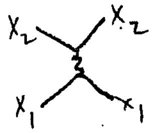
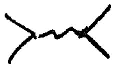
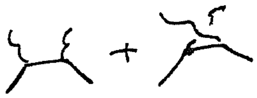
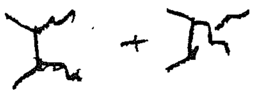
2260 : in 2025

QED processes

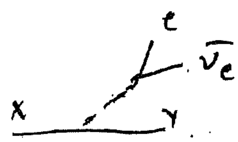
Rutherford  done in class

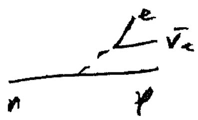
$e^-e^+ \to \mu^-\mu^+$  HW ($\sigma \sim 90$ nb at $s = (6$ GeV

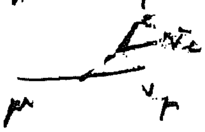
Compton $\gamma e \to \gamma e$  HW: kinematic only ($\sigma = \frac{2}{3}$ barn)

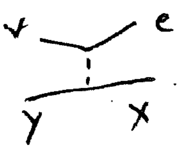
$e^-e^+ \to \gamma\gamma$  exam: kinematics only

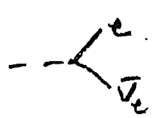
Weak int^n:

β-decay $X \to Y e^- \bar{\nu}_e$  $\left(\frac{1}{60\pi^3} G^2 a^5\right)$ in class

n-decay $n \to p e^- \bar{\nu}_e$  in class

μ-decay $\mu \to e \nu_\mu \bar{\nu}_e$  $\left(\frac{1}{192\pi^3} G^2 m_\mu^5\right)$ in class

$\bar{\nu}$ absorption $\bar{\nu} Y \to X e^+$  HW

W decay $W \to e\bar{\nu}$  in class

[This is just some talking points
to start the 2nd half of semester]

[Up to now, we have focused on]

Possibilities (conservation laws)

[Now we will focus on]

Probabilities (quantum mechanics)

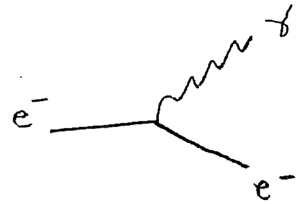
- lifetimes
- cross-sections

[Previously we looked a bit at these
1) α-particle $\Rightarrow$ tunnelling $\Rightarrow$ Geiger-Nuttall
2) Rutherford σ $\Rightarrow$ classical calculation
3) nuclear σ $\sim$ geometric]

QED (quantum electrodynamics)

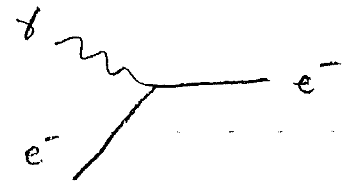
QED is the QFT describing the interaction of the electromagnetic field (photons) with charged particles

The basic QED vertex is

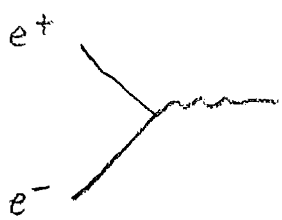


charged particle emits a photon

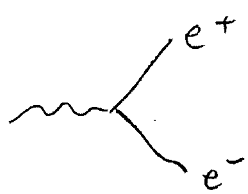
$\longrightarrow$ time [don't take this too literally. Just means incoming particle on left; outgoing on right; Griffiths: think of a line passing from left to right]



charged particle absorbs a photon



particle + antiparticle turn into a photon



vice versa

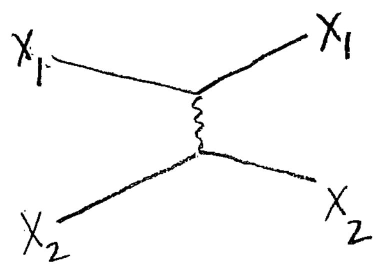
All of these processes, by themselves, obey charge conservation but violate energy momentum conservation if all the particles are on-shell, ie obey $E^2 = \vec{p}^{\,2} + m^2$

[recall problem]

An electromagnetic process can be described by a set of Feynman diagrams assembled from the QED vertices

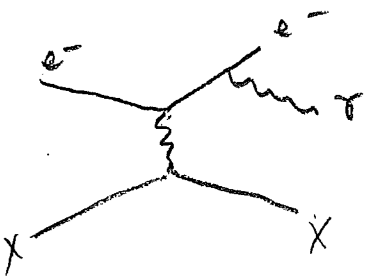
① elastic Coulomb scattering: $X_1 X_2 \to X_1 X_2$ (X = charged particle)

eg. Rutherford scattering

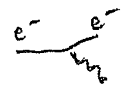


[photon is "virtual" ie not on shell.
which particle emits + which absorbs? not relevant.
photon causes momentum transfer.]

② inelastic electron scattering: $e^- X \to e^- X \gamma$ (Bremsstrahlung) or radiative scattering

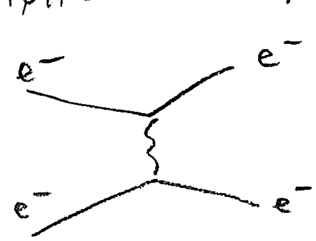


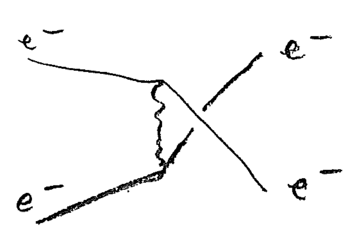
[electron emits because it accelerates more]

[

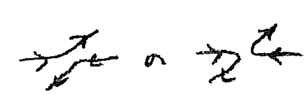
if conserve energy, then violate mom. unless mom can be transferred ("force") from nucleus]

③ Møller scattering: $e^- e^- \to e^- e^-$

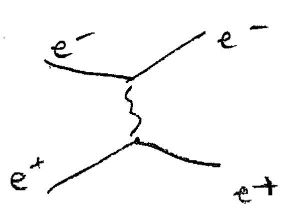


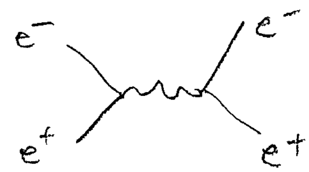


[physically, can't distinguish which particle scattered where


or]

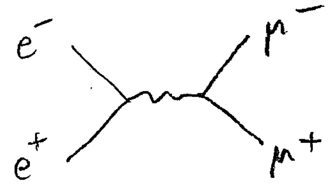
④ Bhabha scattering: $e^- e^+ \to e^- e^+$





[γ exchange also binds together opp. charge pcle as we saw in PI]

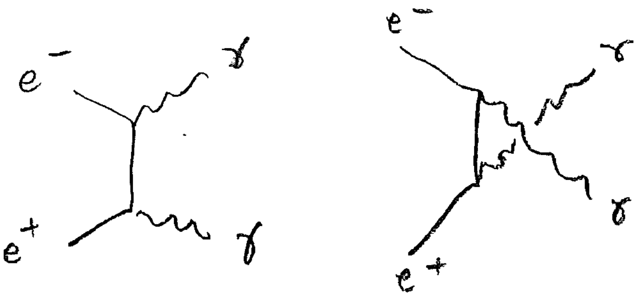
(5) pair annihilation: $e^- e^+ \to \mu^- \mu^+$ (if $e^\pm$ have enough energy)



[recall, can't have $e^+ e^- \to \gamma$

[saw this in β^+ decay]

(6) pair annihilation: $e^- e^+ \to \gamma\gamma$



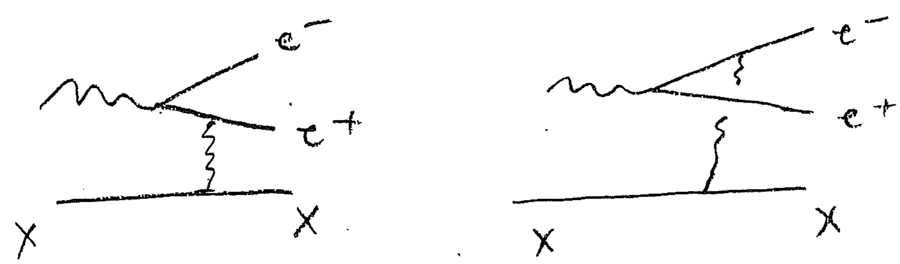
[now the electron is virtual]

All internal lines represent <u>virtual</u> particles which are off-shell: $E^2 \neq \vec{p}^{\,2} + m^2$

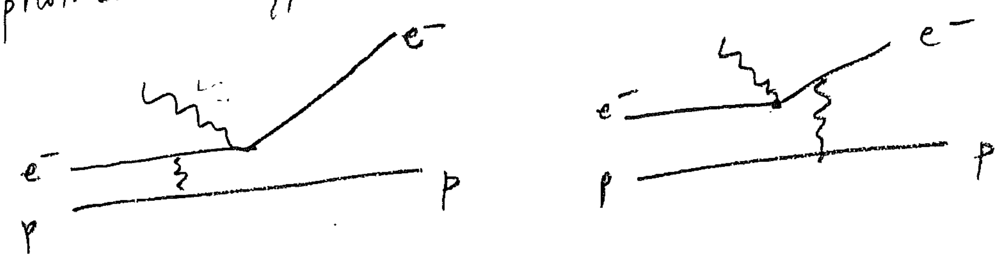
Processes w/ incident photons:

⑦ Compton scattering: $\gamma e^- \to \gamma e^-$ [saw this in photons(γ)

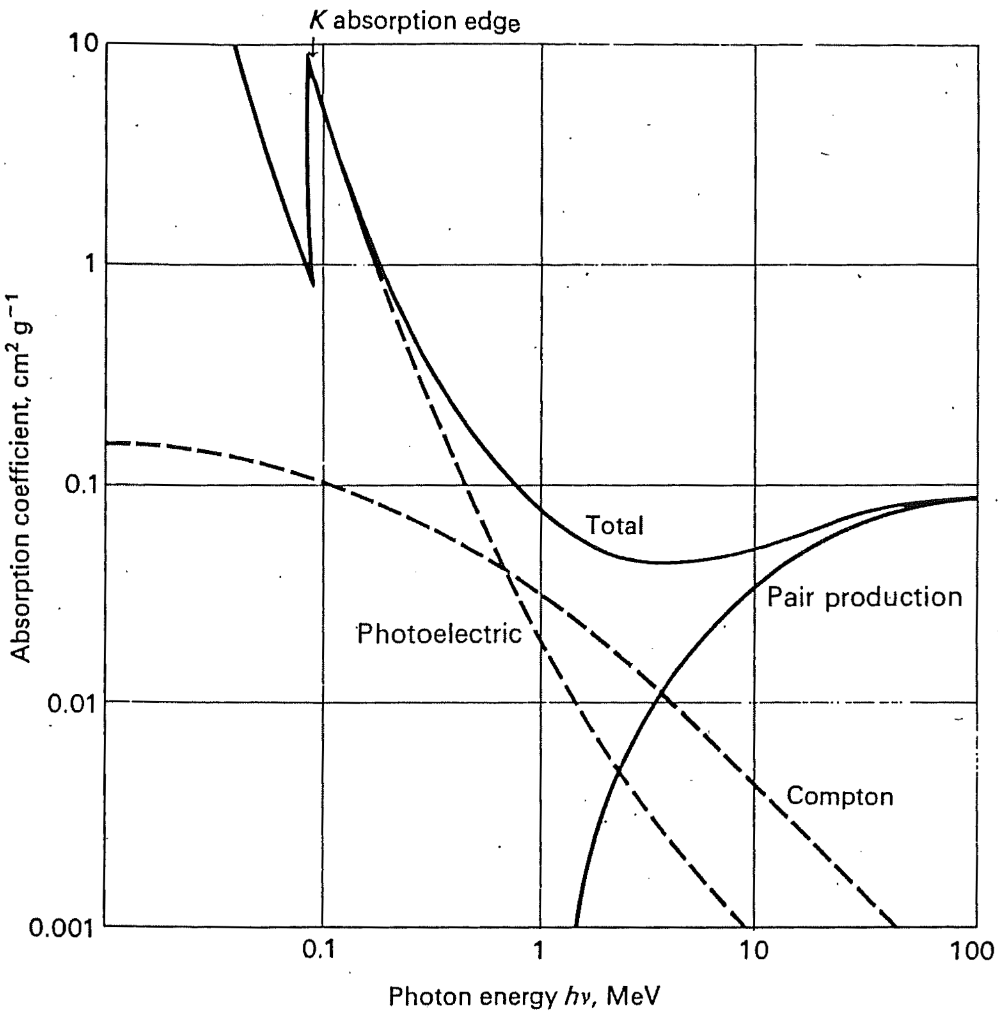
⑧ pair production: $\gamma X \to X e^+ e^-$



⑨ photoelectric effect: γ (atom) $\to$ (ion)$^+$ e^-



[show graph of 3 processes]



The absorption coefficient per g cm^{-2} of lead for γ-rays as a function of energy.

