)}_{1.46}\, \underbrace{\cos^2\theta_c}_{0.948} \qquad (\theta_c = 13.1^\circ)$$

$$= \frac{1}{2\pi^3} \frac{G_F^2}{\hbar} m_e^5 \underbrace{\left(\frac{6.533}{4}\right)}_{1.633} \underbrace{(c_V^2 + 3c_A^2)\cos^2\theta_c}_{|M|^2 = (5.84)(0.948)} \qquad \begin{pmatrix} c_V = 1 \\ c_A = 1.27 \end{pmatrix}$$

$$\Rightarrow |M| = 2.353$$

Povh et al. (3.62) [he neglects $\cos^2\theta_c$]

$$\frac{1}{\tau} = \frac{1}{2\pi^3} \frac{G_F^2}{\hbar} m_e^5\, (1.636)\, \underbrace{(1 + 3g_A^2)}_{|M|^2}$$

it). Track the $e^{\pm}$ density through annihilation assuming $n_{e^\pm} = n^{(0)}_{e^\pm}$. This ity holds during the BBN epoch because electromagnetic interactions (e.g., $e^- \leftrightarrow \gamma + \gamma$) are so strong. When does the density fall to 1% of the photon y density? If $\eta_b \simeq 6\times 10^{-10}$, at what temperature do you expect n_{e^-} to depart $n^{(0)}_{e^-}$?

cise 3. Compute the rate for neutron-to-proton conversion, λ_{np}. Show that it ıal to Eq. (3.29). There are two processes which contribute to λ_{np}: $n + \nu_e \rightarrow$ $p + e^-$ and $n+e^+ \rightarrow p+\bar{\nu}_e$. Assume that all particles can be described by Boltzmann tics and neglect the mass of the electron. With these approximations the two are identical. a bit drastic

ſse Eq. (3.8) to write down the rate for $n+\nu_e \rightarrow p+e^-$. Perform the integrals ıeavy particle momenta to get

$$\lambda_{np} = n^{(0)}_{\nu_e}\langle\sigma v\rangle = \frac{\pi}{4m^2}\int\frac{d^3p_\nu}{(2\pi)^3 2p_\nu}\, e^{-p_\nu/T}$$

$$\times \int \frac{d^3p_e}{(2\pi)^3 2p_e}\delta(\mathcal{Q}+p_\nu - p_e)\,|\mathcal{M}|^2 . \qquad (3.61)$$

The amplitude squared is equal to $|\mathcal{M}|^2 = 32G_F^2(1+3g_A^2)m_p^2 p_\nu p_e$, where g_A is xial-vector coupling of the nucleon. The present best measurement of g_A is via eutron lifetime, $\tau_n = \lambda_0 G_F^2(1+3g_A^2)m_e^5/(2\pi^3)$, where the phase space integral

$$\lambda_0 \equiv \int_1^{\mathcal{Q}/m_e} dx x(x-\mathcal{Q}/m_e)^2(x^2-1)^{1/2} = 1.636. \qquad (3.62)$$

out the integrals in Eq. (3.61) to get the rate, λ_{np} in terms of τ_n. Don't forget ıltiply by 2 for the two different reactions.

cise 4. Solve the rate equation (3.27) numerically to determine the neutron on as a function of temperature. Ignore decays. There are (at least) two ways rf n this computation. The first is to treat it as a simple ordinary differential ion and solve numerically. The second is to proceed analytically and reduce roblem to an evaluation of a single numerical integral. This second method, ı I'll lead you through here, is based on a numerical coincidence noted by stein, Brown, and Feinberg (1988).

Frauenfelder + Henley

p. 279, table 11.1 $\quad n \to p \quad \underline{ft_{\frac{1}{2}}} = \underline{1100 \text{ sec}}$

p. 30: eq 11.74: $|GM|^2 = |\langle n e | H_w | p \bar{\nu} \rangle|^2$

p. 278 eq 11.11

$$= \frac{2\pi^3}{f\tau} \frac{\hbar^7}{m_e^5 c^4}$$

$$= \underbrace{\frac{2\pi^3 \hbar^7 \ln 2}{m_e^5 c^4}} \frac{1}{ft_{\frac{1}{2}}}$$

strangely disappears in eq 11.12

$$= \frac{2\pi^3 \ln 2 \, (\hbar c)^7}{c \, (m_e c^2)^5}$$

$$= \frac{2\pi^3 \ln 2 \, (197.327 \text{ MeV fm})^7}{(3E23 \frac{\text{fm}}{\text{s}})(0.511 \text{ MeV})^5} = (4.8E-5) \text{MeV}^2 \text{fm}^6 \text{ s}$$

