2023 revision

[Next we turn to weak interaction & β-decay]

A decay (or other transformation)
will generally occur provided it
does not violate any conservation laws
["what is not forbidden is mandatory"]

[review of pi-4] Conservation laws

- energy
- momentum
- angular momentum

} a consequence of invariance of laws of physics under translations in space and time, & rotations in space (Noether's thm)

- electric charge

} a consequence of gauge invariance of electromagnetism

[believed to be absolute]

- baryon number (quark number)
- lepton number

} "accidental", (no known symmetry) but valid for standard model
[due to no known symmetry, but satisfied by the 4 known forces; # quarks & leptons don't convert into each other;

If a process obeys energy & momentum conservation, it is said to be "kinematically allowed"
otherwise "kinematically forbidden"

Energy conservation

[types of energy]

- rest energy mc^2
- kinetic energy T.

$$E = \gamma mc^2 \quad \text{where} \quad \gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} \cdots$$

$$T = E - mc^2 = \boxed{(\gamma - 1)mc^2} \text{ exact}$$

$$\text{non rel. approx.} \cong \underbrace{\boxed{\tfrac{1}{2}mv^2}}_{\text{approx}} + \cancel{\frac{3}{8}m\frac{v^4}{c^2}} + \cdots$$

- potential energy V
  associated w/ interactions between particles (EM, strong..)

[Well before and well after the process the particles are well separated
⇒ ignore any potential energies in initial & final states]

$$E_{init} = E_{final}$$

$$\sum_{init} (mc^2 + T) = \sum_{final} (mc^2 + T)$$

Neither kinetic energy nor ~~nor~~ mass is separately conserved
only their sum.

Define the kinetic energy released in a process

$$Q = T_{final} - T_{init}$$

$$= \sum_{init} mc^2 - \sum_{final} mc^2$$

If $Q = 0$, "elastic process" $\Rightarrow$ kinetic energy is conserved
[e.g. scattering of particles which retain their identities]

If $Q < 0$, "inelastic process" $\Rightarrow$ kinetic energy is lost
final state has more mass than initial (collisions)

[lumps of putty: hot object has slightly more mass than a cool one]

[More massive particles can be produced in a collision e.g. cosmic rays, particle accelerators]

If $Q > 0$, "superelastic process"
$\Rightarrow$ mass "converted" to kinetic energy
(eg fission, decay)

If initial state consists of a single particle m_0 at rest ($T_{init} = 0$)

then $Q = T_{final} \geq 0$

$$\Rightarrow \quad m_0 c^2 \geq \sum_{final} mc^2$$

$\Rightarrow$ massive particles can only decay into less massive particles (not vice versa)

Generally, decays w/ larger Q
have larger probability to occur
+ therefore shorter half-lives
though for differing reasons

e.g. α-decay, higher Q $\therefore$ small potential barrier
to tunnel through

β-decay, high Q $\rightarrow$ larger phase space

But in some cases, other considerations
cause the relationship to be violated

e.g. parity violation of the weak force
means that $\pi \rightarrow \mu$ is more likely
than $\pi \rightarrow e$,
despite the smaller phase space available

Conservation laws guarantee stability of certain particles

e^- is stable because it is least massive pcl w/ electric charge

[if it decayed into lighter neutral particles, violates charge cons.]

Why is p stable?

[not the lightest positively charged pcl : $p \to e^+ \gamma$]

1938 Ernst Stuckelberg proposed conservation of baryon # (A)
(quark #)

p is stable because it is lightest pcl w/ baryon #

All forces of standard model conserve baryon # and lepton #
but extensions of standard model (eg GUTs)
typically violate both

---

proton decay would be a signal of new (BSM) physics!