Perkins

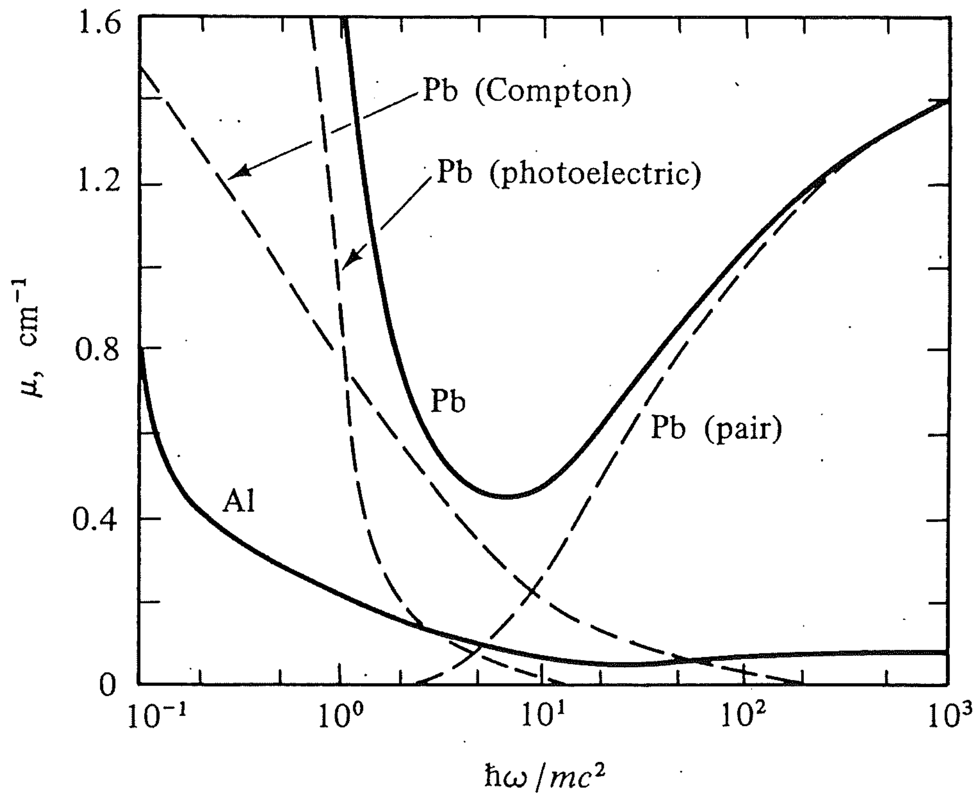
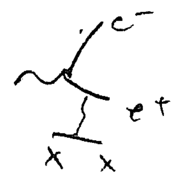
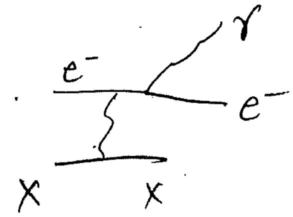


Fig. 3.7. Total absorption coefficients of γ rays by lead and aluminum as a function of energy (solid lines). Photoelectric absorption of aluminum is negligible at the energies considered here. Dashed lines show separately the contributions of photoelectric effect, Compton scattering, and pair production for Pb. Abscissa, logarithmic energy scale; $\hbar\omega/mc^2 = 1$ corresponds to 511 keV. (From W. Heitler, *The Quantum Theory of Radiation*, The Clarendon Press, Oxford, 1936, p. 216.)

Frauenfelder & Henley

[Discuss how pair production 

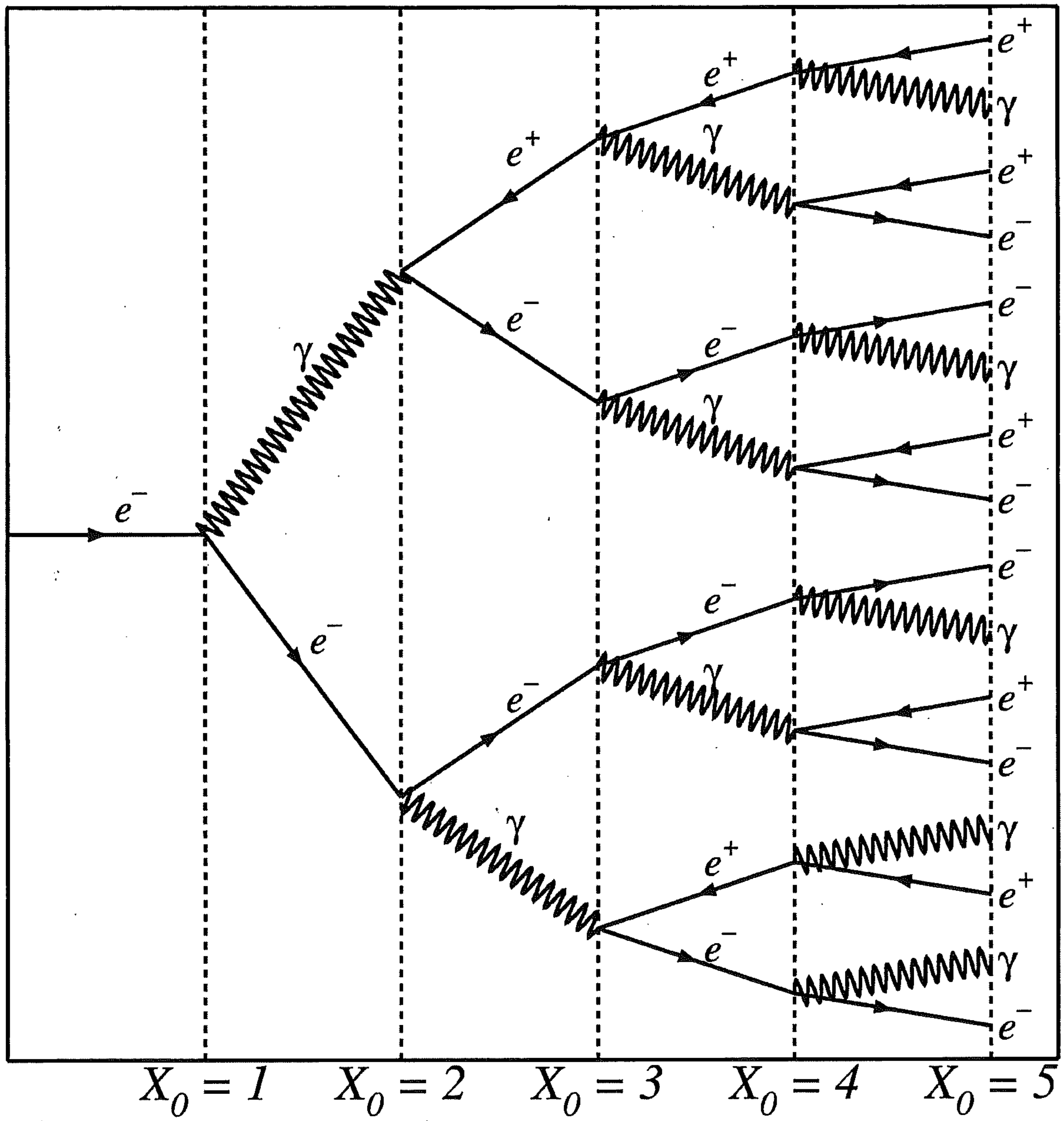
and bremsstrahlung 

in matter can lead to

electromagnetic showers in detectors

& in atmosphere]

$X_0 \sim 0.5\,\text{km}$


e−
γ
e−
e+
e−
e−
γ
e+
γ
e−
γ
e−
γ
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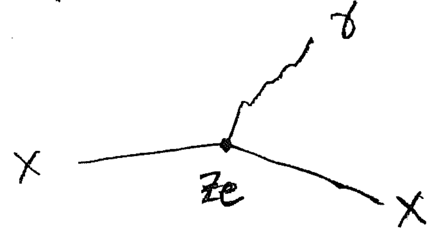
e−
e+
γ
e+
e−
e−
γ
e+
e−
e−
γ
e+
e−
γ
e+
γ
e−
X0 = 1
X0 = 2
X0 = 3
X0 = 4
X0 = 5

e = charge of proton ($e>0$)

e is also called the <u>QED coupling constant</u> because it represents the strength of the coupling between electron/proton + photon



Ze = charge of X

How strong is EM force?

Recall fine structure constant (dimensionless)

$$\alpha = \frac{Ke^2}{\hbar c} = \frac{e^2}{4\pi\epsilon_0 \hbar c} \approx \frac{1}{137}$$

Because $\alpha \ll 1$, EM force is weak

[Particle physicists often use] "rationalized" units
(Heaviside - Lorentz)

$c = 1$ [as usual]
$\hbar = 1$
$\epsilon_0 = 1$

$$\Rightarrow \alpha = \frac{e^2}{4\pi}$$

Later we'll replace $e^2 \rightarrow 4\pi\alpha$

Feynman dgms not just pretty pictures
to help me visualize a process.

They are useful that way because if you can't draw
a Feynman diagram, the process cannot occur.

F. d are a tool to compute likelihood of various processes

The sum of Feynman diagrams gives the amplitude A
for a given process

The probability of the process is proportional to $|A|^2$

E.g. $e^- X \to e^- X$

"tree diagram"

"one loop diagrams"

two-loop

Tree diagram has 2 vertices: $(Ze)e$
1-loop " " 4 vertices: $(Ze)^2 e^2$, $(Ze)e^3$
2-loop " " 6 vertices: $(Ze)^3 e^3$

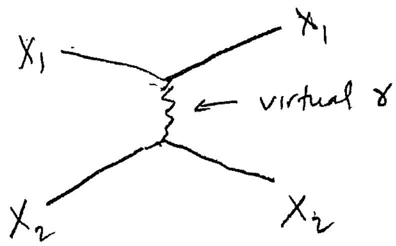
The more powers of e, the smaller the contribution to the amplitude

Since $\alpha \ll 1$, loop diagrams contribution is a small correction

In lowest approximation, need only consider tree diagram! (lowest # of vertices)

Consider Rutherford scattering: $X_1 X_2 \rightarrow X_1 X_2$ where $X_1 = \alpha$ particle

The lowest order Feynman diagram for this process is



- The 2 vertices contribute factors of $Z_1 e$ and $Z_2 e$ where $Z_1 = 2$ for α

- In addition, each <u>internal</u> line contributes a factor of

$$\frac{1}{p^2 - m^2}, \text{ called the propagator}$$

where p = 4-momentum of the internal particle and m is its mass. (which is zero for a photon)

N.B. if the internal particle were on-shell then $p^2 = m^2$ and the propagator $\rightarrow \infty$.

- For Rutherford scattering

$$A = \frac{(Z_1 e)(Z_2 e)}{p_\gamma^2} \begin{pmatrix} \text{spin} \\ \text{stuff} \end{pmatrix}$$

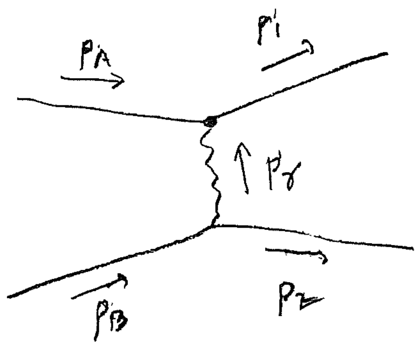
[Dirac γ matrices]

where we will not go into details of spin stuff in this course

How do we determine p_γ ?

Ans: 4-Momentum conservation at the vertices.

Let the initial state particle have 4-momenta p_A and p_B
final state " " 4-momenta p_1 and p_2



$p_A + p_\gamma = p_1$ at top vertex $\Rightarrow p_\gamma = p_1 - p_A$

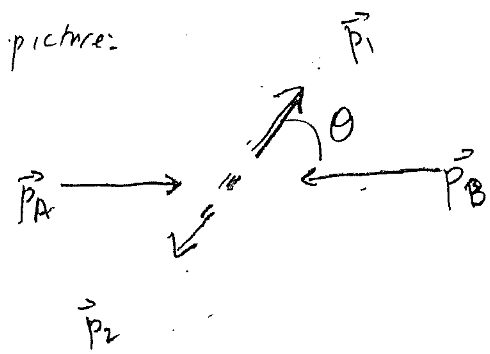
$p_B = p_\gamma + p_2$ at bottom vertex $\Rightarrow p_\gamma = p_B - p_2$

These are the same since $p_A + p_B = p_1 + p_2$.