$$|GM|^2 = 4.4 E-8 \text{ MeV}^2 \text{fm}^6$$

$$GM = 2.1E-4 \text{ MeV fm}^3 \quad \leftarrow (11.13)$$

$$\frac{G}{(\hbar c)^3} = 1.1664E-5 \text{ GeV}^{-2}$$

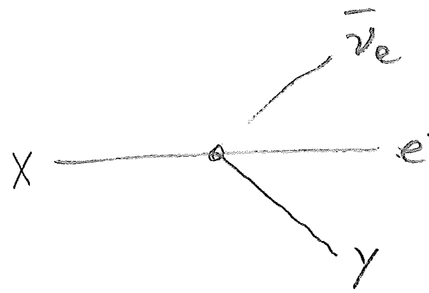
$$= 1.1664E-11 \text{ MeV}^{-2}$$

$$G = (1.1664E-11)(197)^3$$

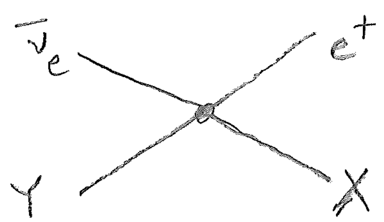
$$= 8.9E5 \text{ MeV} \cdot \text{fm}^3$$

$$M = 2.3$$

Continuous spectrum of β-decay & Kurie plot gives indirect evidence for neutrinos, but can one detect them directly?



$\bar{\nu}$ emission



$\bar{\nu}$ absorption

[Hw: calc $\sigma(\bar{\nu}_e Y \to e^+ X)$. Ans: 1.3×10^{-19} barns for $E_\nu = 2.5$ MeV on protons.

$\bar{\nu}$ absorption cross is very small, approx. 10^{-19} barns for $E_\nu \sim$ few MeV

[a very small barn]

[Hw: calc mean free path of $\bar{\nu}_e$ in lead. Ans. $\ell = \frac{1}{n\sigma} \sim 10^{18}$ m ~ 100 ly]

mean free path of $\bar{\nu}$ in matter $\sim$ light-years

65. **25.xx.** *Antineutrino cross section.*

Consider a process in which a massless electron antineutrino with energy E_ν is absorbed by a stationary nucleus X, converting it into a nucleus Y and a positron: $\bar{\nu}_e X \to e^+ Y$. The positron goes off at an angle θ with respect to the incident antineutrino. Assume that E_ν is low enough that the kinetic energy of Y is negligible. This means that the lab frame and the CM frame are basically the same, so you can calculate everything in the CM frame and completely ignore the kinetic energies of the X and Y (but not their momenta).

(a) Let $Q = m_X - m_Y - m_e$, where m_X and m_Y are the masses of the nuclei. You may assume that $Q < 0$, for otherwise the X would be unstable to β^+ decay ($X \to Y e^+ \nu_e$). Compute the minimum energy E_ν required for $\bar{\nu}_e X \to e^+ Y$ to occur, in terms of Q.

(b) Assume that E_ν is above the threshold calculated in part (a). Compute the energy E_e and momentum p_e of the final-state positron in terms of E_ν, m_e, and Q.

(c) Draw the Feynman diagram for this process and write the amplitude A, making the same assumptions we did in class for the spin and nuclear stuff. Express A solely in terms of G_F, m_X, m_e, E_ν, Q, and the nuclear matrix element M.

(d) Starting from the general $2 \to 2$ formula that we derived in class, write down $(d\sigma/d\Omega)_{\rm cm}$, the differential cross-section for this process in the CM frame, expressing your final answer solely in terms of G_F, m_X, m_e, E_ν, Q, and the nuclear matrix element M.

(e) Simplify your answer by assuming that E_ν, m_e and Q are all much less than m_X.

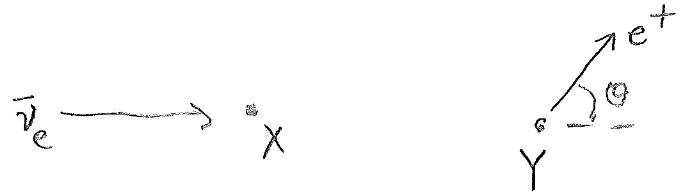
(f) Now assume that M is independent of θ to compute the cross section σ for this process.

(g) Finally, specialize your result to the process $\bar{\nu}_e p \to e^+ n$. You may assume that M takes on the same value that we found in class for neutron decay $n \to p e^- \bar{\nu}_e$, to which this process is closely related. Numerically evaluate σ for an incident antineutrino of energy $E_\nu = 2.5$ MeV, expressing your result in fm^2 and also in barns.

