Recall
Radiation

α-decay: emission of ^4He nucleus

β-decay: emission of e^-, or e^+

γ-decay: emission of γ

$$X \longrightarrow Y + \alpha$$

$$^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

[charge & baryon # cons]

$$^{A+4}_{Z+2}X \longrightarrow {}^{A}_{Z}Y + {}^{4}_{2}He$$

$$Q = m_{nucl}(X) - m_{nucl}(Y) - m_{nucl}(^4He)$$

$$= [(A+4)u + \Delta(X) - (Z+2)m_e] - [Au + \Delta(Y) - Zm_e] - (4u + \Delta(^4He) - 2m_e)$$

$$= \Delta(X) - \Delta(Y) - \Delta(^4He)$$

[makes sense, since $\frac{B}{A} \uparrow$ as $A \downarrow$, and ^4He has large binding energy]

If $Q > 0$, α-decay can occur

Typically $Q \approx 5$ MeV for heavy unstable nuclei

[see next page for ^{238}U]

$Q = T_Y + T_{He}$ [for X at rest]

Because $m_{He} \ll m_Y \Rightarrow T_{He} \gg T_Y$

so α particle gets most of the released energy.

In air, an α-particle travels a few cm.

$\Rightarrow$ [picture]

$\uparrow T \Rightarrow \uparrow$ range

[refer to curve of binding energies]

$\alpha - 2.1$

$\Delta(^{12}C) = 0$ [typical binding]

$\Delta(^{4}He) = 2.4$ MeV [less bound than ^{12}C but pretty bound for such a light nucleus (magic numbers)]

$\Delta(^{56}_{26}Fe) = -60.6$ MeV [tightly bound]

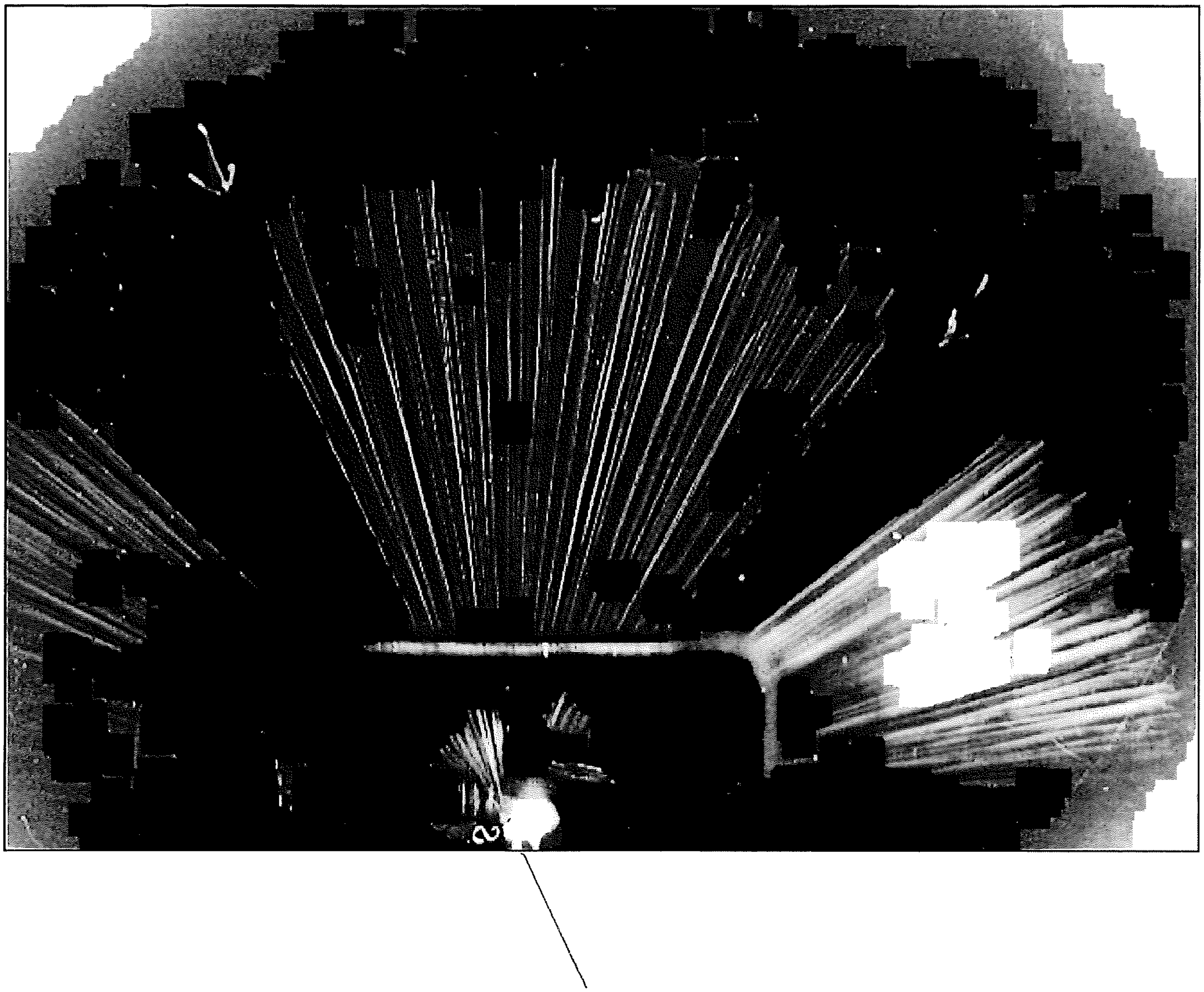
$\Delta(^{238}_{92}U) = 47.3$ MeV [not too tight]

$\Delta(^{234}_{90}Th) = 40.6$ MeV

difference of ~ 7 MeV,

$Q = \Delta(^{238}U) - \Delta(^{234}Th) - \Delta(^{4}He) = 4.3$ MeV

[Recall $\Delta(p) = 7.3$
$\Delta(n) = 8.1$]