Proton decay expts since 1970's [problem]
(Super Kamiokande)
null results so far

$\tau_p > 2 \times 10^{29}$ yrs [need a lot of protons]

(2023)
Abby Chris(?) asked about lifetime limits + confidence levels quoted in PPB

prot decay τ limits

Decay rate $R = \frac{1}{\tau}$

Prob of decay in time of experiment T: $p = RT$

N nuclei

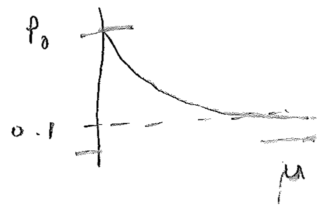
mean # nuclei decaying in time T is $\mu = Np = \frac{NT}{\tau}$

Poisson distrib. P_n: prob of n nuclei decaying

$$= e^{-\mu}\frac{\mu^n}{n!}$$

Prob. of no nuclei decaying $P_0 = e^{-\mu}$

Suppose no nuclei decay.



This is very likely for small μ.
↳ large τ

Unlikely for large μ
↳ small τ

Set $e^{-\mu} = 0.1$,

$e^{\mu} = 10$

$\mu = \log(10) = 2.3$

W/ 90% confidence we can say that $\mu < 2.3$

↳ i.e. only a 10% chance that no particles would decay in time T

$$\frac{NT}{\tau} < \ln 10$$

$$\tau > \frac{NT}{\ln 10}$$

Mass/energy conversions

Masses of atoms or nuclei are specified in terms of unified atomic mass unit (u).

$$1\,u \equiv \frac{(\text{mass of neutral } ^{12}C \text{ atom in ground state})}{12}$$

[amu was based on ^{16}O; u is based on ^{12}C]

A mole of particles = Avogadro's number of particles

$N_A = 6.022\,140\,76 \times 10^{23}$ (exact) omit [cf PPB]

A mole of particles of mass 1u has mass of 1 g (approx.) omit

omit [used to be a definition, & N_A was determined experimentally. Now N_A is defined exactly, so the statement is only approximate. Molar mass of ^{12}C is $11.999\,999\,995\,8(36)$ g.]

$$1\,u = \frac{10^{-3}\,kg}{N_A} = \underbrace{1.660539}... \times 10^{-27}\,kg \text{ (approx.)}$$ omit

Using exact $c = 299\,792\,458\,\frac{m}{s}$, a particle of mass 1u has rest energy

$$mc^2 = 1.492418... \times 10^{-10}\,J$$ (approx) omit

(exact)

$c = 299792458 \frac{m}{s}$

$h = 6.626\ 070\ 15 \times 10^{-34}$ J/Hz

$e = 1.602\ 176\ 634 \times 10^{-19}$ C

$k_B = 1.380\ 649 \times 10^{-23}$ J/K

do not discuss

1 second defined by
ν=9,192,631,770 cycles of hyperfine transition of ^{133}Cs.

$E = h\nu = 6.091102297\ldots E{-}24$ J
so 1 J = 1.64 E23 photons of ^{133}Cs

$E = \frac{h\nu}{e} = 3.80171702\ldots E\ 5$ eV
ie 1 eV = 26,300 photons

$m = \frac{h\nu}{c^2}$ 1.0136…E-42 kg
= mass equivalent of photon
This defines the kg.

- We'll use eV rather than J

$e = 1.602\ 176\ [634] \times 10^{-19}$ C (exact) omit

$1\ eV = 1.602\ 176\ [634] \times 10^{-19}$ J (exact) [recall J = C·V]

omit

A particle of mass 1 u has rest energy

$mc^2 = 9.31494 \times 10^8$ eV (approx).

$= 931.494$ MeV (approx) omit

$$\boxed{1\ u \Leftrightarrow 931.494\ldots \text{MeV}}$$

need this precision to talk about mass differences

proton [1.00727654 → 938.272 MeV

neutron [1.008664.. → 939.565 MeV

electron [0.00054860 → 0.511 MeV [about $\frac{1}{1800} m_p$]

Neutron decay

$n \to p$ [baryon # conserved but violates charge cons.]