Solution

$$\boxed{\bar{\nu}_e X \to e^+ Y}$$ in cm frame

w/ kinetic energies of X, Y negligible



$Q \equiv m_X - m_Y - m_e < 0$

(a) $E_\nu + m_X = E_e + m_Y$

$$E_\nu = -m_X + m_Y + m_e + T_e$$

$$\geq -(m_X - m_Y - m_e) \quad \text{since } T_e \geq 0$$

$$\boxed{E_\nu \geq -Q}$$

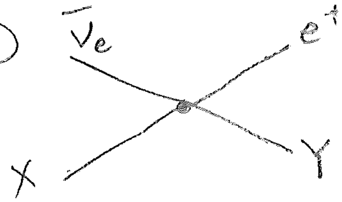
(b) $E_e = E_\nu + m_X - m_Y$

$$\boxed{E_e = E_\nu + Q + m_e}$$

$$p_e = \sqrt{E_e^2 - m_e^2} = \sqrt{(E_\nu + Q + m_e)^2 - m_e^2}$$

$$= \sqrt{(E_\nu + Q)^2 + 2m_e(E_\nu + Q)}$$

(c)



$$A = G_F \sqrt{(2E_X)(2E_Y)(2E_\nu)(2E_e)}\, M$$

$$= 4G_F \sqrt{m_X m_Y E_\nu E_e}\, M$$

$$= 4G_F \sqrt{m_X (m_X - Q - m_e) E_\nu (E_\nu + Q + m_e)}\, M$$

$\boxed{\bar{\nu}_e X \to e^+ Y \quad (\text{cont})}$

(d) $$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \left(\frac{\hbar}{8\pi E_{cm}}\right)^2 \frac{p_e}{p_\nu} |A|^2 \qquad \text{where } E_{cm} = m_X + E_\nu$$

$$= \left(\frac{\hbar}{8\pi(m_X+E_\nu)}\right)^2 \frac{\sqrt{(E_\nu+Q+m_e)^2 - m_e^2}}{\cancel{E_\nu}} (4G_F)^2 m_X (m_X - Q - m_e) \cancel{E_\nu} (E_\nu + Q + m_e) |M|^2$$

$$= \left(\frac{\hbar G_F}{2\pi}\right)^2 \frac{m_X(m_X - Q - m_e)}{(m_X+E_\nu)^2} \sqrt{(E_\nu+Q+m_e)^2 - m_e^2}\, (E_\nu + Q + m_e) |M|^2$$

(e) $$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \left(\frac{\hbar G_F}{2\pi}\right)^2 \sqrt{(E_\nu+Q+m_e)^2 - m_e^2}\, (E_\nu + Q + m_e) |M|^2$$

[usually written as $\left(\frac{\hbar G_F}{2\pi}\right)^2 p_e E_e |M|^2$]

(f) $$\sigma = \int \left(\frac{d\sigma}{d\Omega}\right)_{cm} d\Omega = \frac{\hbar^2 G_F^2}{\pi} p_e E_e |M|^2$$

(g) $$\sigma = \frac{(\hbar c)^2 G_F^2}{\pi} (cp_e) E_e |M|^2 = (2.2 \times 10^{-18}\ fm^2) |M|^2$$

$$= 1.3 \times 10^{-17}\ fm^2$$

$$\boxed{\sigma = 1.3 \times 10^{-19}\ \text{barns}}$$

$1.3 \times 10^{-43}\ cm^2$

$Q = m_p - m_n - m_e = -1.8$ MeV

$E_\nu = 2.5$ MeV

$E_e = E_\nu + Q + m_e = 1.2$ MeV

$cp_e = \sqrt{E_e^2 - m_e^2} \quad 1.1$ MeV

$G_F = 1.1664 \times 10^{-11}\ \text{MeV}^{-2} \approx 10^{-5}\ \text{GeV}^{-2}$

$\hbar c = 197$ MeV fm

$M = 2.4$

Perkins, p. 221

Pel astroph 1.276

$$\left(\frac{G_F^2 (cp_e) E_e}{\pi} = 5.7E-17\ \text{GeV}^{-2}\right)$$

$1\ \text{barn} = 2568\ \text{GeV}^{-2}$

66. **23.09. 19.08. Since 2005.** *Mean free path of antineutrinos in matter.* As you calculated in the previous problem, neutrinos (or antineutrinos) have a very small absorption cross section in matter, of the order of 10^{-19} barns for incident neutrino energies of a few MeV.

(a) Show that in general the rate of depletion of neutrino flux as it passes through an absorbing material is given by $dF/dx = -\mu F$ where μ is a constant, and compute μ in terms of the cross section σ and the number density of targets n.

(b) Show that the flux diminishes as $F(x) = F_0 \exp(-\mu x)$, where F_0 is the flux as the beam enters the material and x is the distance into the material.

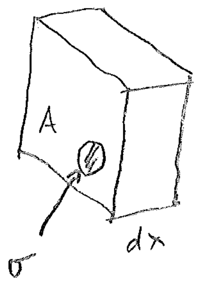
(c) Calculate ℓ, the *mean free path* of the neutrino, i.e., the average distance it travels before being absorbed.

(d) Assuming $\sigma = 10^{-19}$ barns and $n = 10^{29}$ targets per cubic meter, numerically evaluate the mean free path, expressing your result in light-years.