Because all α particles get same kinetic energy they all travel ~ the same distance in air, a few cm

but different nuclei have diff ranges

Asimov: neutrino
Bragg 1904

^{232}Th	2.8 cm
^{226}Ra	3.3 cm
^{252}Po	8.6 cm

(see picture →)

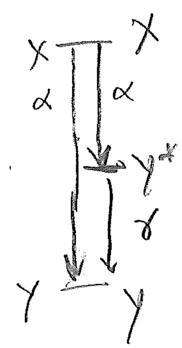
Not strictly true e.g. see ^{227}Th decay

∃ maximum K_α = 6.04 MeV

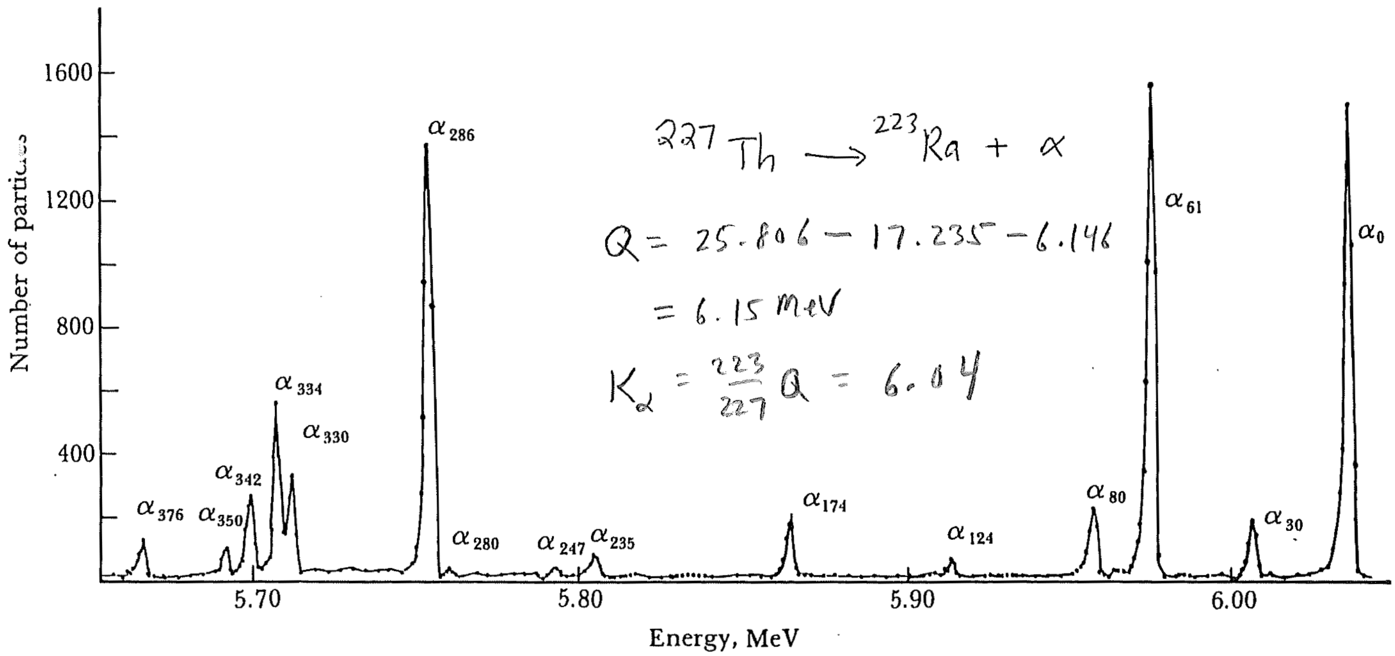
but also smaller K_α, due to decay into an excited state of ^{223}Ra.