Kinematics of 2 → 2 scattering: AB → 12

Easiest to determine momenta in the CM frame

Physical picture:



[use 3-momenta]

Because in CM frame, total 3-momentum vanishes

$$\vec{p}_A + \vec{p}_B = 0 \quad \Rightarrow \quad |\vec{p}_A| = |\vec{p}_B| \equiv p_i$$

[NB magnitude of 3-vector, not 4-momentum]

$$\vec{p}_1 + \vec{p}_2 = 0 \quad \Rightarrow \quad |\vec{p}_1| = |\vec{p}_2| \equiv p_f$$

Energy conservation

$$E_A + E_B = E_1 + E_2$$

$$\sqrt{p_i^2 + m_A^2} + \sqrt{p_i^2 + m_B^2} = \sqrt{p_f^2 + m_1^2} + \sqrt{p_f^2 + m_2^2}$$

If collision is elastic (identities don't change)

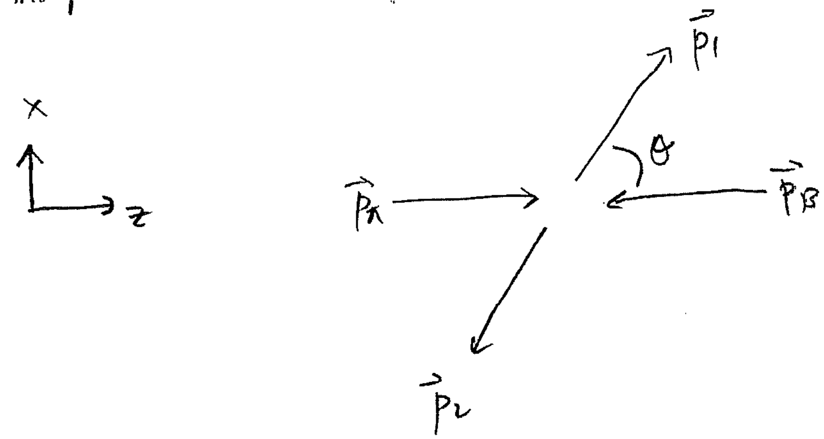
$$m_A = m_1 \quad \text{and} \quad m_B = m_2$$

Therefore, $p_i = p_f \equiv p$ (all momenta equal in magnitude)

Also
$$E_1 = E_A = \sqrt{p^2 + m_A^2}$$
$$E_2 = E_B = \sqrt{p^2 + m_B^2}$$

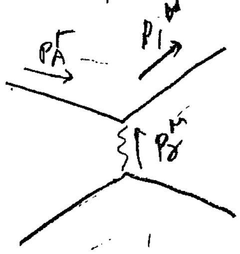
$$E_{cm} \equiv E_A + E_B = E_1 + E_2$$

Independent variable: p, θ, ϕ



The four momenta $p^\mu = (E, p_x, p_y, p_z)$ are

$$p_A^\mu = (E_A, 0, 0, p)$$
$$p_B^\mu = (E_B, 0, 0, -p)$$
$$p_1^\mu = (E_A, p\sin\theta, 0, p\cos\theta)$$
$$p_2^\mu = (E_B, -p\sin\theta, 0, -p\cos\theta)$$



$$p_\gamma^\mu = p_1^\mu - p_A^\mu$$
$$= (0, p\sin\theta, 0, p(\cos\theta - 1))$$

Virtual photon has 3-momentum but no energy in CM frame
("space like momentum")
Because $E_\gamma \neq |\vec{p}_\gamma|$, photon is off-shell

$$p_\gamma^2 = 0 - (p\sin\theta)^2 - p^2(\cos\theta - 1)^2$$
$$= 2p^2(\cos\theta - 1) \qquad \left(\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)$$
$$= -4p^2\sin^2\left(\frac{\theta}{2}\right)$$

$$\boxed{A = \frac{Z_1 Z_2 e^2}{-4p^2\sin^2\left(\frac{\theta}{2}\right)}\left(\begin{matrix}\text{spin}\\ \text{stuff}\end{matrix}\right)}$$

QED

54. **25.?** *Kinematics of Compton scattering.*
Consider Compton scattering $\gamma e^- \to \gamma e^-$ in the CM frame. The initial state photon moves in the $+z$ direction with energy E. The final state photon moves in the xz plane, making angle θ with respect to the incident photon.

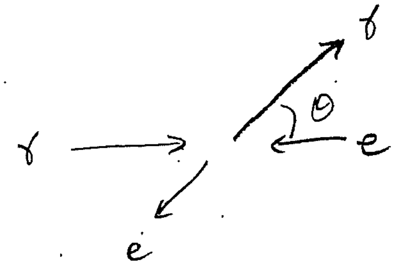
(a) Write the four-momenta of each of the particles involved in terms of E and θ.

(b) There are two Feynman diagrams contributing to this process. For each one, compute the four-momentum of the virtual electron.

(c) Compute the propagator $1/(p_e^2 - m_e^2)$ for each diagram, simplifying as much as possible.

Solution

$\boxed{\text{Compton scattering kinematics}}$ $\gamma e^- \to \gamma e^-$ in CM frame



(a)

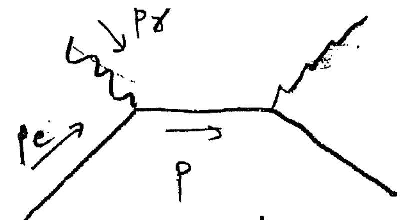
$$p_\gamma = (E,\ 0,\ 0,\ E)$$

$$p_e = (\sqrt{E^2+m_e^2},\ 0,\ 0,\ -E)$$

$$p_\gamma' = (E,\ E\sin\theta,\ 0,\ E\cos\theta)$$

$$p_e' = (\sqrt{E^2+m_e^2},\ -E\sin\theta,\ 0,\ -E\cos\theta)$$

(b)



$$p = p_e + p_\gamma$$

$$= (E+\sqrt{E^2+m_e^2},\ 0,\ 0,\ 0)$$

$$p = p_e - p_\gamma'$$

$$= (\sqrt{E^2+m_e^2} - E,\ -E\sin\theta,\ 0,\ -E(1+\cos\theta))$$

(c) $p^2 - m_e^2 = (E + \sqrt{E^2+m_e^2})^2 - m_e^2 = \boxed{2E(E + \sqrt{E^2+m_e^2})} > 0$

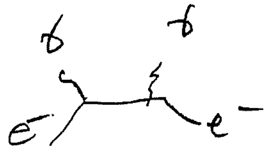
see check

$$p^2 - m_e^2 = (2E^2 + m_e^2 - 2E\sqrt{E^2+m_e^2}) - E^2\sin^2\theta - E^2(1+2\cos\theta+\cos^2\theta) - m_e^2$$

$$= \boxed{-2E(\sqrt{E^2+m_e^2} + E\cos\theta)} < 0$$

Not for class

Compton



we just found

cm frame: $p_e^2 - m^2 = 2E(E + \sqrt{E^2+m^2})$

E = energy of γ in cm frame

FT frame $p_e^2 - m^2 = (E'+m)^2 - E'^2 - m^2 = 2mE$

therefore invariance of propagator implies

$$\Rightarrow E' = \frac{E(E+\sqrt{E^2+m^2})}{m}$$

energy of incident γ in FT frame

Let's verify that!

cm frame: $p_\gamma = (E, 0, 0, E)$

$p_e = (\sqrt{E^2+m^2}, 0, 0, -E)$

$$v_e = \frac{E}{\sqrt{E^2+m^2}} \Rightarrow \gamma_e = \frac{\sqrt{E^2+m^2}}{m}$$

rel. vel. of cm + FT frame is velocity of electron in cm frame

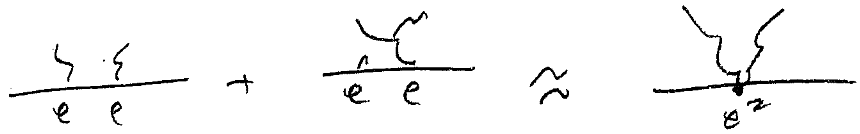
$$E' = \gamma(E + vE) =$$

$$= \frac{\sqrt{E^2+m^2}}{m}\left(1 + \frac{E}{\sqrt{E^2+m^2}}\right) E$$

$$= \frac{E}{m}\left(\sqrt{E^2+m^2} + E\right) \checkmark$$

Not for class

Compton → Thomson



classical electron radius

$$A \cong e^2 = 4\pi\alpha$$

$$\sigma = \frac{\hbar^2}{16\pi S}|A|^2 \sim \frac{\hbar^2}{16\pi m_e^2}(4\pi\alpha)^2 = \pi\left(\frac{\hbar\alpha}{m_e}\right)^2 = \pi r_e^2$$

$|p_f| = |p_i|$

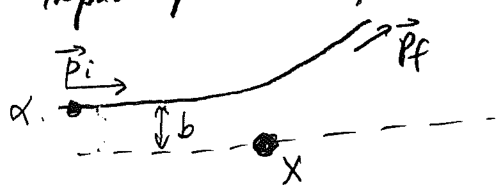
$$r_e = \frac{\hbar\alpha}{m_e c} = \frac{(197 \text{ MeV fm})}{137\,(0.511 \text{ MeV})} = 2.8 \text{ fm}$$

$$\sigma = 25 \text{ fm}^2 = \frac{1}{4} \text{ barn}$$

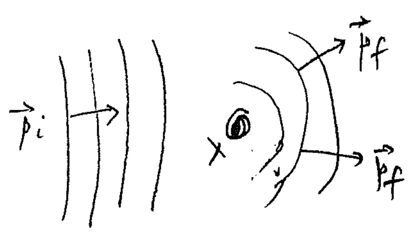
(actual result $= \frac{8\pi}{3} r_e^2 = 0.666$ barns)

How do we go from amplitude A to decay rate or scattering rate?

Classically, given the initial state (eg. initial momentum and impact parameter), the final state is determined



Quantum mechanically, initial state represented by a wave (or wavepacket), and many final states are possible



Rate is given by a sum over final states.

probability of a given final state is the modulus squared of the amplitude A

$$\text{Rate} = \sum_{\substack{\text{final} \\ \text{states}}} |A|^2$$

Therefore, rate depends on
① the strength of the interaction (EM, weak, strong)
② # number of final states (which increases w/ amt of energy released Q)

State of a single particle determined by its momentum (+ spin)

Recall: particle in a box L^3 has discrete momenta $\vec{p} = \frac{h}{L}\vec{n}$

Sum over final states = sum over momenta (and spin)

[we'll suppress this]

$$= \sum_{\vec{n}}$$

$$\approx \int d^3\vec{n}$$

$$= \frac{L^3}{h^3}\int d^3\vec{p} \qquad (h = 2\pi\hbar)$$

$$= \frac{L^3}{\hbar^3}\int \frac{d^3\vec{p}}{(2\pi)^3}$$

strategem: put the system into a large box.

In the end, the size of box is irrelevant, all factors of L cancel.

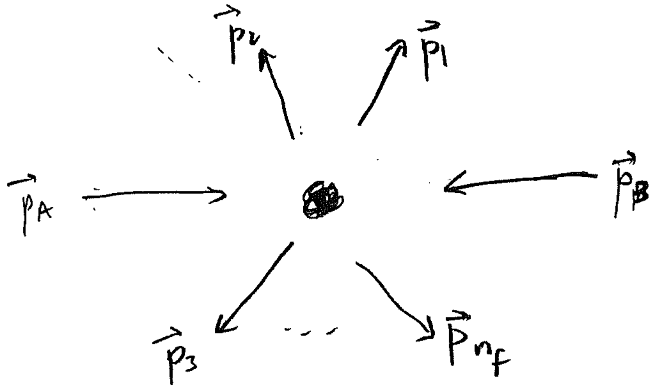
Also, set $\hbar = 1$. (Restore factors in the end)

$\Rightarrow$ final state phase space of a <u>single</u> particle

$$\int \frac{d^3\vec{p}}{(2\pi)^3}$$

$$\boxed{2 \rightarrow n_f \quad \text{scattering process}}$$

(usually $n_f = 2$ or 3)



Energy-momentum conservation

$$p_A + p_B = \sum_{j=1}^{n_f} p_j$$

[4-momenta]

Final state phase space

$$\int \prod_{j=1}^{n_f} \frac{d^3 p_j}{(2\pi)^3} \cdot \underbrace{(2\pi)^4 \, \delta^{(4)}(\Delta p)}_{\text{enforces energy/momentum conservation}}$$

Not all final momenta $\vec{p}_j$ are independent due to mom. cons.

so we include a momentum-conserving δ-function,

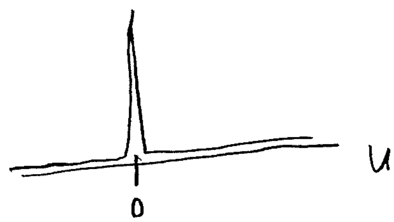
$$\Delta p \equiv \sum_{j=1}^{n_f} p_j - (p_A + p_B) = (\Delta E, \Delta p_x, \Delta p_y, \Delta p_z)$$

$$\delta^{(4)}(\Delta p) = \delta(\Delta E)\, \delta(\Delta p_x)\, \delta(\Delta p_y)\, \delta(\Delta p_z)$$

$\delta(u)$ forces u to vanish. How?

Dirac delta function

Roughly, $\delta(u) = \begin{cases} 0 & \text{if } u \neq 0 \\ \infty & \text{if } u = 0 \end{cases}$



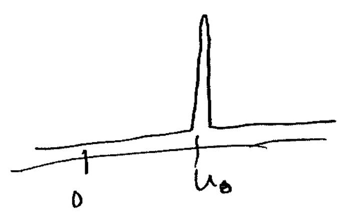
"spike" function

δ only appears inside of integrals such that

$$\int_{-\infty}^{\infty} du \; \delta(u) = 1$$

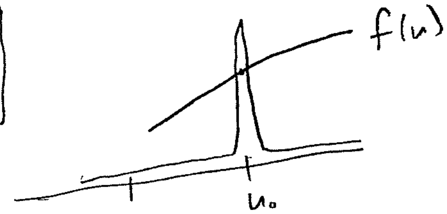
"Area under curve" = 1. must be infinitely high because it's infinitely narrow.

$$\delta(u-u_0) = \begin{cases} 0 & \text{if } u \neq u_0 \\ \infty & \text{if } u = u_0 \end{cases}$$



Also, for any function $f(u)$

$$\boxed{\int_{-\infty}^{\infty} du \; f(u)\, \delta(u-u_0) = f(u_0)}$$



All we need to know!