Solution

mean free path of antineutrinos

[see ru-13 notes]

(a) 

$n = \#$ density of targets

$N = n A\, dx$

$$\text{prob. of absorption} = \frac{N\sigma}{A} = n\sigma\, dx$$

$$dF = -(n\sigma\, dx) F$$

$$\frac{dF}{dx} = -n\sigma F \quad \text{so} \quad \boxed{\mu = n\sigma}$$

(b) $$\frac{dF}{F} = -\mu\, dx$$

$$\ln F \Big|_{F_0}^{F} = -\mu x \Big|_0^x$$

$$F = F_0 e^{-\mu x}$$

(c) $$l = \langle x \rangle = \frac{1}{F_0}\int_0^\infty dx\, \underbrace{\text{prob}(x)}_{F(x)\cdot\mu}\, x = \mu \int_0^\infty dx\, e^{-\mu x} x$$

$$= -\mu \frac{d}{d\mu} \underbrace{\int_0^\infty dx\, e^{-\mu x}}_{\frac{1}{\mu}} = \frac{1}{\mu} = \frac{1}{n\sigma} \qquad \boxed{l = \frac{1}{n\sigma}}$$

(d) Assume $\sigma = 10^{-19}$ barns $= 10^{-17}\ \text{fm}^2 = 10^{-47}\ \text{m}^3$

$n = 10^{29}$ targets/m^3

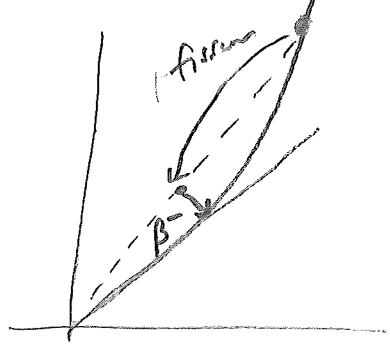
$$l = \frac{1}{n\sigma} = 10^{18}\ \text{m} = \boxed{100\ \text{ly} = l}$$

$$1\ \text{ly} = (3E8\ \tfrac{m}{s})(3E7\ \tfrac{s}{yr}) \doteq 10^{16}\ \text{m}$$

To detect $\bar{\nu}$ need a lot of them

Fission reactors

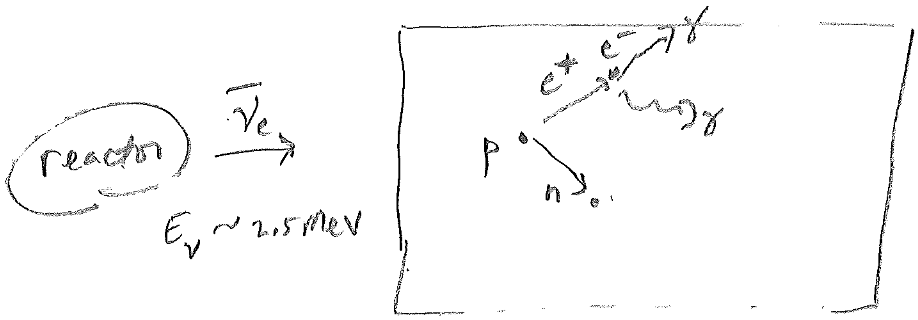
$X \to Y e^- \bar{\nu}_e$



The $\bar{\nu}_e p \to e^+ n$ provided E_ν is large enough.

1956 Cowan-Reines antineutrino detection experiment at Savannah River, So. Carolina, commercial reactor

[show picture]



- positron annihilates to 2γ w/ energy 0.511 MeV each
- neutron absorbed by $CdCl_2$ which then emits 3 to 4 γ w/ $E_{tot} \sim 9$ MeV after 10^{-6} sec

Recall $\sigma = \frac{R}{F}$

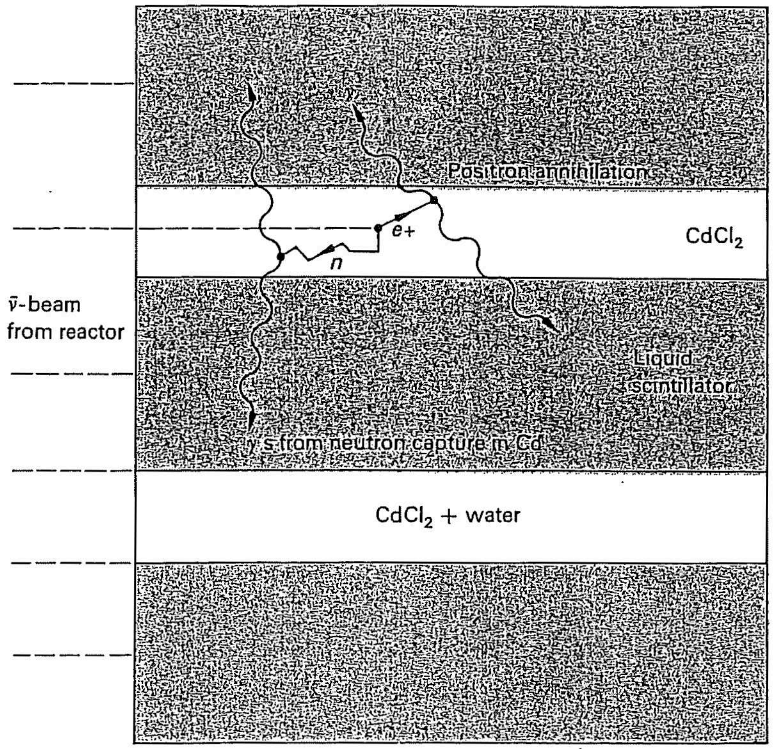
Absorption rate $R = \sigma F$ per target