These are followed by emission of γ

$Q = m_{nucl}(X) - m_{nucl}(Y^*) - m_{nucl}(\alpha)$



Tipler, p. 390-1



$^{227}Th \longrightarrow {}^{223}Ra + \alpha$

$Q = 25.806 - 17.235 - 6.146$

$= 6.15$ MeV

$K_\alpha = \frac{223}{227} Q = 6.04$

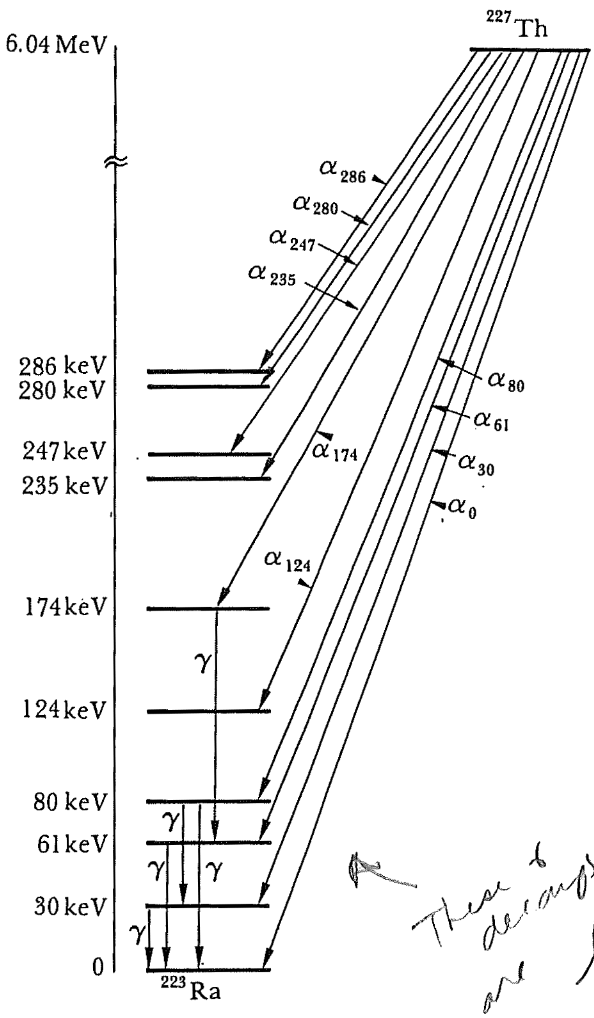
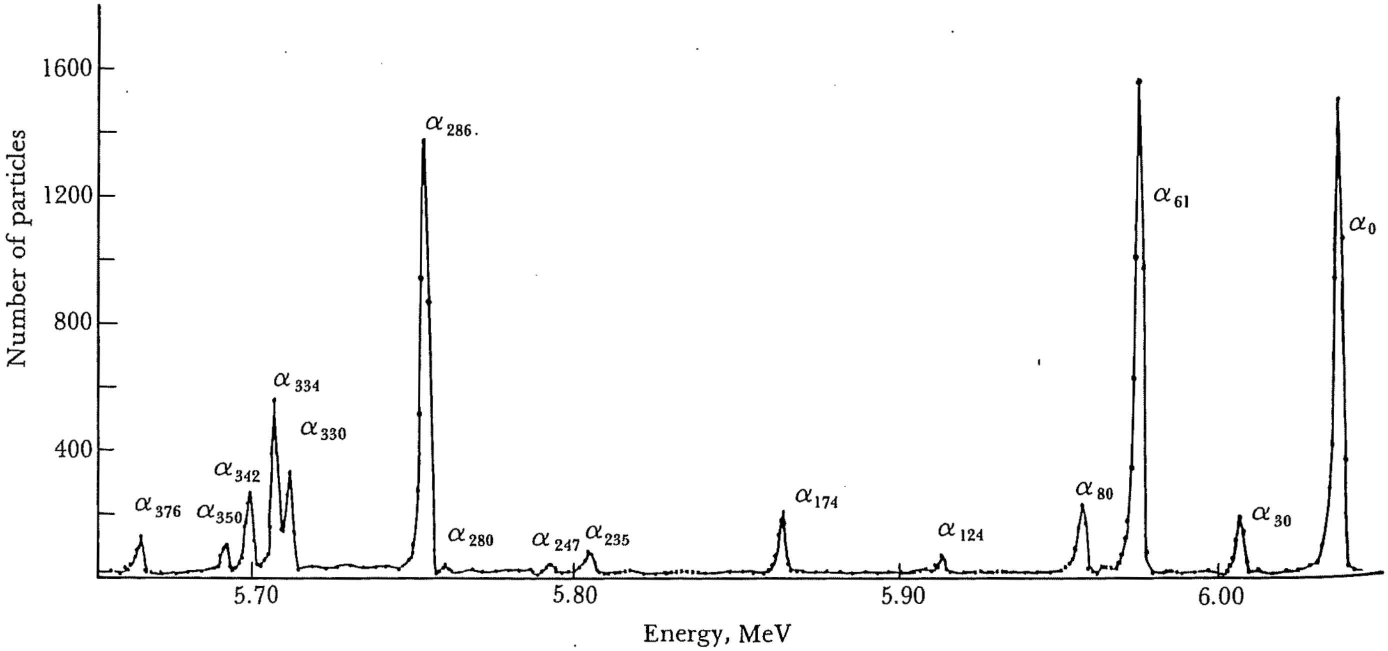


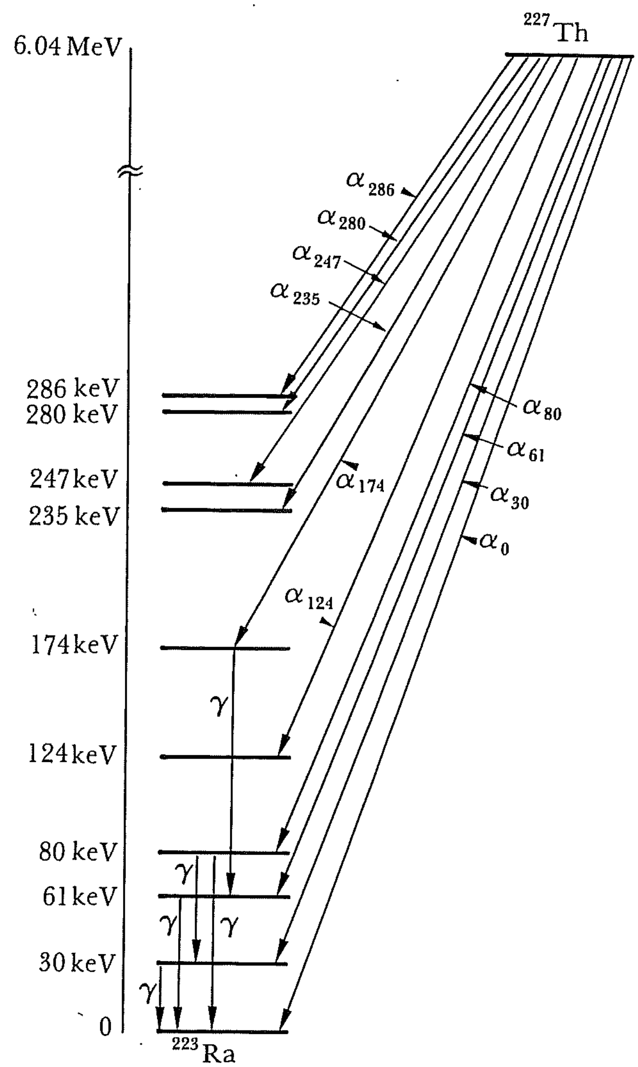
Figure 11-13
Energy levels of ^{223}Ra determined by measurement of α-particle energies from ^{227}Th, as shown in Figure 11-12. Only the lowest-lying levels and some of the γ-ray transitions are shown.

Widths due to experimental resolution, not to finite width due to finite lifetime

These decays are between 0.1 ns and 1 ns (see Table 11-...)