$\delta(u-u_0)$ forces $f(u)$ to be $f(u_0)$

Define <u>Lorentz invariant phase space</u> of n_f particles

$$(LIPS)_{n_f} = \prod_{j=1}^{n_f} \frac{d^3 p_j}{(2\pi)^3 (2E_j)} (2\pi)^4 \delta^{(4)}(\Delta p)$$

only change is that we've divided each integration measure by $2E_j$. This factor is needed to make the final state wavefunction invariant under Lorentz boosts.

At low energies this reduces to $\frac{1}{2m_j}$, a constant

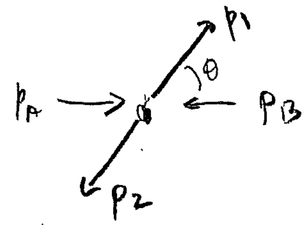
At high energies it accounts for Lorentz contraction

[You don't need to understand this.]

Scattering rate $R = \sum\limits_{\substack{\text{final}\\\text{states}}} |A|^2$

$$= \int (LIPS)_{n_f} |A|^2$$

Two-particle final state



$$\int (LIPS)_2 = \int \frac{d^3p_1}{(2\pi)^3(2E_1)} \frac{d^3p_2}{(2\pi)^3(2E_2)} (2\pi)^4 \delta^{(4)}(\Delta p)$$

$$\Delta p = p_1 + p_2 - (p_A + p_B)$$

$$\begin{cases} \Delta E = E_1 + E_2 - \underbrace{(E_A + E_B)}_{E_{cm}} \\ \Delta\vec{p} = \vec{p}_1 + \vec{p}_2 - \underbrace{\vec{p}_{cm}}_{0} \end{cases}$$

$$\therefore \delta^{(4)}(\Delta p) = \delta(E_1 + E_2 - E_{cm})\, \delta^{(3)}(\vec{p}_1 + \vec{p}_2)$$

$$\int (LIPS)_2 = \frac{1}{(2\pi)^2} \int \frac{d^3p_1}{(2E_1)(2E_2)} \underbrace{\int d^3p_2\, \delta^{(3)}(\vec{p}_1 + \vec{p}_2)}_{1,\ \text{enforcing } \vec{p}_2 = -\vec{p}_1} \delta(E_1 + E_2 - E_{cm})$$

Let $p_f = |\vec{p}_1|$

$$d^3p_1 = p_f^2\, dp_f \underbrace{\sin\theta\, d\theta\, d\phi}_{d\Omega}$$

θ = angle $\vec{p}_1$ makes wrt incident

$$\int (LIPS)_2 = \frac{1}{(2\pi)^2} \int d\Omega \frac{p_f^2\, dp_f}{(2E_1)(2E_2)} \delta(E_1 + E_2 - E_{cm})$$

Observe that p_f is fixed by the energy δ-funct

$$\sqrt{p_f^2+m_1^2} + \sqrt{p_f^2+m_2^2} = E_{cm}$$

[HW: solve for p_f]

Define $u = E_1 + E_2 - E_{cm} = \sqrt{p_f^2+m_1^2} + \sqrt{p_f^2+m_2^2} - E_{cm}$

Consider $\dfrac{du}{dp_f} = \dfrac{p_f}{\sqrt{p_f^2+m_1^2}} + \dfrac{p_f}{\sqrt{p_f^2+m_2^2}} = \dfrac{p_f}{E_1} + \dfrac{p_f}{E_2}$

$$= \frac{(E_1+E_2)\,p_f}{E_1 E_2} = \frac{E_{cm}\,p_f}{E_1 E_2}$$

$$\Rightarrow dp_f = \frac{E_1 E_2}{E_{cm}\,p_f}\,du$$

$$\int (\text{LIPS})_2 = \frac{1}{(2\pi)^2}\int d\Omega\, \frac{p_f}{4E_{cm}} \underbrace{\int du\, \delta(u)}_{1}$$

$$\boxed{\int (\text{LIPS})_2 = \frac{p_f}{(4\pi)^2 E_{cm}} \int d\Omega}$$

cross section

Recall from Rutherford scattering

$$\sigma = \frac{R}{F} = \frac{\text{scattering rate}}{\text{incident flux}} = \int d\Omega \left(\frac{d\sigma}{d\Omega}\right)$$

We just found R; what is F?

---

2 particle initial state

incident flux depends on difference in velocities $\vec{v}_A - \vec{v}_B$

$\vec{v}_A - \vec{v}_B$ is invariant under Galilean boosts, but not Lorentz!

Lorentz invariant incident flux: $F = (2E_A)(2E_B)\,|\vec{v}_A - \vec{v}_B|$

$$\Rightarrow \boxed{\sigma = \frac{\hbar^2}{(2E_A)(2E_B)\,|\vec{v}_A - \vec{v}_B|} \int (\mathrm{LIPS})_{n_f} |A|^2}$$

Griffiths calls this the golden rule for scattering

[restored correct factor of $\hbar$]
[so σ has units of m^2]
$(\hbar c) = \text{MeV fm}$

valid in any frame (CCM or fixed target) because all ingredients are Lorentz invariant

$\xrightarrow{\quad} v_A \qquad \vec{v}_B$

$\boxed{E_A E_B (v_A - v_B)}$ is Lorentz invariant under boosts in x-direction

Proof: $(m_A \cosh\gamma_A)(m_B \cosh\gamma_B)(\tanh\gamma_A - \tanh\gamma_B)$

$= m_A m_B (\cosh\gamma_A \sinh\gamma_B - \cosh\gamma_B \sinh\gamma_A)$

$= m_A m_B \sinh(\gamma_A - \gamma_B)$

$= m_B \; m_A \cosh\gamma_{AB} \tanh\gamma_{AB}$

$= m_B (E_A \underbrace{V_{AB}}_{\text{relative velocity in fixed target frame}})$ (ie $\tanh\gamma_{AB} = \tanh(\gamma_A - \gamma_B)$)

$\xrightarrow{V_{AB}} \quad \overset{\circ}{m_B}$

In FT frame $= m_B p_A$ (lab frame)

In cm frame: $p_A = -p_B$

$m_A \sinh\gamma_A = -m_B \sinh\gamma_B$

so $\underline{E_A E_B (v_A - v_B)} = m_A \sinh\gamma_A \; m_B \cosh\gamma_B - m_B \sinh\gamma_B \, m_A \cosh\gamma_A$

$= m_A \sinh\gamma_A (m_B \cosh\gamma_B + m_A \cosh\gamma_A)$

$= p(E_B + E_A)$

$= \underline{p\, E_{cm}} =$

where V_1 is the normalization volume for particle 1, and v_1 its velocity. In fact by (103)

$$V_1 = \frac{mc^2}{E_1} \qquad V_2 = \frac{mc^2}{E_2} \qquad (v_1 - v_2) = \frac{c^2 p_1}{E_1} - \frac{c^2 p_2}{E_2} \tag{151}$$

Hence the cross-section becomes

$$\sigma = \frac{w\,(mc^2)^2}{c^2\,|p_1 E_2 - p_2 E_1|} = |K|^2 \frac{(mc^2)^4}{c^2\,|E_2 p_{13} - E_1 p_{23}|\,|E_2' p_{13}' - E_1' p_{23}'|} \frac{1}{(2\pi\hbar)^2}\, dp_{11}'\, dp_{12}' \tag{152}$$

It is worth noting that the factor $p_1 E_2 - p_2 E_1$ is invariant under Lorentz transformations leaving the x_1 and x_2 components unchanged (e.g.boosts parallel to the x_3 axis.)[15] To prove this, we have to show that $p_{13}E_2 - p_{23}E_1 = \tilde{p}_{13}\tilde{E}_2 - \tilde{p}_{23}\tilde{E}_1$ (where ~ denotes the quantities after the Lorentz transformation) because we have chosen a Lorentz system in which the direction of the momentum vector is the x_3 axis. Then

$$\tilde{E} = E\cosh\theta - cp\sinh\theta$$
$$\tilde{p} = p\cosh\theta - \frac{E}{c}\sinh\theta$$

Since $E^2 = p^2c^2 + m^2c^4$, we can write

$$E = mc^2\cosh\phi \qquad pc = mc^2\sinh\phi, \qquad \text{which makes}$$
$$\tilde{E} = mc^2\cosh(\phi - \theta) \qquad \tilde{p}c = mc^2\sinh(\phi - \theta) \qquad \text{and thus}$$

$$\begin{aligned}\tilde{E}_2\tilde{p}_{13} - \tilde{E}_1\tilde{p}_{23} &= m^2c^3\left\{\cosh(\phi_2 - \theta)\sinh(\phi_1 - \theta) - \cosh(\phi_1 - \theta)\sinh(\phi_2 - \theta)\right\} \\ &= m^2c^3\sinh(\phi_1 - \phi_2)\end{aligned}$$

independently of θ. Hence we see that σ is invariant under Lorentz transformations parallel to the x_3 axis.

Results for Møller Scattering

One electron initially at rest, the other initially with energy $E = \gamma mc^2$;

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$$

$$\begin{aligned}\text{scattering angle} &= \theta \text{ in the lab system} \\ &= \theta^* \text{ in the center-of-mass system}\end{aligned}$$

Then the differential cross-section is (Mott and Massey, *Theory of Atomic Collisions*, 2nd ed., p. 368)

$$2\pi\sigma(\theta)\,d\theta = 4\pi\left(\frac{e^2}{mv^2}\right)^2\left(\frac{\gamma+1}{\gamma^2}\right)dx\left\{\frac{4}{(1-x^2)^2} - \frac{3}{1-x^2} + \left(\frac{\gamma-1}{2\gamma}\right)^2\left(1 + \frac{4}{1-x^2}\right)\right\} \tag{153}$$

with

$$x = \cos\theta^* = \frac{2 - (\gamma+3)\sin^2\theta}{2 + (\gamma-1)\sin^2\theta}$$

Without spin you get simply

$$4\pi\left(\frac{e^2}{mv^2}\right)^2\left(\frac{\gamma+1}{\gamma^2}\right)dx\left\{\frac{4}{(1-x^2)^2} - \frac{3}{1-x^2}\right\}$$

Effect of spin is a measurable *increase* of scattering over the Mott formula. Effect of exchange is roughly the $\frac{3}{1-x^2}$ term. Positron-electron scattering is very similar. Only the exchange effect is different because of annihilation possibility.

[we'll almost always be in cm frame]

<u>initial state in cm frame</u>

$$\vec{p}_A \longrightarrow \qquad\qquad \longleftarrow \vec{p}_B$$

$$\vec{p}_{cm} = \vec{p}_A + \vec{p}_B = 0$$
$$E_{cm} = E_A + E_B$$

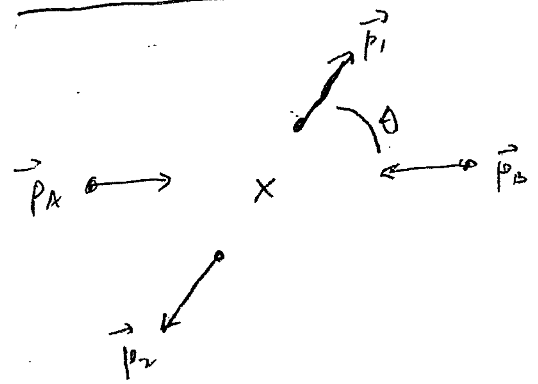
Consider $(2E_A)(2E_B)(\vec{v}_A - \vec{v}_B) = 4E_B\,(\underbrace{E_A \vec{v}_A}_{\vec{p}_A}) - 4E_A\,(\underbrace{E_B \vec{v}_B}_{\vec{p}_B})$

$$= 4(E_B + E_A)\,\vec{p}_A$$
$$= 4E_{cm}\,\vec{p}_A$$

initial flux: $F = 4E_{cm}\,p_i$ where $p_i = |\vec{p}_A| = |\vec{p}_B|$

[in fixed target frame $F = (2E_A)(2E_B)|\vec{v}_A - \vec{v}_B| = 4m_B E_A v_A = 4m_B p_A$]

2 → 2 scattering in CM frame



$$|\vec{p}_A| = |\vec{p}_B| = p_i$$

$$|\vec{p}_1| = |\vec{p}_2| = p_f$$

If collision is elastic, then $p_f = p_i$

$$\sigma = \frac{\hbar^2}{(2E_A)(2E_B)|\vec{v}_A - \vec{v}_B|} \int (LIPS)_2 \, |A|^2$$

$$= \left(\frac{\hbar^2}{4E_{cm}\, p_i}\right) \frac{p_f}{(4\pi)^2 E_{cm}} \int d\Omega \, |A|^2$$

$$= \left(\frac{\hbar}{8E_{cm}\, p_i}\right)^2 \frac{p_f}{p_i} \int d\Omega \, |A|^2$$