$n \to p e^-$

[violates lepton #]

$n \to p e^- \bar{\nu}_e$

[actually lepton # was not known until 1950's energy conservation led Pauli to predict ν. We'll discuss later, but meanwhile want to be correct so we'll include it now]

$m_{\bar{\nu}} \lesssim 1\ eV$

[actually need to check if kinematically allowed]

$$Q = m_n c^2 - m_p c^2 - m_e c^2 - m_{\bar{\nu}} c^2$$

$= 0.782$ MeV, kinematically allowed [recall $\tau = 15$ min]

A. nuclear process involving e^- or e^+ is called β-decay

[not ionization of atom]

most unstable light nuclei decay via β-decay

Mass excess

[It is generally easier to measure the masses of neutral atoms than of bare nuclei]

A neutral atom of Z protons, N neutrons, Z electrons has an approximate mass $m \approx (Z+N)\,u = A\,u$

ie rest energy

$$mc^2 \approx A\,(931.494\text{ MeV})$$

It is convenient to characterize the discrepancy as

$$mc^2 \equiv A\,(931.494\text{ MeV}) + \Delta$$ ← atomic mass (incl. electrons)

Δ = "mass excess" [actually rest energy excess]

$$m = \left(A + \frac{\Delta}{931.494}\right) u$$ [nuclear wallet cards]

Recall: the unified mass unit u is defined so that $\Delta \equiv 0$ for ${}^{12}_{6}C$

[Using our earlier numbers] $p = 938.272$
$n = 939.565$

- proton $\Delta = 6.778$ MeV
- neutron $\Delta = 8.071$ MeV

[neutron agrees w/ nuclear wallet cards, but not proton] [why not??]

- hydrogen = proton + electron $\Delta = 7.289$ MeV ✓

$n \longrightarrow p\, e^-\, \bar{\nu}_e$

From now on, we can omit the factor of c^2:

$$Q = m_n - m_p - m_e - \cancel{m_{\bar{\nu}}}$$
$$= m_n - (m_H - m_e) - m_e$$
$$= m_n - m_H$$
$$= (1\,u + \Delta(n)) - (1\,u - \Delta(^1H))$$
$$= \Delta(n) - \Delta(^1H)$$
$$= 0.782 \text{ MeV} \quad ✓$$

[Drop the c^2, w/ this freedom comes responsibility]

(need restore units when do calculation ⋮)

Bound states

Deuteron $D = pn$

[Simplest composite nucleus
p and n bound together by strong force
(residue of color force between quarks]

Composite objects hold together due to their binding energy.

Binding energy = difference between the sum of (rest) energies of the components and the (rest) energy of the composite object

Deuteron binding energy:

$$B_D = m_n + m_p - m_D$$

$$= m_n + [m(^1H) - \cancel{m_e}] - [m(^2H) - \cancel{m_e}]$$

$$= [1u + \Delta(n)] + [1u + \Delta(^1H)] - [2u + \Delta(^2H)]$$

$$= \Delta(n) + \Delta(^1H) - \Delta(^2H) \qquad [\text{NB. } B \neq \Delta]$$

$$= 8.071 + 7.289 - 13.136$$

$$= 2.224 \text{ MeV} \quad (\text{about } 0.1\% \text{ of } m_d \sim 2\text{ GeV})$$

$$\Rightarrow m_D = m_n + m_p - B_D$$

Deuteron mass is less than its constituents

[How to understand B ?]

Binding energy is due to attractive potential energy minus kinetic energy

Consider a bound state of 2 particles (eg hydrogen atom, or deuteron)

$$\underbrace{E_{12}}_{\text{bound state energy}} = \underbrace{E_1 + E_2}_{\text{energy of each particle}} + \underbrace{V_{12}}_{\text{(negative) potential energy due to attractive force}}$$

$E_1 = m_1 + T_1$

$E_2 = m_2 + T_2$

$V_{12} = -|V_{12}|$

Bound state at rest $E_{12} = m_{12}$

$$\Rightarrow m_{12} = m_1 + m_2 + \underbrace{(T_1 + T_2 - |V_{12}|)}_{-B_{12}}$$

$$\Rightarrow B_{12} = |V_{12}| - T_1 - T_2$$

Why do individual particles have kinetic energy?