1600
1200
800
400
Number of particles
α_{376}
α_{350}
α_{342}
α_{334}
α_{330}
α_{286}
α_{280}
α_{247}
α_{235}
α_{174}
α_{124}
α_{80}
α_{61}
α_{30}
α_0
5.70
5.80
5.90
6.00
Energy, MeV



^{227}Th
6.04 MeV
α_{286}
α_{280}
α_{247}
α_{235}
α_{174}
α_{124}
α_{80}
α_{61}
α_{30}
α_{0}
286 keV
280 keV
247 keV
235 keV
174 keV
124 keV
80 keV
61 keV
30 keV
0
γ
^{223}Ra

Nuclear Wallet Cards

Isotope Z El A	Jπ	Δ (MeV)	T½, Γ, or Abundance	Decay Mode
91 Pa 229	(5/2+)	29.890	1.50 d *5*	ε 99.52%, α 0.48%
230	(2−)	32.167	17.4 d *5*	ε 91.6%, β− 8.4%, α 3.2×10^{-3}%
231	3/2−	33.421	32760 y *110*	α, Ne 13×10^{-10}%, SF$<2\times10^{-11}$%
232	(2−)	35.939	1.31 d *2*	β−, ε 3.0×10^{-3}%
233	3/2−	37.484	26.967 d *2*	β−
234	4+	40.336	6.70 h *5*	β−
234m	(0−)	40.410	1.17 m *3*	β− 99.84%, IT 0.16%
235	(3/2−)	42.32	24.5 m *2*	β−
236	1(−)	45.3	9.1 m *1*	β−
237	(1/2+)	47.6	8.7 m *2*	β−
238	(3−)	50.76	2.3 m *1*	β−, SF$<2.6\times10^{-6}$%
239	(1/2+)	53.2s	106 m *30*	β−
240		56.8s	≈2 m	β−?
92 U 218	0+	21.88s	1.5 ms *+73−7*	α
219	(9/2+)	23.21	42 μs *+34−13*	α
220	0+	23.0s	≈60 ns	α?, ε?
221		24.5s	≈0.7 μs	α?, ε?
222	0+	24.3s	1.0 μs *+10−4*	α
223	(7/2+)	25.82	55 μs *10*	α
224	0+	25.70	0.9 ms *3*	α
225		27.37	60 ms *10*	α
226	0+	27.33	0.35 s *15*	α
227	(3/2+)	29.01	1.1 m *1*	α
228	0+	29.22	9.1 m *2*	α>95%, ε<5%
229	(3/2+)	31.201	58 m *3*	ε≈80%, α≈20%
230	0+	31.603	20.8 d	α, SF$<1\times10^{-10}$%
231	(5/2−)	33.803	4.2 d *1*	ε
231	(3/2+,5/2+)	33.803	4.2 d *1*	α≈4×10^{-3}%
232	0+	34.602	68.9 y *4*	α, Ne 9×10^{-10}%, SF$<1\times10^{-12}$%
233	5/2+	36.913	1.592×10^{5} y *2*	α, SF$<6\times10^{-11}$%, Ne 7×10^{-11}%
234	0+	38.141	2.455×10^{5} y *6* **0.0054% *5***	α, SF 1.6×10^{-9}%, Mg 1×10^{-11}%, Ne 9×10^{-12}%
235	7/2−	40.914	703.8×10^{6} y *5* **0.7204% *6***	α, SF 7.0×10^{-9}%, Ne 8×10^{-10}%
235m	1/2+	40.914	≈25 m	IT
236	0+	42.441	2.342×10^{7} y *3*	α, SF 9.4×10^{-8}%, ^{30}Mg
237	1/2+	45.386	6.75 d *1*	β−
238	0+	47.304	4.468×10^{9} y *3* **99.2742% *10***	α, SF 5.4×10^{-5}%
239	5/2+	50.569	23.45 m *2*	β−
240	0+	52.709	14.1 h *1*	β−
241		56.2s	≈5 m	β−?
242	0+	58.6s	16.8 m *5*	β−
93 Np 225	(9/2−)	31.58	>2 μs	α

Half-life of α-emitters depends strongly on Q

	Q	$T_{1/2}$	
^{227}U	7.2 MeV	1 minutes	
^{229}U	6.5 MeV	58 minutes	
^{230}U	6.0 MeV	21 days	
^{232}U	5.4 MeV	69 yrs	
^{233}U	4.9 MeV	1.6×10^{5} yrs	
^{235}U	4.7 MeV	7.0×10^{8} years	0.7%
^{238}U	4.3 MeV	4.5×10^{9} yrs	99.3%

As $Q\downarrow$, $T_{\frac{1}{2}}\uparrow$

[see plot]

1912 Geiger Nuttall rule: $\ln T_{\frac{1}{2}} = c_1 \frac{Z}{\sqrt{Q}} + c_2$

(empirical!) (not understood until QM invented)

Can we understand this?