Since $\sigma = \int d\Omega \left(\frac{d\sigma}{d\Omega}\right)$ we have

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \left(\frac{\hbar}{8\pi E_{cm}}\right)^2 \frac{p_f}{p_i} |A|^2}$$

valid for any 2→2 scattering in cm frame

[We'll use it for Rutherford, $e^- e^+ \to \mu^- \mu^+$, $\bar{\nu} p \to n e^+$]

Dimensional analysis

$\frac{d\sigma}{d\Omega} \sim (\text{length})^2$

$\hbar \sim (\text{length})(\text{energy})$

$\Rightarrow |A| \sim$ dimensionless for any 2 → 2 scattering connection 7 nonrelativistic scattering formula:

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \left(\frac{\hbar}{8\pi E_{cm}}\right)^2 \frac{p_f}{p_i} |A|^2$$

Let $|A|^2 = (2E_A)(2E_B)(2E_1)(2E_2)\,|\tilde{V}|^2$ $\quad \tilde{V}$ = nonrel. Fourier tx

(Since $\dfrac{2E_1\,2E_2}{4E_{cm}} = \dfrac{p_f}{|v_1 - v_2|}$ and $\dfrac{2E_A\,2E_B}{4E_{cm}} = \dfrac{p_i}{|v_A - v_B|}$

$$\left[\text{Proof: } \frac{E_1E_2(v_1-v_2)}{E_{cm}} = \frac{E_2(E_1v_1) - E_1(E_2v_2)}{E_{cm}} = \frac{E_2p_1 - E_1p_2}{E_{cm}} = p_f\right]$$

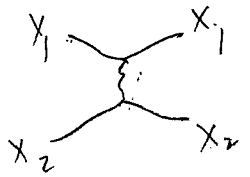
$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \left(\frac{\hbar}{2\pi}\right)^2 \frac{p_f}{p_i}\,\frac{p_f}{(v_1-v_2)}\,\frac{p_i}{(v_A-v_B)}\,|\vec{V}|^2$$

$$= \left(\frac{\hbar}{2\pi}\right)^2 \frac{p_f^2}{(v_1-v_2)(v_A-v_B)} |\tilde{V}|^2 \quad \leftarrow \text{nonrel. formula}$$

<u>Rutherford scattering</u> $\quad X_1 X_2 \to X_1 X_2$

Elastic, so $p_f = p_i = |\vec{p}|$

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \left(\frac{\hbar}{8\pi E_{cm}}\right)^2 |A|^2$$



$$A = \frac{(Z_1 e)(Z_2 e)}{p_\gamma^2} \begin{pmatrix}\text{spin}\\ \text{stuff}\end{pmatrix}$$

$$p_\gamma^2 = -4|\vec{p}|^2 \sin^2\frac{\theta}{2}$$

$$e^2 = 4\pi\alpha$$

Since A dimensionless, $\begin{pmatrix}\text{spin}\\ \text{stuff}\end{pmatrix}$ has dimns of $(\text{energy})^2$

Treating all particles democratically, let's write

"matrix element"

$$\begin{pmatrix}\text{spin}\\ \text{stuff}\end{pmatrix} = \sqrt{(2E_A)(2E_B)(2E_1)(2E_2)}\; M$$

Just a parametrization, w/ M dimensionless.

In this case $\begin{pmatrix}\text{spin}\\ \text{stuff}\end{pmatrix} = 4E_1E_2 M$ (Since $E_A = E_1$, $E_B = E_2$)

$$A = -\frac{4\pi\alpha\, Z_1 Z_2 E_1 E_2}{|\vec{p}|^2 \sin^2\frac{\theta}{2}} M$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \left(\frac{\hbar Z_1 Z_2 \alpha}{2|\vec{p}|^2 \sin^2\frac{\theta}{2}} \frac{E_1 E_2}{(E_1+E_2)}\right)^2 |M|^2$$

$|M|$ can depend on energies, angles + spins

but in the nonrelativistic limit, $|M| \to 1$

[not obvious, but true]

Also in nonrel limit

$p \to m_1 v_1$

$E_1 \to m_1$

$E_2 \to m_2$

nonrel: $\left(\frac{d\sigma}{d\Omega}\right)_{cm} \to \left(\frac{\hbar Z_1 Z_2 \alpha \quad m_1 m_2}{2 m_1^2 v_1^2 \sin^2\frac{\theta}{2} \, (m_1+m_2)}\right)^2$

Furthermore if $m_1 \ll m_2$ (eg α-particle on heavy nucleus)

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} \to \left(\frac{\hbar Z_1 Z_2 \alpha}{2 m_1 v_1^2 \sin^2\frac{\theta}{2}}\right)^2$$

exact agreement w/ classical Rutherford result

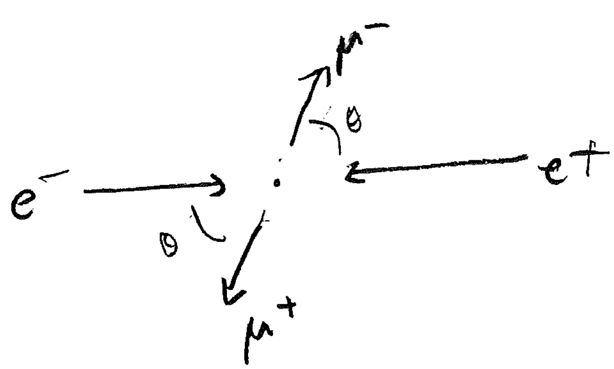
[$\exists$ corrections at relativistic speeds]

$$\left[\ \sigma(\theta > \theta_0) = \pi \left(\frac{\hbar Z_1 Z_2 \alpha}{2 T_1}\right)^2 \cot^2\left(\frac{\theta_0}{2}\right) \right]$$

[Another 2 → 2 process]

$e^- e^+ \longrightarrow \mu^- \mu^+$ [not just scattering but involves creation of new particles]

In cm frame



outgoing $\mu^-\mu^+$ are slower because some of e^-e^+ kinetic energy converted to rest energy of $\mu^-\mu^+$

(inelastic) $p_f \neq p_i$

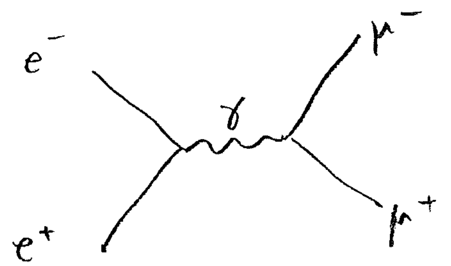
Incoming e^-, e^+ have equal & opposite momenta & ∴ same energy E

process can occur iff $E_{cm} = 2E \geq 2m_\mu$

ie $E \geq m_\mu \approx 106$ MeV

[HW: compute 4-momenta of each particle, fcn of E and θ]

Lowest order Feynman diagram



HW: compute the 4-momentum of the virtual photon

!!! calculate A.

Assume $\left(\begin{matrix}\text{spin}\\ \text{stuff}\end{matrix}\right) = \sqrt{(2E_A)(2E_B)(2E_1)(2E_2)}\; M$

calculate $\left(\frac{d\sigma}{d\Omega}\right)_{cm}$

[check: your answer should vanish when $E = m_\mu$.]

Let $|M|^2 = 1 + \cos^2\theta + \left(\frac{m_\mu}{E}\right)^2 (1 - \cos^2\theta)$ $\quad (m_e \approx 0)$

compute σ.

[Check: if $E = 1$ GeV, then $\sigma = 22$ nb.]

55. **25.0?.** 23.07. *Amplitude for the process $e^-e^+ \to \mu^-\mu^+$.*
Consider the process $e^-e^+ \to \mu^-\mu^+$ in the CM frame. The initial state particles e^- and e^+ are moving in the $+z$ and $-z$ directions respectively, and each has energy E. The final state particles μ^- and μ^+ are moving in the xz plane, with the direction of μ^- making an angle θ with respect to the direction of e^-.

(a) Write the four-momenta of each of the particles involved in terms of E and θ.

(b) Using the lowest energy Feynman diagram contributing to this process, compute the four-momentum of the virtual photon.

(c) Compute the amplitude A, assuming that

$$(\text{spin stuff}) = \sqrt{(2E_A)(2E_B)(2E_1)(2E_2)}M\,.$$

(d) Compute $(d\sigma/d\Omega)_{\text{cm}}$, the differential cross section in the CM frame.
[Check: your answer should vanish as $E \to m_\mu$, which is the threshold for this process.]

(e) If one neglects the mass of the electron, it can be shown that

$$|M|^2 = 1 + \cos^2\theta + \left(\frac{m_\mu}{E}\right)^2 (1 - \cos^2\theta)$$

Use this to compute the cross section σ. Restoring any necessary factors of c, evaluate σ numerically for $E = 1$ GeV in terms of nb.
[Check: For $E = 0.5$ GeV, you should get 87 nb.]

56. **EXAM?** New in 2025. *Kinematics of the process $e^-e^+ \to \gamma\gamma$.*
Consider pair annihilation to two photons, $e^-e^+ \to \gamma\gamma$, in the CM frame. The initial state particles e^- and e^+ are moving in the $+z$ and $-z$ directions respectively, and each has energy E. The final state photons are moving in the xz plane, with one of the photons making an angle θ with respect to the direction of e^-.

(a) Write the four-momenta of each of the particles involved in terms of E and θ.

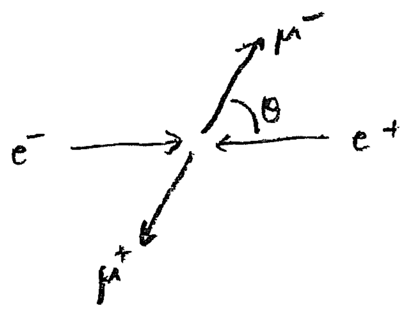
(b) There are two Feynman diagrams contributing to this process. For each one, compute the four-momentum of the virtual electron.

(c) Compute the propagator $1/(p_e^2 - m_e^2)$ for each diagram, simplifying as much as possible.

Solution

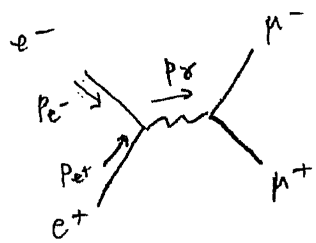
$$\boxed{e^- e^+ \to \mu^- \mu^+}$$

cf Griffiths (2e) p. 277
Peskin (Concepts) p. 122



(a)
$$p_{e^-} = (E,\ 0,\ 0,\ p_e)$$
$$p_{e^+} = (E,\ 0,\ 0,\ -p_e)$$
where $p_e = \sqrt{E^2 - m_e^2}$

$$p_{\mu^-} = (E,\ p_\mu \sin\theta,\ 0,\ p_\mu \cos\theta)$$
$$p_{\mu^+} = (E,\ -p_\mu \sin\theta,\ 0,\ -p_\mu \cos\theta)$$
where $p_\mu = \sqrt{E^2 - m_\mu^2}$



(b) $p_\gamma = p_{e^-} + p_{e^+} = \boxed{(2E,\ 0,\ 0,\ 0)}$

(c) (Spin stuff) $= \sqrt{(2E_A)(2E_B)(2E_1)(2E_2)}\, M = (2E)^2 M$

$$p_\gamma^2 = (2E)^2$$

$$\Rightarrow A = \frac{e^2 (2E)^2 M}{(2E)^2} = e^2 M = \boxed{4\pi\alpha M}$$

(d) $$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \left(\frac{\hbar}{8\pi E_{cm}}\right)^2 \frac{p_f}{p_i} |A|^2 = \left(\frac{\hbar}{16\pi E}\right)^2 \frac{\sqrt{E^2 - m_\mu^2}}{\sqrt{E^2 - m_e^2}} (4\pi\alpha M)^2$$

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \left(\frac{\hbar\alpha}{4E}\right)^2 \sqrt{\frac{E^2 - m_\mu^2}{E^2 - m_e^2}}\, |M|^2}$$

[check: vanishes if $E = m_\mu$]

(e) Let $|M|^2 = 1 + \cos^2\theta + \left(\frac{m_\mu}{E}\right)^2 (1 - \cos^2\theta)$ if neglect m_e

Use $\int d\Omega \cos^2\theta = \int 2\pi \sin\theta\, d\theta \cos^2\theta = 4\pi\left(\frac{1}{3}\right)$

$$\sigma = \left(\frac{\hbar\alpha}{4E}\right)^2 \sqrt{1 - \left(\frac{m_\mu}{E}\right)^2}\, 4\pi \left[1 + \frac{1}{3} + \left(\frac{m_\mu}{E}\right)^2 \left(1 - \frac{1}{3}\right)\right]$$

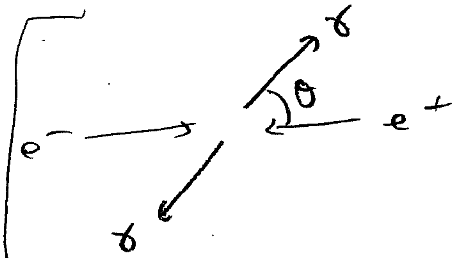
$$\boxed{\sigma = \frac{\pi}{3}\left(\frac{\hbar\alpha}{E}\right)^2 \sqrt{1 - \left(\frac{m_\mu}{E}\right)^2} \left[1 + \frac{1}{2}\left(\frac{m_\mu}{E}\right)^2\right]} \approx \frac{\pi}{3}\left(\frac{\hbar\alpha}{E}\right)^2 \left(1 - \underbrace{\frac{3}{8}\left(\frac{m_\mu}{E}\right)^4}_{< 10^{-3} \text{ for } E = 500\text{ MeV}} + \ldots\right)$$