Heisenberg uncertainty principle due to confinement to small space

$$\Delta p \, \Delta x \gtrsim \frac{\hbar}{2}$$

$$\Delta p \gtrsim \frac{\hbar}{2\Delta x} \quad \text{where } \Delta x \sim \text{size of bound state}$$

$\langle p^2 \rangle - \langle p \rangle^2 = (\Delta p)^2$

$$\text{Nonrelativistic: } \left.\begin{array}{l} p = mv \\ T = \frac{1}{2}mv^2 \end{array}\right\} \Rightarrow T = \frac{p^2}{2m}$$

$$T \sim \frac{(\Delta p)^2}{2m} \sim \frac{\hbar^2}{2m(\Delta x)^2}$$

---

electron in hydrogen atom: $\Delta x \approx a_0$ (Bohr radius)

$$T = \frac{\hbar^2}{2ma_0^2} = 13.6\,\text{eV}$$

$$|V_{12}| = 2T \quad \text{(virial theorem)}$$

$$\Rightarrow \underline{B = 13.6\,\text{eV}} \text{ (binding energy of hydrogen)}$$

---

deuteron: $\Delta x \approx 1$ fm

$$T_p \approx T_n \sim \frac{\hbar^2}{2m(\Delta x)^2} = \frac{(\hbar c)^2}{2mc^2(\Delta x)^2} = \frac{(200\,\text{MeV fm})^2}{2(1000\,\text{MeV})(1\,\text{fm})^2} = \underline{\underline{20\,\text{MeV}}}$$

Since $B_D \approx |V_{pn}| - T_p - T_n \approx 2$ MeV

we infer $|V_{pn}| \approx 42$ MeV

old notes

~~B~~ 10

First, let's understand binding energy for hydrogen atom.

$$E_H = (m_p + T_p) + (m_e + T_e) + V_{ep}$$

Coulomb potential energy between p and e

$T_e = \frac{p^2}{2m_e}$, $T_p = \frac{p^2}{2m_p} \ll T_e$ since $m_p \gg m_e$

Estimate T_e. Uncertainty principle $\Rightarrow p \sim \frac{\hbar}{\Delta x}$

where Δx = uncertainty in position of electron $\sim a_0$ = Bohr radius

$$T_e \approx \frac{1}{2m_e}\left(\frac{\hbar}{a_0}\right)^2 = 13.6\,eV$$

more precisely

$T_e = \frac{1}{2m}\left(\frac{h}{\lambda}\right)^2 = \frac{1}{2m}\left(\frac{\hbar}{a_0}\right)^2$

$a_0 = \frac{\hbar^2}{mke^2}$

$T_e = \frac{1}{2}mc^2\left(\frac{ke^2}{\hbar c}\right)^2 = \frac{1}{2}mc^2\alpha^2$

Attractive inverse square force

$\Rightarrow V_{ep} = -2T_e$ (virial theorem)

$\Rightarrow T_e + T_p + V_{ep} \approx T_e + 0 - 2T_e = -T_e = -13.6\,eV$

$E_H = m_p + m_e + T_p + T_e + V_{ep} = m_p + m_e - 13.6\,eV$

But $m_H = m_p + m_e - B_{atomic}$ so $B_{atomic} = 13.6\,eV$

$\approx 1\,GeV + \frac{1}{2}MeV - 10\,eV$

B is only 10^{-8} of total mass

old notes

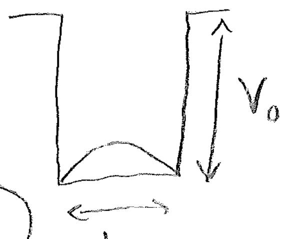
Binding energy of deuteron

strong interaction potential energy

$$m_d = m_p + m_n + \underbrace{T_p + T_n + V_{pn}}_{-B_d} \qquad \text{where } B_d \approx 2.2 \text{ MeV}$$