Recall $T_{\frac{1}{2}} = (\ln 2)\,\tau = \frac{\ln 2}{R}$

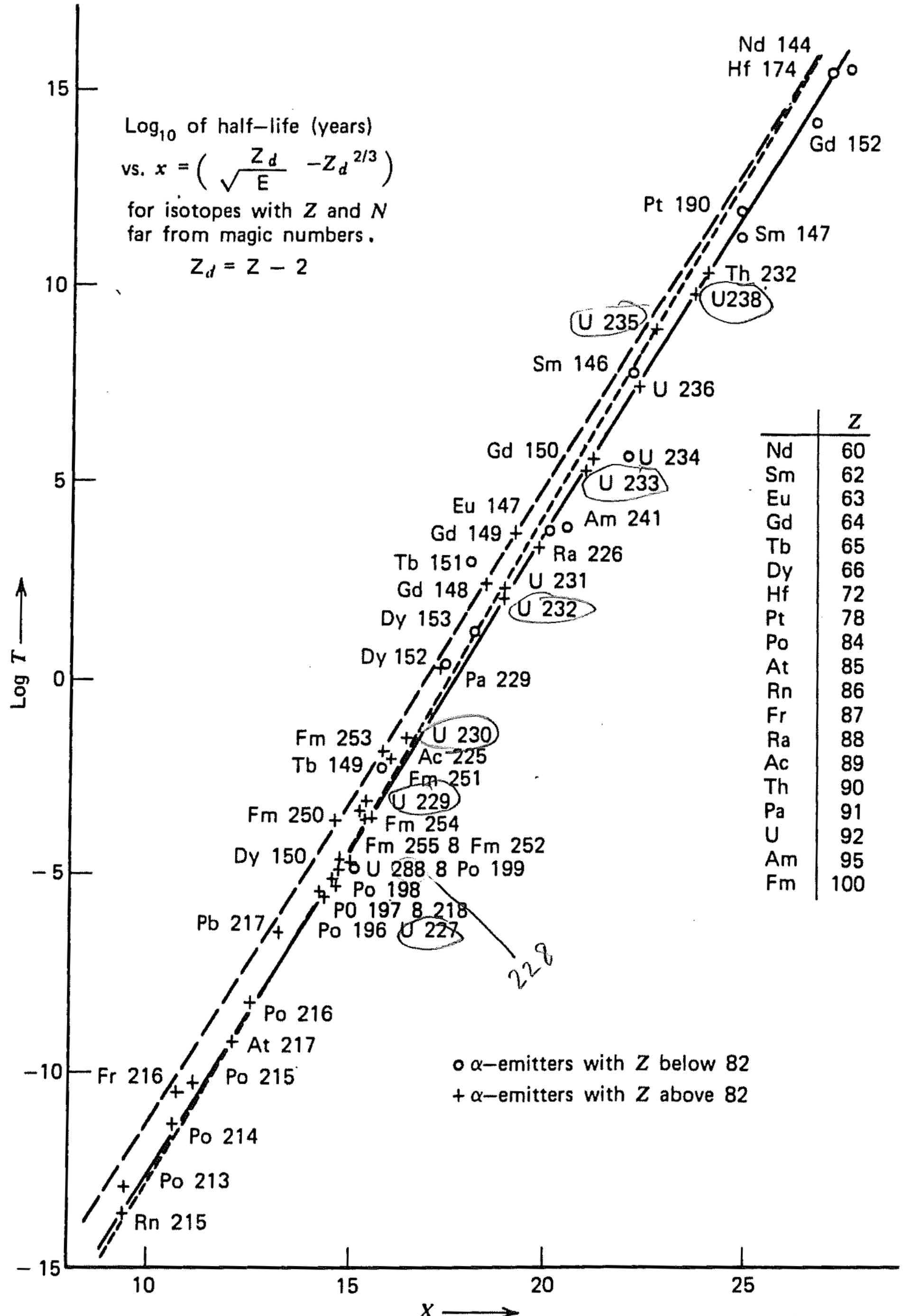
(from HW)

$R = \frac{\text{prob. of decay}}{\text{time}}$ ← inherently quantum mechanical (what is the process?)

How do we calculate the probability?

$U \rightarrow Th + \alpha$

Imagine a uranium nucleus as an α particle trapped inside a thorium nucleus



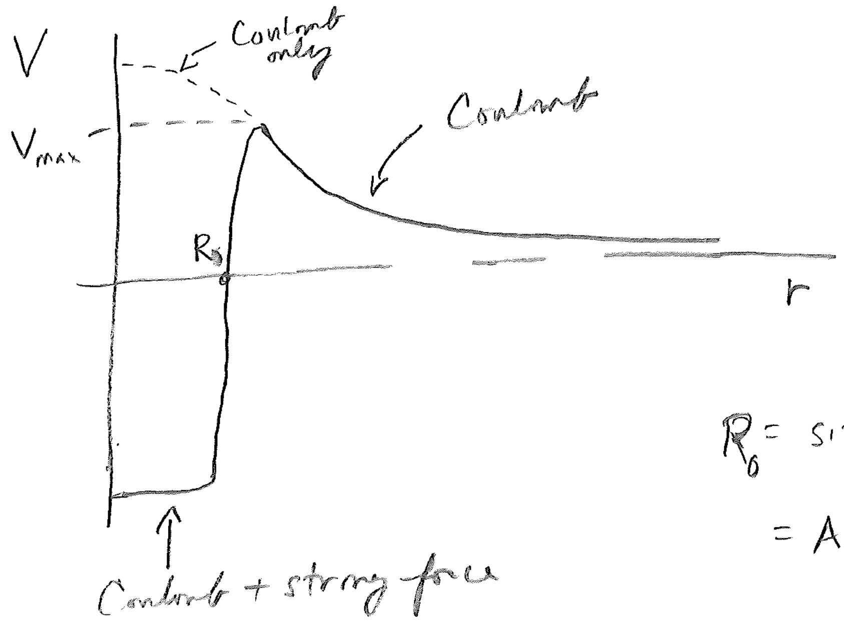
	Z
Nd	60
Sm	62
Eu	63
Gd	64
Tb	65
Dy	66
Hf	72
Pt	78
Po	84
At	85
Rn	86
Fr	87
Ra	88
Ac	89
Th	90
Pa	91
U	92
Am	95
Fm	100

Figure 5-9. Plot of $\log_{10} 1/\tau$ versus $C_2 - C_1 Z_1/\sqrt{E}$ with $C_1 = 1.61$ and a slowly varying $C_2 = 28.9 + 1.6Z_1^{2/3}$. (From E. K. Hyde, I. Perlman, and G. T. Seaborg, *The Nuclear Properties of the Heavy Elements,* Vol. 1, Prentice-Hall, Englewood Cliffs, N.J. (1964), reprinted by permission.)