targets $N = nV$

n = target density (# hydrogen atoms/volume)

Total rate $= RN$

[HW: calc # events per day]



Positron annihilation
e+
n
CdCl₂
ν̄-beam from reactor
Liquid scintillator
γ's from neutron capture in Cd
CdCl₂ + water

← on Canvas

183 cm × 132 cm

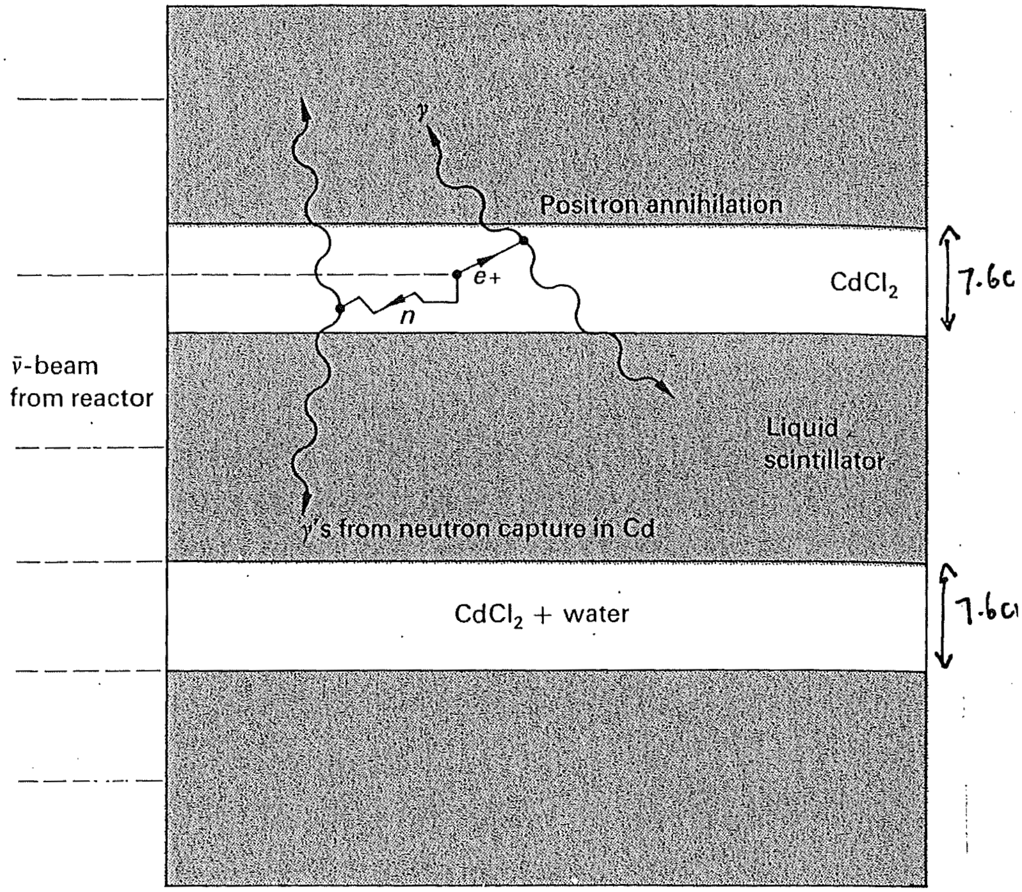


Fig. 6.4 Schematic diagram of the experiment by Reines and Cowan (1959), interactions of free antineutrinos from a reactor.

Perkins

water volume ~ 400,000 cm^3

6′ $4\frac{1}{3}$′

183cm × 132cm × 2(7.6cm) = 367,000 cm^3

Asimov: $6\frac{1}{4}$′ × $4\frac{1}{2}$′ × 6″

67. **23.08. 19.08. Since 2015.** *Cowan-Reines antineutrino detection experiment.*
In the first direct neutrino detection experiment in 1956, a beam of antineutrinos from a nuclear reactor, with average energy of about 2.5 MeV, traversed a tank of water.