Restore c: $\sigma = \frac{\pi}{3}\left(\frac{\hbar c \alpha}{E}\right)^2 = \frac{\pi}{3}\left(\frac{197.327 \text{ MeV fm}}{137.036\ (1000 \text{ MeV})}\right)^2 = 2.17\text{E-}6 \text{ fm}^2 = \boxed{21.7 \text{ nb}}$ if $E = 1$ GeV

$\boxed{87 \text{ nb}}$ if $E = 500$ MeV

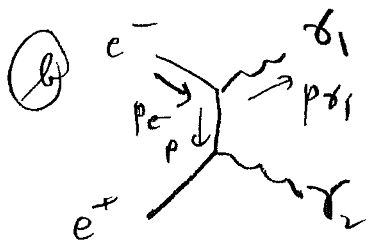
[Solution to exam problem]

$\boxed{\text{Kinematics of process } e^-e^+ \to \gamma\gamma \text{ in cm frame}}$

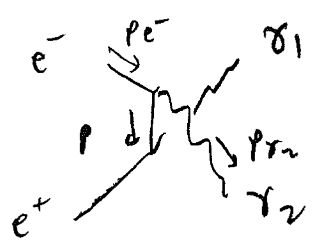


(a)
$$p_{e^-} = (E,\ 0,\ 0;\ \sqrt{E^2 - m_e^2})$$
$$p_{e^+} = (E,\ 0,\ 0,\ -\sqrt{E^2 - m_e^2})$$
$$p_{\gamma 1} = (E,\ E\sin\theta,\ 0,\ E\cos\theta)$$
$$p_{\gamma 2} = (E,\ -E\sin\theta,\ 0,\ -E\cos\theta)$$

(b)



$$p = p_{e^-} - p_{\gamma 1}$$
$$= (0,\ -E\sin\theta,\ 0,\ \sqrt{E^2 - m_e^2} - E\cos\theta)$$



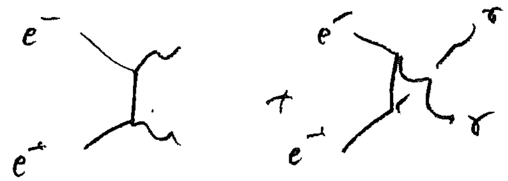
$$p = p_{e^-} - p_{\gamma 2}$$
$$= (0,\ E\sin\theta,\ 0,\ \sqrt{E^2 - m_e^2} + E\cos\theta)$$

(c)
$$p^2 - m_e^2 = -(E^2\sin^2\theta) - (E^2 - m_e^2 + E^2\cos^2\theta - 2E\sqrt{E^2 - m_e^2}\cos\theta) - m_e^2$$
$$= 2E\left(\sqrt{E^2 - m_e^2}\cos\theta - E\right) < 0$$

The other one is
$$p^2 - m_e^2 = 2E\left(-\sqrt{E^2 - m_e^2}\cos\theta - E\right) < 0$$

3-21-25

$$\boxed{e^- e^+ \to \gamma\gamma}$$



For low energy e^-, e^+ → $A = 4e^2 = 16\pi\alpha$

(Griffiths (2e) 7.163, p. 260)

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi E_{cm}}\right)^2 \frac{p_f}{p_i} |A|^2 \cdot \frac{1}{2} \leftarrow \text{(symmetry factor)}$$

---

slow moving initial state: $p_i = m_e v_i = \frac{1}{2} m_e v$ $\left(\xrightarrow{v_i} \xleftarrow{v_i}\right)$

$p_f = m_e c$

$E_{cm} = 2 m_e c^2$

relative velocity

$$\sigma = 4\pi \left(\frac{\hbar c (16\pi)\alpha}{8\pi (2 m_e c^2)}\right)^2 \frac{2c}{v} \cdot \frac{1}{2}$$

$$\boxed{\sigma = \left(\frac{\hbar\alpha}{m_e c}\right)^2 4\pi\left(\frac{c}{v}\right)} \leftarrow \text{eq (7.168) of Griffiths (2e)}$$

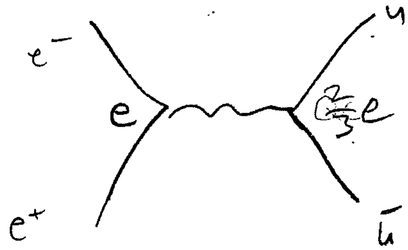
$$= r_e^2 \, 4\pi \left(\frac{c}{v}\right)$$

↓

use to compute positronium lifetime

If $E \gtrsim 2m_\mu \approx 210\,\text{MeV}$ then $e^-e^+ \to \mu^-\mu^+$ possible

Also possible $e^-e^+ \to u\bar{u}$



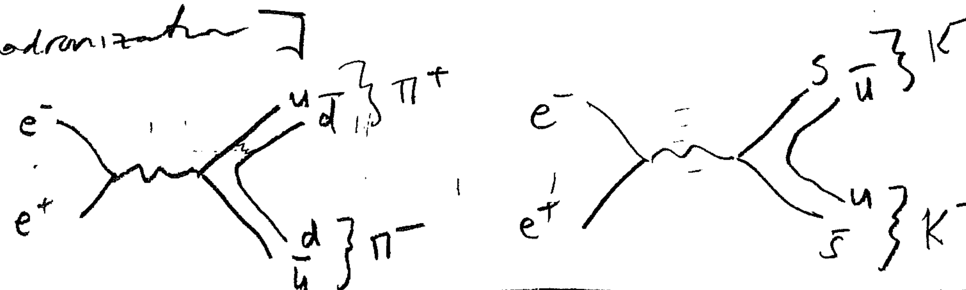
$$\frac{A(e^-e^+ \to u\bar{u})}{A(e^-e^+ \to \mu^-\mu^+)} = \frac{\left(\frac{2}{3}e\right)e}{e^2} = \frac{2}{3}$$

$$\frac{\sigma(e^-e^+ \to u\bar{u})}{\sigma(e^-e^+ \to \mu^-\mu^+)} = \frac{4}{9}$$

But also $\to d\bar{d}$ + (if $E \gtrsim 1\,\text{GeV}$) $s\bar{s}$

$$\frac{\sigma(e^+e^- \to u\bar{u} \text{ or } d\bar{d} \text{ or } s\bar{s})}{\sigma(e^-e^+ \to \mu^-\mu^+)} = \frac{4}{9} + \frac{1}{9} + \frac{1}{9} = \frac{2}{3}$$

[hadronization]



→ explain

At $E \gtrsim 1\,\text{GeV}$,

$$\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^-e^+ \to \mu^-\mu^+)} = \frac{2}{3}$$

But no! much larger (see plot)

Quarks come in $N_c = 3$ colors so actually

$$\frac{\sigma(e^+e^- \to \text{had})}{\sigma(e^-e^+ \to \mu^-\mu^+)} = \frac{2}{3}N_c = 2$$

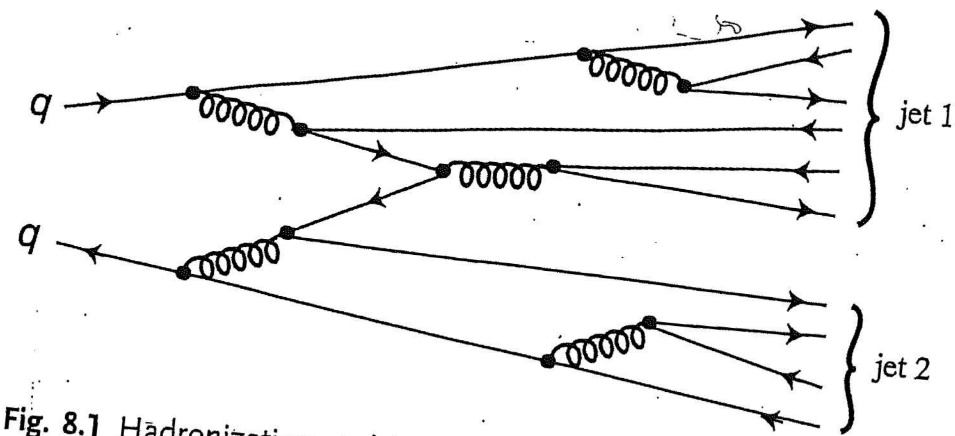
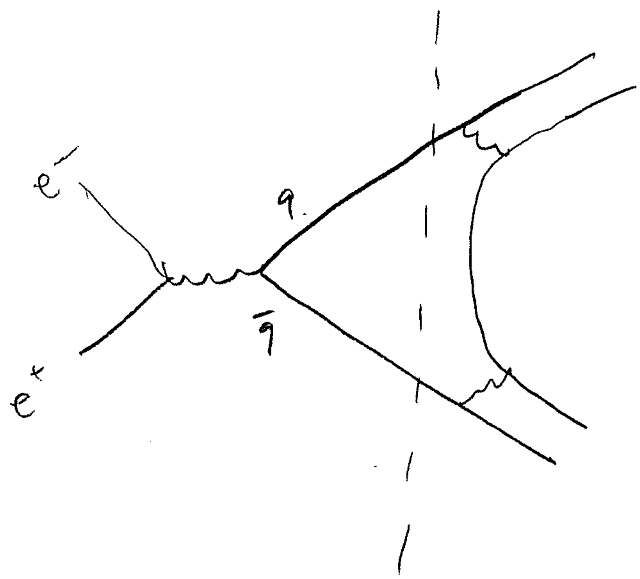


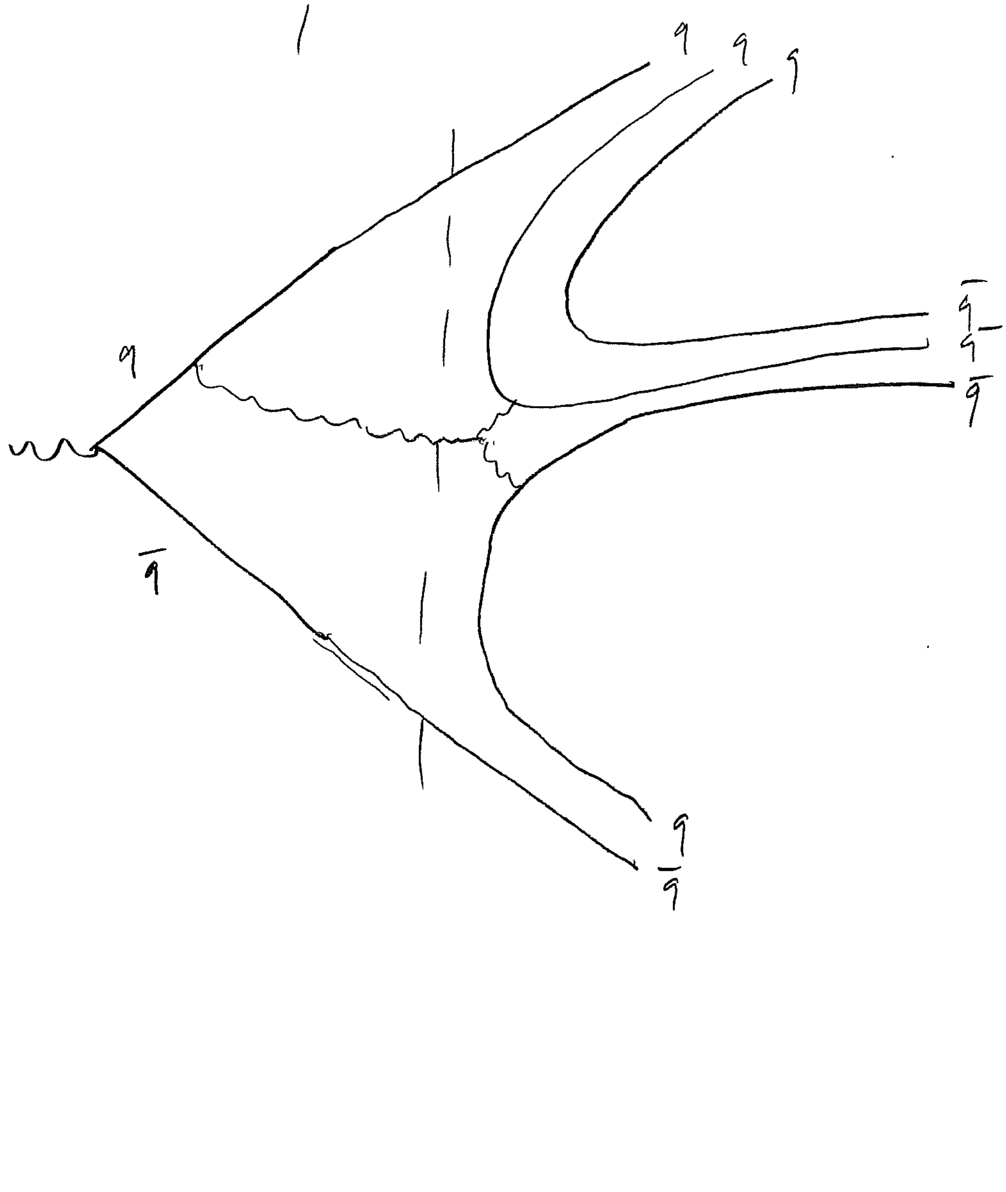
Fig. 8.1 Hadronization and jet formation.