$\ln \frac{1}{\tau} \approx -3.71$

$$^{A+4}_{Z+2}X \longrightarrow \ ^{A}_{Z}Y \quad \alpha$$

Consider the potential energy between Y and α as a function of separation r



R_0 = size of Y nucleus
$= A^{1/3} r_0$

Rough estimate of height of barrier

$$V_{max} \sim \frac{K q_1 q_2}{R} = \frac{K(Ze)(2e)}{A^{1/3} r_0} = 2\left(\frac{Ke^2}{r_0}\right)\frac{Z}{A^{1/3}}$$

$$\sim 2\,\frac{(1.44\ \text{MeV fm})}{(1.2\ \text{fm})}\,\frac{Z}{A^{1/3}}$$

$$\sim (2.4\ \text{MeV})\,\frac{Z}{A^{1/3}}$$

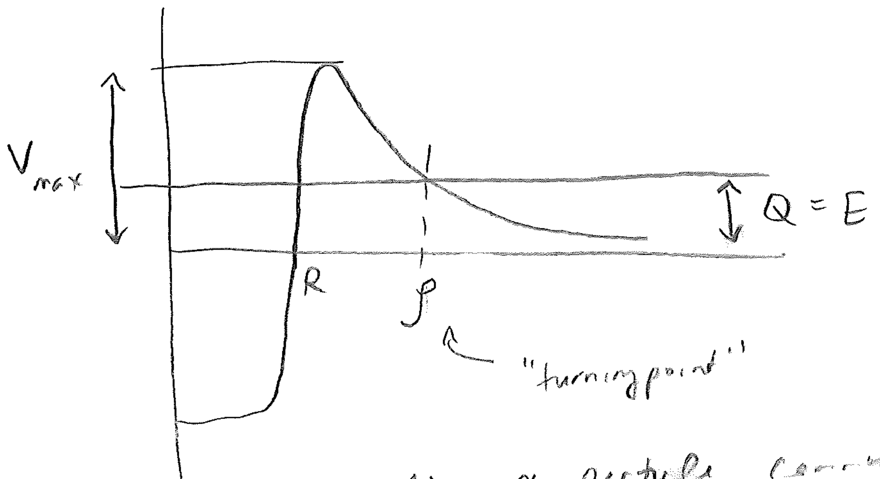
For ^{238}U decay, $Z = 90$, $A = 234$ $\rightarrow$ $V_{max} \sim 35$ MeV

[actual $V_{max} \sim 25$ MeV]

What is energy of α particle?

After it escapes, $E = T + V$

$\approx Q + 0$



Classically, α particle cannot escape

Quantum mechanically, it can "tunnel" through barrier

[Gamow (1929)
Gurney-Condon]

Tunnelling rate

$$R = (\text{const})\, e^{-\frac{2}{\hbar}\int_R^{\rho} dr \sqrt{2m(V(r)-E)}}$$

[HW] calculate this to find $R \approx e^{-c_1 \frac{Z}{\sqrt{Q}}}$

[HW: const = 3.96
expt. 3.68 (Gasiorowicz)]

$$T_{\frac{1}{2}} \sim \frac{\ln 2}{R}$$

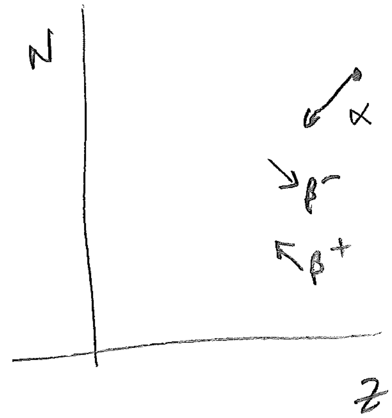
$$\ln T_{\frac{1}{2}} \sim -\ln R + \text{const}$$

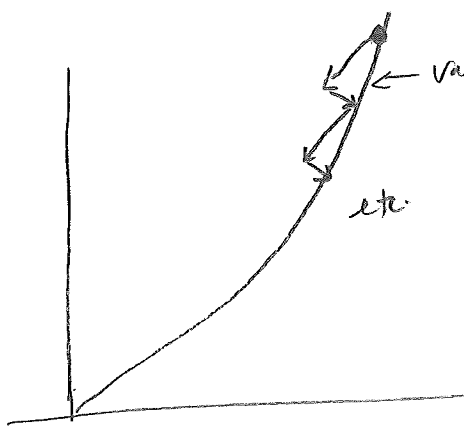
$$\sim c_1 \frac{Z}{\sqrt{Q}} + \text{const} \qquad \text{(Geiger-Nuttall)}$$