(a) Show that the antineutrinos are sufficiently energetic to be absorbed by the hydrogen nuclei in water.

(b) Show that the antineutrinos are not sufficiently energetic to be absorbed by the oxygen nuclei.

(c) The cross-section for antineutrino absorption by a proton at the relevant energy scale is about 1.1×10^{-19} barns. The beam flux through the tank is $F = 1.2 \times 10^{17}$ antineutrinos per second per meter squared. The lateral dimensions of the tank of water are ~~1.83~~ 183 cm m $\times$
132 cm ~~1.32~~ m, and the height of each of the two water-filled sections is 7.6 cm. (A schematic of the experiment can be found on Canvas.) If the efficiency of detecting each event is $\epsilon = 0.025$, compute the number of events expected to be detected in one day.

Cowan-Reines experiment solution

(a) $\bar{\nu} p \rightarrow e^+ n$

$$Q = m_p - m_n - m_e$$

$$= \Delta(^1H) - \Delta(n) - 2m_e = -1.804 \text{ MeV}$$

(-2) if ignored $m_e \Rightarrow Q = -1.3$

(b) $\bar{\nu}\ ^{16}O \rightarrow e^+\ ^{16}N$

$$Q = \underbrace{\Delta(^{16}O)}_{-4.737} - \underbrace{\Delta(^{16}N)}_{5.685} - 2m_e = -11.44 \text{ MeV}$$

(-1) if ignored $2m_e \Rightarrow Q = -10.4$

(c) From class

$$\text{Events detected} = \epsilon \overset{\text{absorption rate per target}}{R} \overset{\leftarrow \#\text{ targets}}{N}$$

$$= \epsilon(\sigma F)(nV)$$

Given
$\epsilon = 0.025$
$\sigma = 1.1 \times 10^{-19}$ barns $= 1.1 \times 10^{-47} m^2$
$F = 1.2 \times 10^{17}\ s^{-1} m^{-2}$
$V = (1.83 m)(1.32 m) \cdot (0.076 m) \cdot 2 = 0.367 m^3$

[1956: $\sigma_{exp} = 6 \times 10^{-48} m^2 \pm 25\%$]

$$n = \left(\frac{2 \text{ protons}}{H_2O}\right)\left(\frac{6E23\ H_2O}{\text{mole}}\right)\left(\frac{\text{mole}}{18g}\right)\left(1\frac{g}{cm^3}\right)\left(\frac{10^6 cm^3}{m^3}\right) = 6.7 \times 10^{28} \frac{\text{protons}}{m^3}$$

[nV = 2.5E28 protons paper says 2.2E28?]

$$\text{Events} = (0.025)(1.1E-47)(1.2E17)(6.7E28)(0.367) = 8.1 \times 10^{-4}\ s^{-1}$$

$$= 70 \frac{\text{events}}{\text{day}}$$

WI-24

μ^- decay

[→ In problem they work out maximum ($\frac{(m_\mu - m_e)^2}{2m_\mu} = 52.32$) and minimum (0) kinetic energy for the electron]

[PPB] $\mu^- \to e^- \bar{\nu}_e \nu_\mu$

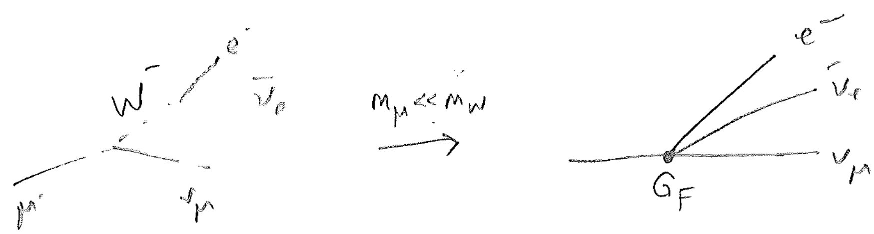
$m_\mu = 105.6$ MeV

m_e 0.5 MeV

$\tau = 2.197 \times 10^{-6}$ s

$R_{exp} = \frac{1}{\tau} = 4.552 \times 10^5\ s^{-1}$ (much faster than neutrons)

Recall



Sargent: $R = \frac{G_F^2}{60\pi^3\hbar} |M|^2 Q^5$ $\qquad Q = m_\mu - m_e \simeq m_\mu$

not valid because assumes $X \to Y e^- \bar{\nu}_e$ w/ $M_Y \gg m_e$ (so $e, \bar{\nu}$ get all the energy)

whereas $m_{\nu_\mu} \ll m_e$ (so split 3 ways, cf problem above)

An accurate calculation gives

$$R = \frac{G_F^2}{192\pi^3\hbar} m_\mu^5$$ in massless electron approximation (+ doing spin correctly)

$$= \frac{(0.2067\ \text{MeV}^{-5}\ s^{-1})(105.66\ \text{MeV})^5}{192\pi^3}$$

$R_{th} = 4.572 \times 10^5\ s^{-1}$

R_{th} + $R_{\cdot}$ very close! ($\frac{1}{2}$%)