Hadronization

~~QCD~~-6



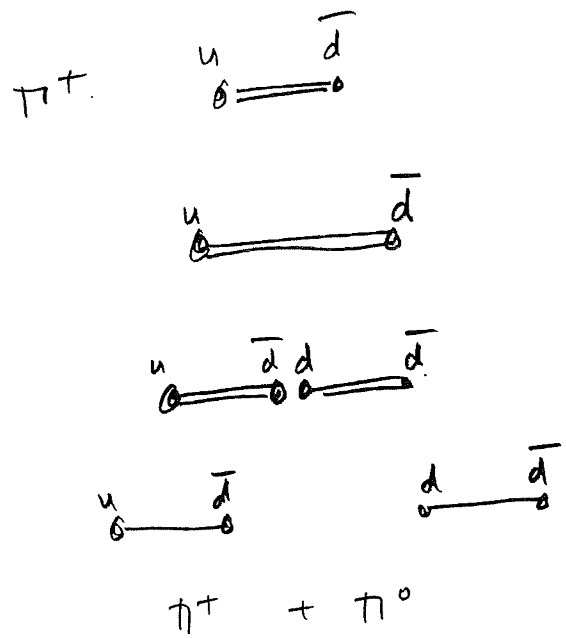
[or show picture in Griffiths (20)]



~~QCD 7~~

Quark confinement

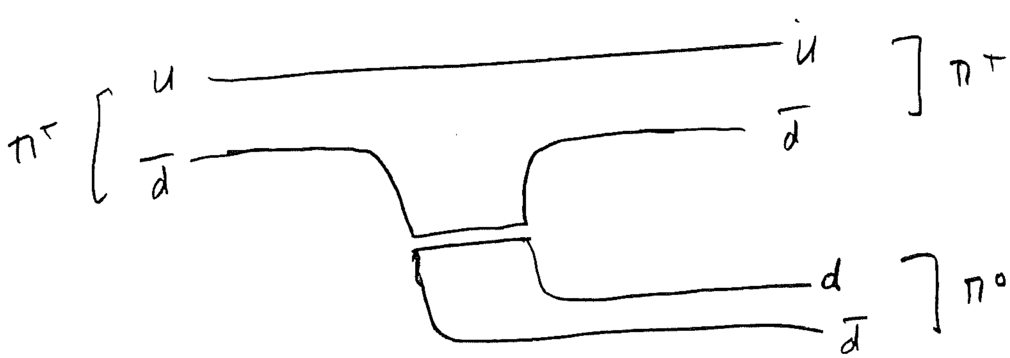
Try to pull two quarks apart



[increased energy]

$F_0 \approx 16 \text{ tons} = 1.5\text{E}5 \text{ N}$

$= 10^{18} \frac{\text{MeV}}{\text{m}} = \frac{1000 \text{ MeV}}{\text{fm}}$



"hadronization"

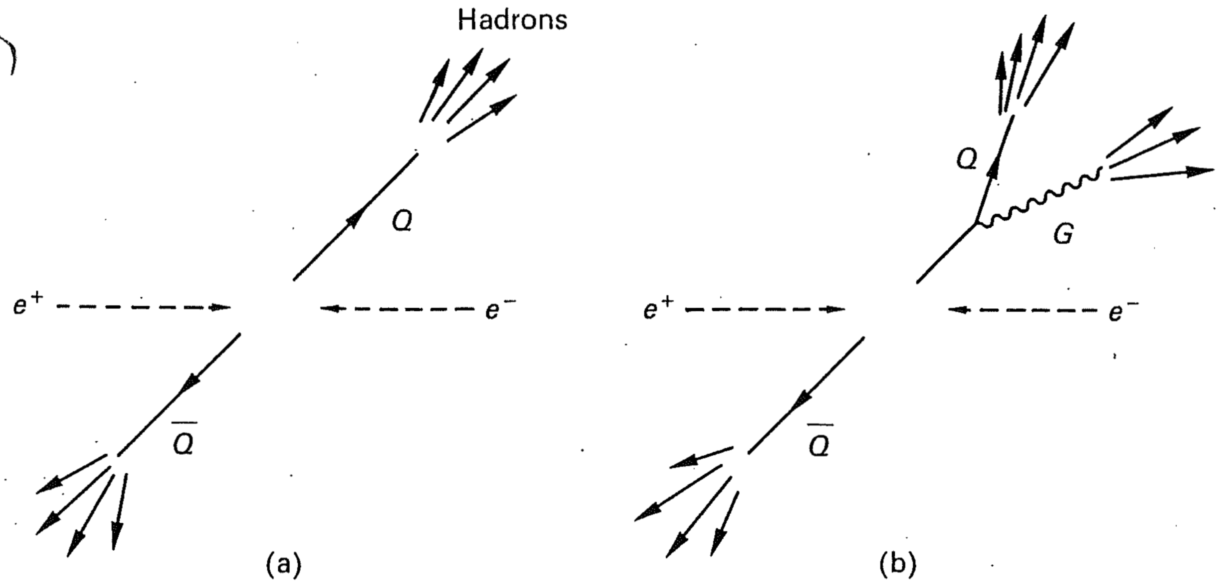


Fig. 8.27

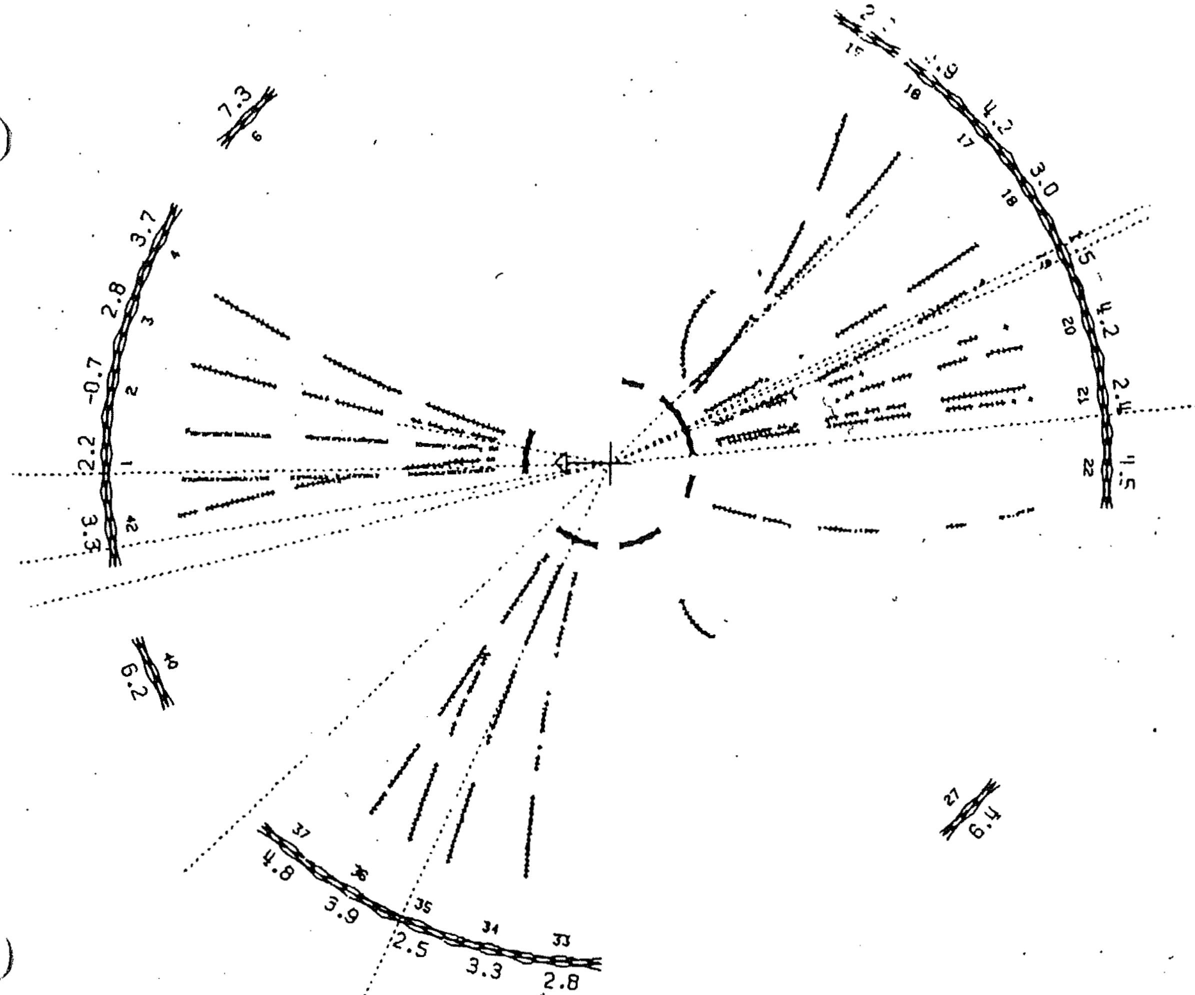


Fig. 8.28 Example of a three-jet event observed in the JADE detector at the PETRA e^+e^- collider

Perkins

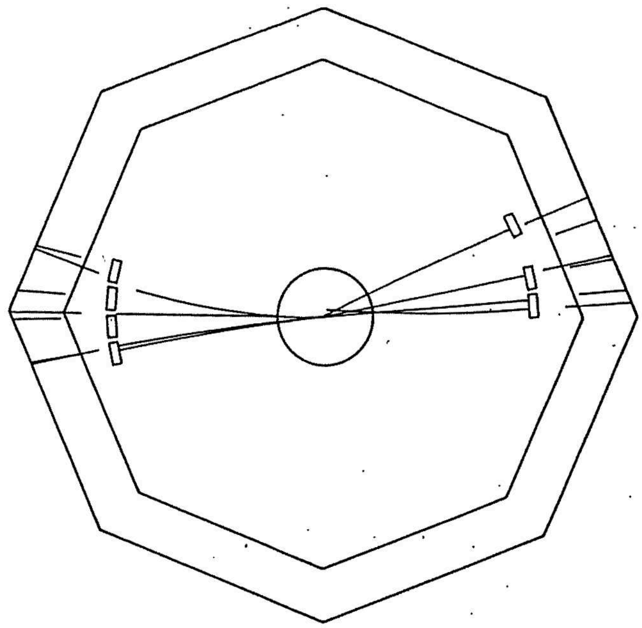


Figure 8.1 A typical two-jet event. (Courtesy J. Dorfan, SLAC.)

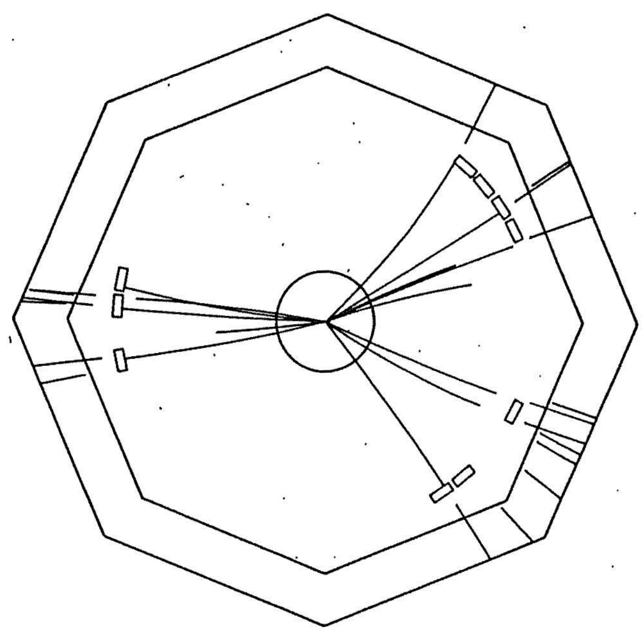
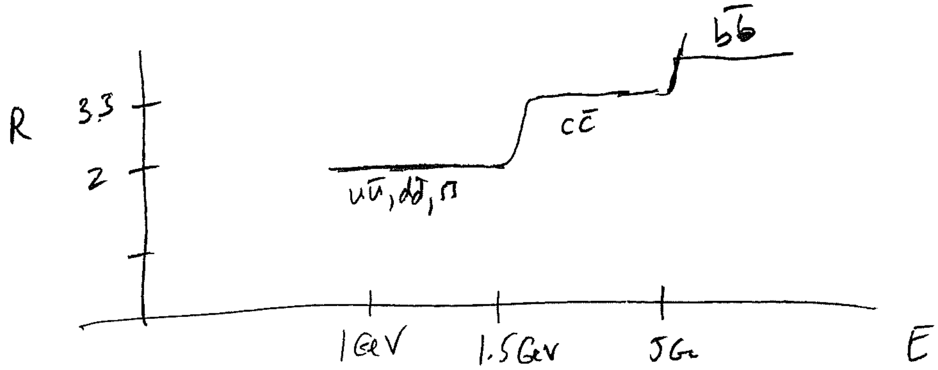


Figure 8.2 A three-jet event. (Courtesy J. Dorfan, SLAC.)