α: $Z \to Z-2$
$A \to A-4$

β^-: $Z \to Z+1$
$A \to A$

β^+, EC: $Z \to Z-1$
$A \to A$





Since α, β conserve A mod 4,
there are 4 distinct radioactive series

$A = 4n+2$ $\quad {}^{238}U$ (4.5 E9 yrs) $\longrightarrow$ ${}^{206}Pb$

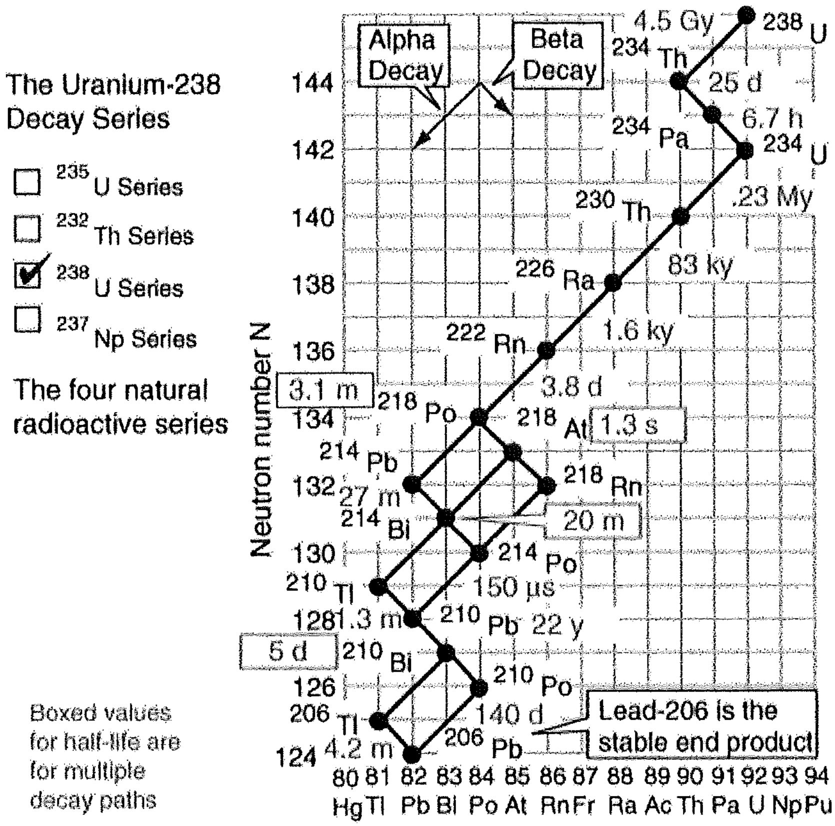
$A = 4n+3$ $\quad {}^{235}U$ (7 E8 yrs) $\longrightarrow$ ${}^{207}Pb$

$A = 4n$ $\quad {}^{232}Th$ (1.4 E10 yrs) $\longrightarrow$ ${}^{208}Pb$

$A = 4n+1$ $\quad {}^{233}U$ (1.6 E7 yrs) $\longrightarrow$ ${}^{209}Bi \xrightarrow{1.9E19\ \text{yrs}} {}^{205}Tl$

hyperphysics. phy-astr. gsu.edu/hbase/nuclear

(georgia state u.)

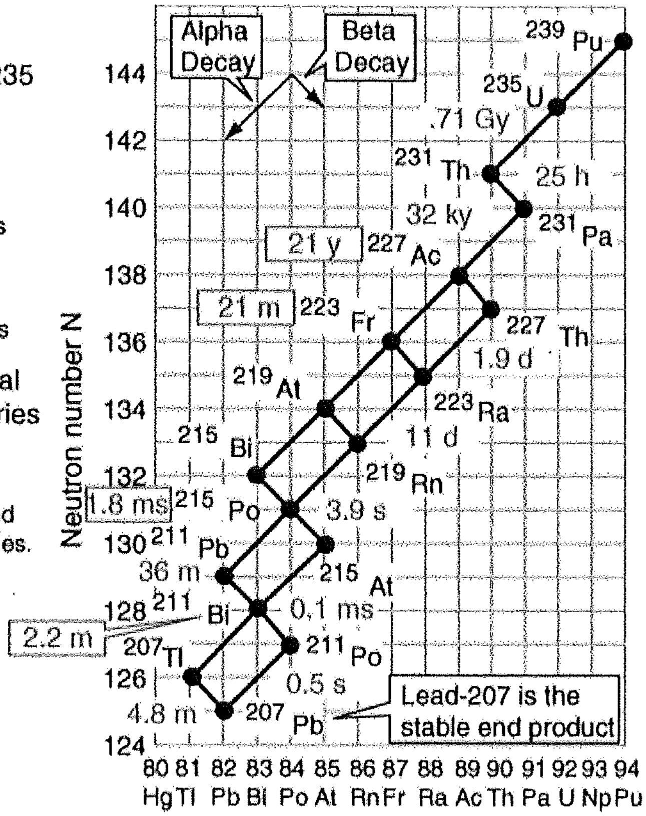


The Uranium-235 Decay Series

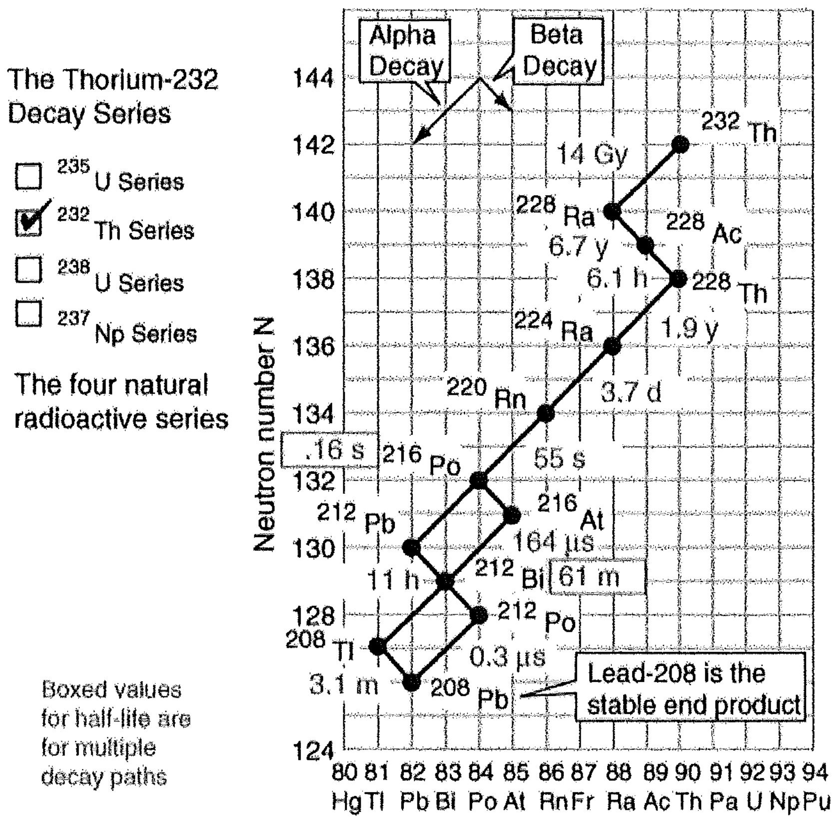
- [x] 235 U Series
- [ ] 232 Th Series
- [ ] 238 U Series
- [ ] 237 Np Series