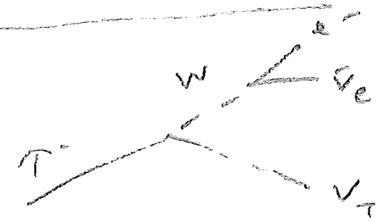
[including $m_e \neq 0$ does not improve]

τ^- decay

$$\tau^- \to e^- \bar{\nu}_e \nu_\tau$$

expt $\tau = 2.90 \times 10^{-13}$ s

$R = 3.45 \times 10^{12}\ s^{-1}$



$$R(\tau \to e^- \bar{\nu}_e \nu_\tau) = \frac{G_F^2}{192\pi^3 \hbar^3} m_\tau^5 = 6.15 \times 10^{11}\ s^{-1}$$

about 6 times too slow

τ decay channels

PPB:

$\tau \to e^- \bar{\nu}_e \nu_\tau$ 17.8%

$\to \mu \bar{\nu}_\mu \nu_\tau$ 17.4%

$\to \pi^- \nu_\tau$ 10.8%

etc

more decay channels $\Rightarrow$ faster rate

$R_{tot} = \sum_i R_i$ $R_i \equiv$ "partial rates"

Branching ratio $(BR)_i = \frac{R_i}{R_{tot}}$ $\sum (BR)_i = 1$

$$\Rightarrow R_{tot} = \frac{R_i}{(BR)_i} = \frac{6.15 \times 10^{11}\ s^{-1}}{0.178} = 3.45 \times 10^{12}\ s^{-1}$$ ✓

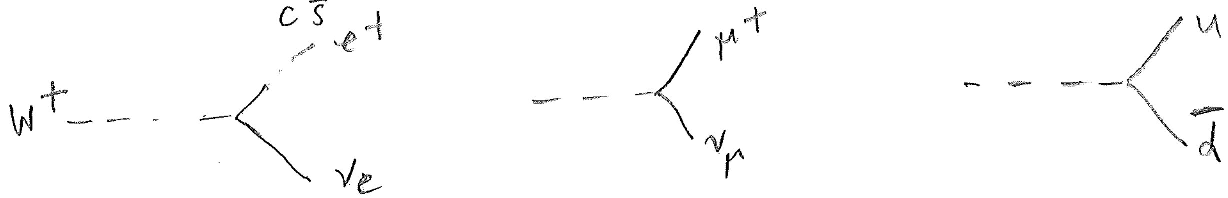
W^- decay

W, Z first directly detected 1983 at CERN

[PPB] $m_W = 80.4$ GeV

$$
\begin{aligned}
W^+ \to\ & e^+\nu_e \quad (10.7\%) \\
& \mu^+\nu_\mu \\
& \tau^+\nu_\tau \\
& u\bar{d} \\
& c\bar{s}
\end{aligned}
$$

- might expect 20%
- but quarks have color so ∃ multiplicity by # colors



kinematics:

$$p_W = (m_W, 0, 0, 0)$$
$$p_e = (E_e, 0, 0, p)$$
$$p_\nu = (E_\nu, 0, 0, -p)$$

$\nu \longleftarrow \overset{W}{\bullet} \longrightarrow e$

$$m_W = E_e + E_\nu = \sqrt{p^2 + m_e^2} + \sqrt{p^2 + m_\nu^2} \approx 2p$$

$m_\nu, m_e \ll m_W$

decay rate $R = \dfrac{1}{2m_W \hbar} \int (LIPS)_2 \, |A|^2$

$$(LIPS)_2 = \frac{p_f}{(4\pi)^2 E_{cm}} d\Omega \approx \frac{1}{2(4\pi)^2} d\Omega$$

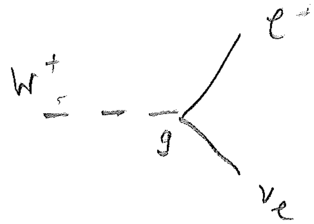
$$R = \frac{1}{4(4\pi)^2 m_W \hbar} \int d\Omega \, |A|^2$$

[assume $|A|$ indep of $\measuredangle$: not really true because W^+ has spin]

$$R = \frac{|A|^2}{16\pi m_W \hbar}$$

Observe $|A|$ must have dimensions of energy

[E^{4-n}: E^1]



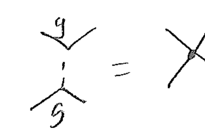
$A = g \left(\begin{smallmatrix} \text{spin} \\ \text{stuff} \end{smallmatrix} \right)$ [no propagator!]

g dimensionless, so $\left(\begin{smallmatrix} \text{spin} \\ \text{stuff} \end{smallmatrix} \right)$ has dimens of energy