Griffiths

[R = ratio, not rate!]

$$R = \frac{\sigma(e^-e^+ \to \text{hadrons})}{\sigma(e^-e^+ \to \mu^-\mu^+)}$$ depends on E



$$R = \begin{cases} E \lesssim 1.5\text{ GeV} & \left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}\right)N_c = \frac{6}{9}N_c = 2 \\ 1.5\text{ GeV} \sim 5\text{ GeV} & \left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}\right)N_c = \frac{10}{9}N_c = 3\frac{1}{3} \\ \cancel{5\text{GeV}} > 5\text{GeV} & \left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}+\frac{1}{9}\right)N_c = \frac{11}{9}N_c = 3\frac{2}{3} \end{cases}$$

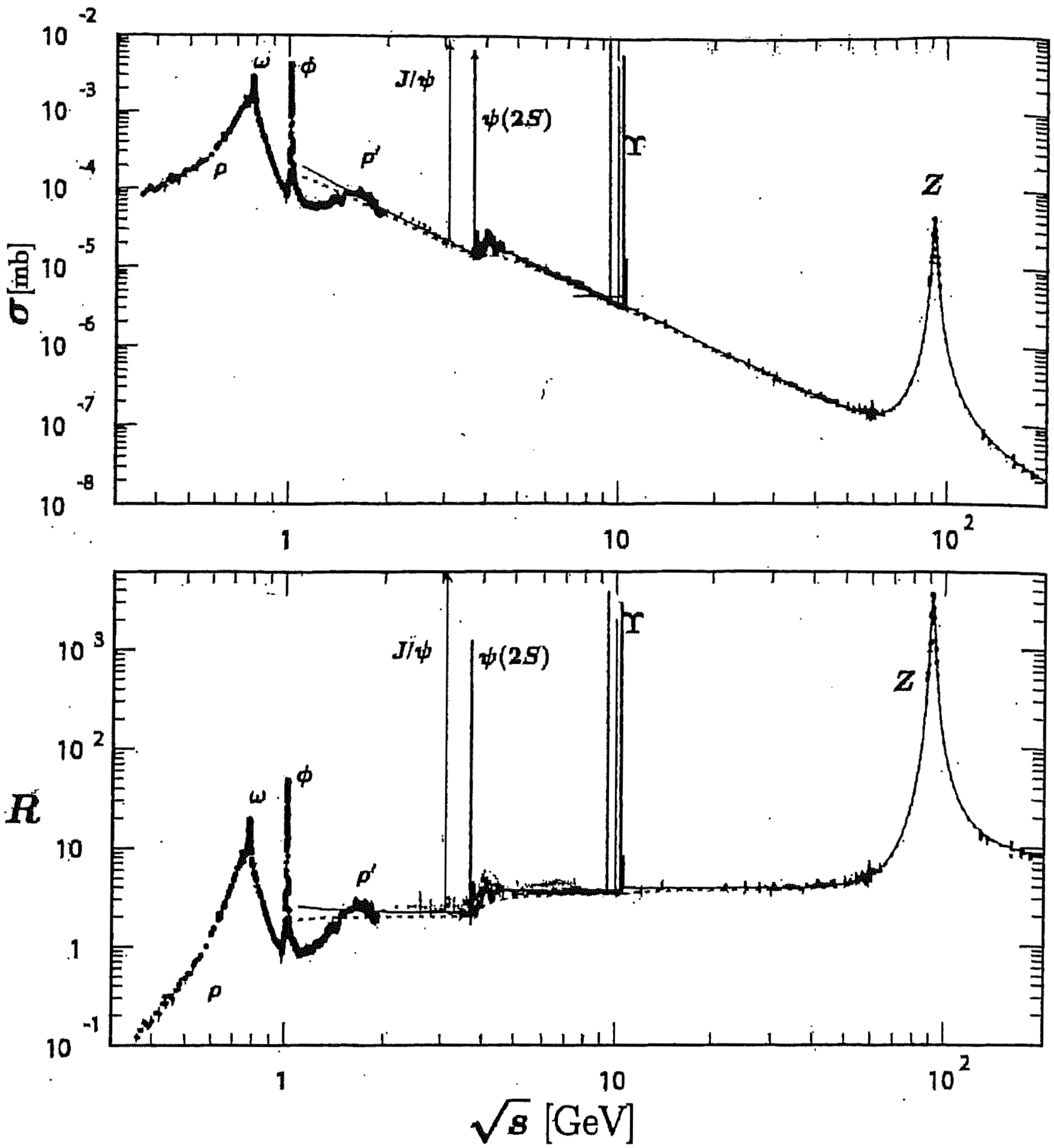


Fig. 8.1 Measurements of the total cross section for e^+e^- annihilation to hadrons as a function of energy, compiled in (Patrignani 2016). The lower figure shows the ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$. The green dotted curve is the prediction (8.56). The vertical red lines show the ψ and Υ resonances. The horizontal red curve is the prediction (11.72).

Peskin, Concepts in EP physics

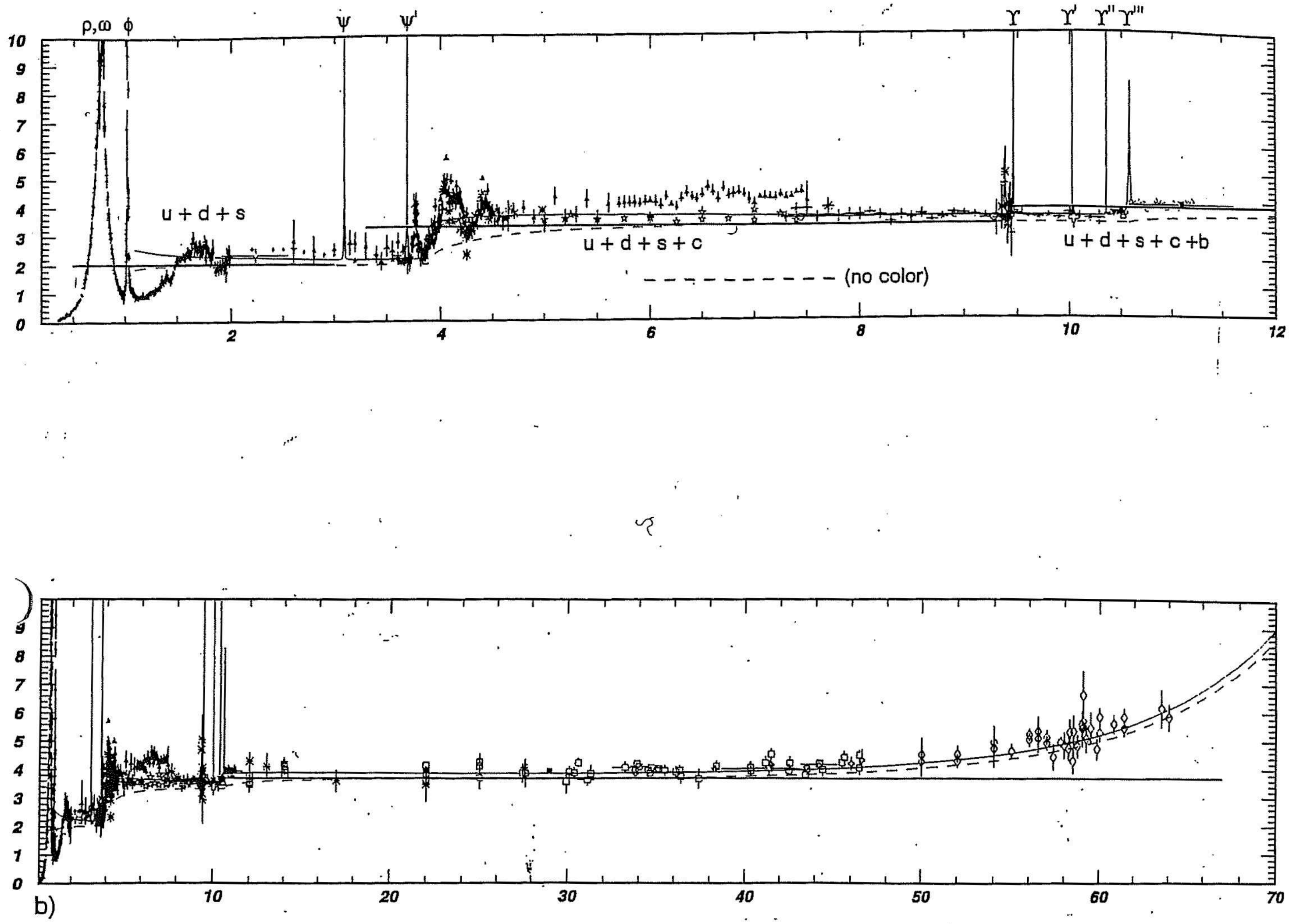


Fig. 8.4 Graph of R, based on experimental data, plotted against total energy ($2E$), in GeV. (Courtesy of the COMPAS (IHEP, Russia) and HEPDATA (Durham, UK) groups, with corrections by P. Janot (CERN) and M. Schmitt (Northwestern).)

Griffiths

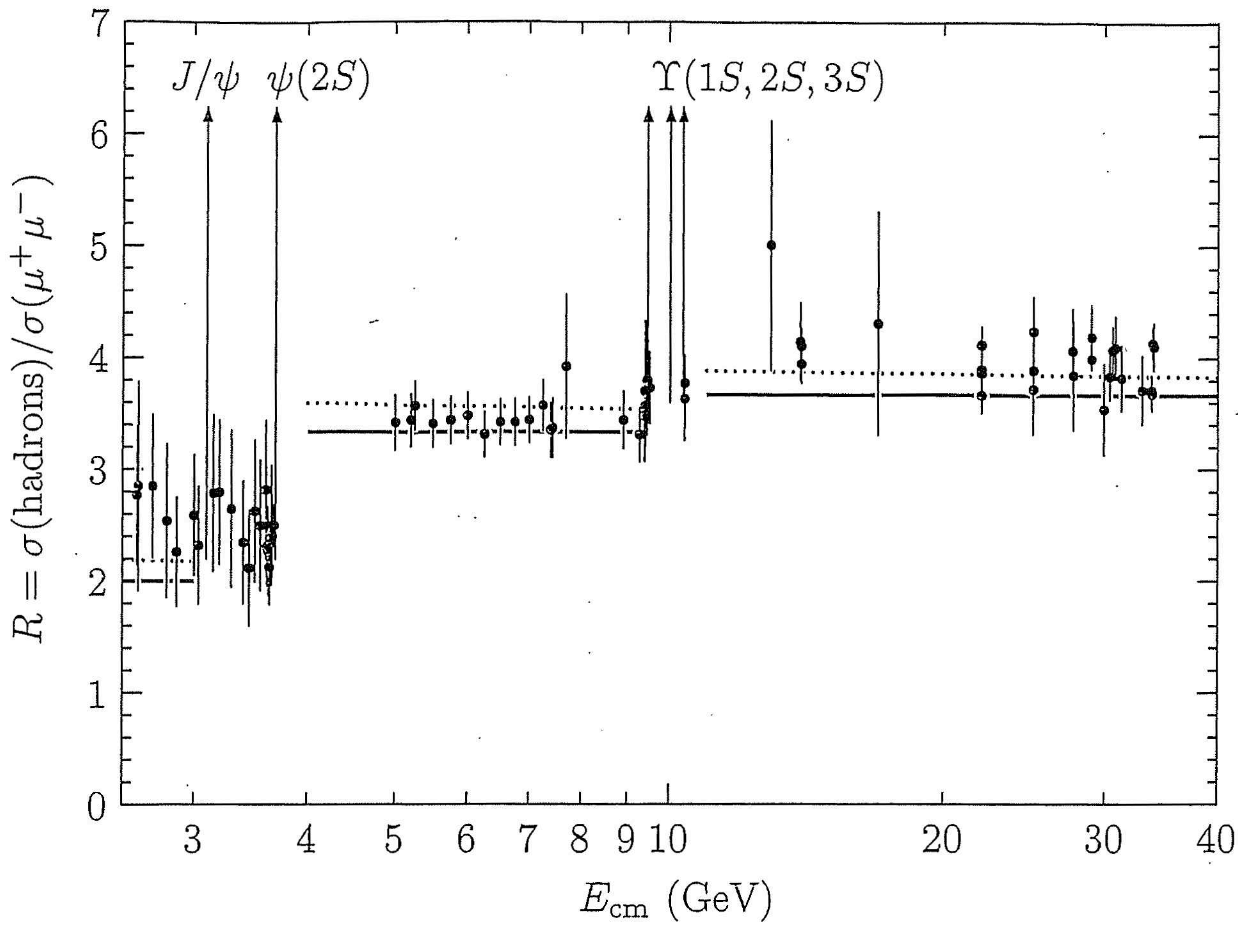


Figure 5.3. Experimental measurements of the total cross section for the reaction $e^+e^- \to$ hadrons, from the data compilation of M. Swartz, *Phys. Rev.* $\mathbb{D}$ (to appear). Complete references to the various experiments are given there. The measurements are compared to theoretical predictions from Quantum Chromodynamics, as explained in the text. The solid line is the simple prediction (5.16).

Peskin + Schroeder

cf 2nd ed of Griffiths