The four natural radioactive series

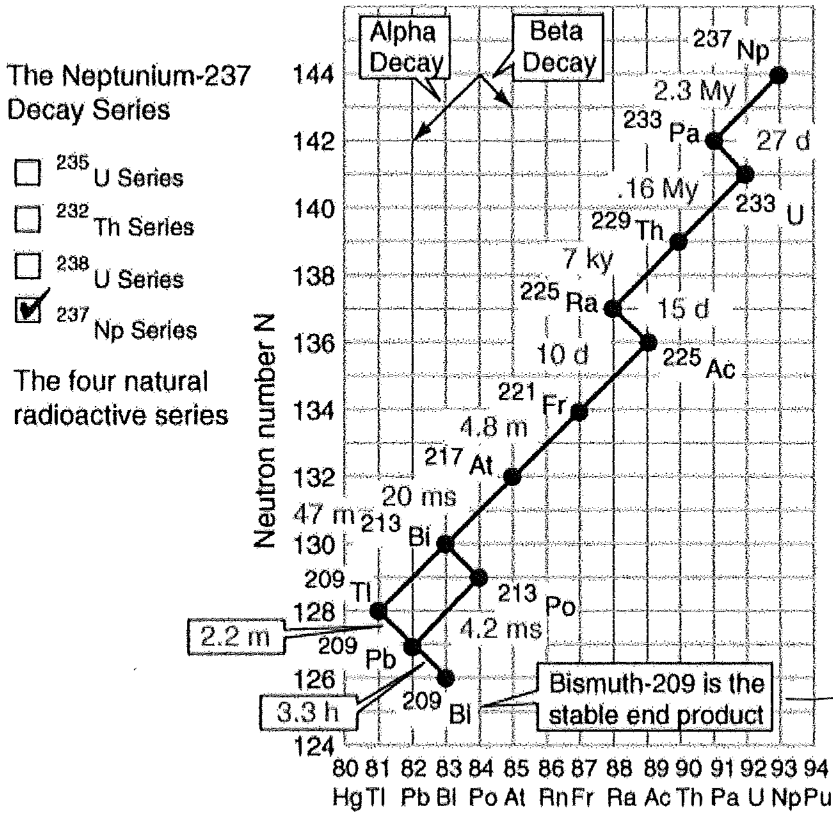
This series is traditionally called the Actinium series.



Boxed values for half-life are for multiple decay paths



The Thorium-232 Decay Series
235 U Series
232 Th Series
238 U Series
237 Np Series
The four natural radioactive series
Boxed values for half-life are for multiple decay paths
Alpha Decay
Beta Decay
Neutron number N
144
142
140
138
136
134
132
130
128
124
232 Th
14 Gy
228 Ra
6.7 y
228 Ac
6.1 h
228 Th
1.9 y
224 Ra
3.7 d
220 Rn
55 s
.16 s
216 Po
216 At
164 μs
212 Pb
11 h
212 Bi
61 m
212 Po
0.3 μs
208 Tl
3.1 m
208 Pb
Lead-208 is the stable end product
80 81 82 83 84 85 86 87 88 89 90 91 92 93 94
Hg Tl Pb Bi Po At Rn Fr Ra Ac Th Pa U Np Pu



$\xrightarrow{\alpha} {}^{205}Tl$

$T_{\frac{1}{2}} = 1.9 \times 10^{19}$ yrs

[2003]

Try to estimate energy released in α-decay using ~~semi-empirical~~

accurate to about 15%

$$^{238}U \longrightarrow {}^{234}Th + \alpha$$

$\Delta = 47.304$ $\quad \Delta = 40.604$ $\quad \Delta = 2.425 \Rightarrow Q = 4.270$ MeV

$\underbrace{\qquad\qquad}_{6.695\text{ MeV}}$ ← actual difference

use binding energy formula

$B(^{238}U) = 1778.63 \Rightarrow \Delta = 71.05$

$B(^{234}Th) = 1756.03 \Rightarrow \Delta = 62.93$

} not too close

$\underbrace{\qquad}_{22.60}$ $\quad \underbrace{\qquad}_{8.12}$ ← not too bad, but not too good

Now do Taylor expansion

$$B = c_1 A - c_2 A^{2/3} - c_3 \frac{Z^2}{A^{1/3}} - c_4 \frac{(A-2Z)^2}{A}$$

$$B(^A_Z X) - B(^{A-4}_{Z-2} Y) = 4c_1 - \frac{8c_2}{3A^{1/3}} - c_3 \frac{Z^2}{A^{1/3}}\left[\frac{4}{Z} - \frac{4}{3A}\right]$$

$$+ 4c_4 \frac{(A-2Z)^2}{A^2}$$

$$= 62 - \frac{44.8}{A^{1/3}} - \frac{2.88\, Z^2}{A^{1/3}}\left(\frac{1}{Z} - \frac{1}{3A}\right) + \frac{92(A-2Z)^2}{A^2}$$

0.464

$^{238}_{92}U \longrightarrow 62 - 7.23 - 37.25 + 4.74 = 22.3$

Since $\Delta = Zm_H + (A-Z)m_n - A(931.5) - B$

$$\Delta(^A_Z X) - \Delta(^{A-4}_{Z-2} Y) = \underbrace{2m_H + 2m_n - 4(931.5)}_{30.73} - \Delta B$$

$$= -31.3 - \frac{44.8}{A^{1/3}} - 2.88 \frac{Z^2}{A^{1/3}}\left[\frac{1}{Z} - \frac{1}{3A}\right] + 92\frac{(A-2Z)^2}{A^2}$$

$$= -31.3 + 7.2 + 37.2 - 4.7 = 8.4$$