Parametrize $\left(\begin{smallmatrix} \text{spin} \\ \text{stuff} \end{smallmatrix} \right) = m_W f,$ f dimensionless

$$A = g\, m_W f$$

$$R = \frac{g^2 m_W f^2}{16\pi\hbar}$$

Recall  $\Rightarrow G_F = \frac{g^2}{m_W^2} \Rightarrow g^2 = G m_W^2$

$$R(W^+ \to e^+ \nu_e) : \frac{G_F m_W^3}{16\pi\hbar} f^2$$

$G_F = 1.166E-11 \text{ MeV}^{-2}$
$m_W : 80.4 \text{ GeV}$
$\frac{G_F m_W^3}{16\pi} = 0.12 \text{ GeV}$

exact : $f^2 = \frac{16}{6\sqrt{2}} = 1.8856$

Analogy w/ previous case suggests $\sqrt{(2m_W)(2E_e)(2E_\nu)}$ but wrong dimensions
One could do $\sqrt{(2E_e)(2E_\nu)}$ but this is same as above
since $2E_e$ $2E_\nu$ m_W

partial rates

$$R = \sum R_i$$

$$R_i = (BR)_i \, R$$

$$R(W^+) = \frac{R(W^+ \to e^+ \nu_e)}{BR(W^+ \to e^+ \nu_e)}$$

width 'Γ' $= \hbar R$

Define partial width $\Gamma_i = \hbar R_i$

$$(BR)_i = \frac{\Gamma_i}{\Gamma}$$

Experiment $\Rightarrow$ $\Gamma = 2.085$ GeV

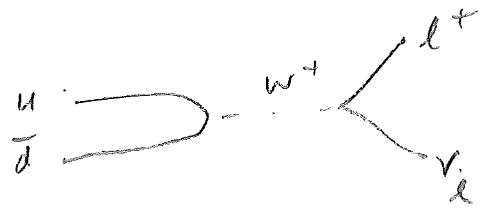
$\tau = \frac{\hbar}{\Gamma} = 3 \times 10^{-25}$ s [shorter than 10^{-23} s even though weak intn!]

[Hw: use Γ and BR to calculate f]

[exact result: $f^2 = \frac{16}{6\sqrt{7}} = 1.8856 \Rightarrow f = 1.373$]

[not for class, maybe problem]

π decay



$$m_\pi = E_\ell + E_\nu = \sqrt{p^2 + m_\ell^2} + p$$

$$p = \frac{m_\pi^2 - m_\ell^2}{2m_\pi}$$

$$R = \frac{1}{2\hbar m_\pi} \int (LIPS)_2 \, |A|^2$$

$$(LIPS)_2 = \frac{p_f}{(4\pi)^2 E_{cm}} \, d\Omega$$

$$\Rightarrow R = \frac{p_f}{8\pi\hbar m_\pi} |A|^2 = \frac{m_\pi^2 - m_\ell^2}{16\pi\hbar m_\pi^3} |A|^2 = \frac{1}{16\pi\hbar m_\pi}\left(1 - \frac{m_\ell^2}{m_\pi^2}\right)|A|^2$$

$$A = \frac{g^2}{m_W^2}\left(\text{spin stuff}\right) = G_F\left(\text{spin stuff}\right)$$

Since A has dimns of energy, let $(\text{spin stuff}) = m_\pi^3 f$

$$R = \frac{G_F^2 m_\pi^5 f^2}{16\pi\hbar}\left(1 - \frac{m_\ell^2}{m_\pi^2}\right) \quad \text{(our naive prediction)}$$

$$\hbar R(\pi^+ \to \mu^+ \nu_\mu)_{expt} = 2.545E{-}17 \text{ GeV}$$

$$\underbrace{\frac{G_F^2 m_\pi^2}{16\pi}}_{1.433E-16}\underbrace{\left(1 - \frac{m_\ell^2}{m_\pi^2}\right)}_{0.4269} = 6.119E{-}17 \text{ GeV} \Rightarrow f = 0.645$$

works pretty well for $\pi \to \mu\nu$
fails badly for $\pi \to e\nu$, of course
→ because misses dependence on m_ℓ due to helicity...

Griffiths: $f = \sqrt{2}\,\frac{f_\pi}{m_\pi}\,\frac{m_\ell}{m_\pi}\sqrt{1 - \frac{m_\ell^2}{m_\pi^2}}$

[see blogged note]

[not for class]

How does decay rate depend on phase space, i.e. on Q?

3-body decay: SARGENT RULE $R \sim G_F^2 Q^5$

2-body decay

W-decay $R \sim g^2 \boxed{M_W} \sim G_F M_W^3$

π-decay $R \sim G_F^2 m_\pi^5$ (naive)

really should be $R \sim G_F^2 f_\pi^2 m_\ell^2 \boxed{m_\pi}$

K-decay $R \sim G_F^2 f_K^2 m_\ell^2 \boxed{m_K}$