[Unlike Coulomb force, we have] No simple formula for V_{pn}
[nor do we expect one, since p and n are composites]

Use simplistic potential well model to estimate kinetic energy



$$T = T_p + T_n = 2 \cdot \frac{\vec{p}^{\,2}}{2m} = 2 \cdot \frac{3}{2m}\left(\frac{h}{\lambda}\right)^2$$

λ = de Broglie wavelength

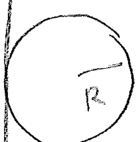
$$\lambda \simeq 2L$$

probably overkill

$$T = \frac{3}{m}\left(\frac{2\pi\hbar}{2L}\right)^2 = \frac{3\pi^2}{mc^2}\left(\frac{\hbar c}{L}\right)^2$$

$$= \frac{3\pi^2}{(1000\,\text{MeV})}\left(\frac{200\,\text{MeV}\cdot\text{fm}}{L}\right)^2 = 1200\,\text{MeV}\left(\frac{\text{fm}}{L}\right)^2$$

skip



$L = 2R, \quad R \approx 2^{1/3} r_0 \simeq 1.5\,\text{fm} \Rightarrow L \sim 3\,\text{fm}$

$$T \simeq 130\,\text{MeV}$$

This is a very crude estimate (probably too high by factor of 2 or 4)

but suggests $V_0 \simeq -T \simeq -130\,\text{MeV}$

Simpler: $T = T_p + T_n \simeq \frac{p^2}{m} \approx \frac{1}{m}\left(\frac{\hbar}{\Delta x}\right)^2 = \frac{1}{mc^2}\left(\frac{\hbar c}{\Delta x}\right)^2 = \frac{1}{1000\,\text{MeV}}\left(\frac{200\,\text{MeV fm}}{1\,\text{fm}}\right)^2 \approx 40\,\text{MeV}$

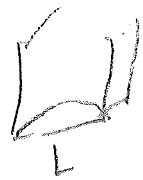
suggests $V \approx -42\,\text{MeV} \Rightarrow B = -T - V \approx 2\,\text{MeV}$

old notes

~~β-H.1~~

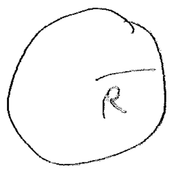
background

particle in 3d box



$$T = 3\cdot\frac{1}{2m}\left(\frac{h}{\lambda}\right) = \frac{3}{2m}\left(\frac{2\pi\hbar}{2L}\right)^2 = \frac{3\pi^2\hbar^2}{2mL^2}$$

particle in a sphere



$$-\frac{\hbar^2}{2m}\frac{1}{r}\frac{d^2}{dr^2}(r\psi) = E\psi$$

$$\frac{d^2}{dr^2}(r\psi) + \frac{2mE}{\hbar^2}(r\psi) = 0$$

$$\psi = \frac{\sin kr}{r} \quad \text{where} \quad \frac{\hbar^2 k^2}{2m} = E$$

$$\psi(R)=0 \Rightarrow kR=\pi \Rightarrow k=\frac{\pi}{R} \Rightarrow \boxed{E=\frac{\hbar^2\pi^2}{2mR^2}}$$

If $L = \underbrace{2R}_{\text{diameter}}$ then 3d box $\Rightarrow \frac{3}{8}\frac{\pi^2\hbar^2}{mR^2}$, slightly less

If volumes equal $L^3 = \frac{4\pi}{3}R^3$

$$\text{the 3d box} = \frac{3\pi^2\hbar^2}{2m}\left(\frac{3}{4\pi}\right)^{2/3}\frac{1}{R^2} = 0.577\,\frac{\hbar^2\pi^2}{mR^2}$$

slightly more (by 15%)

[Because $B_d > 0$, deuteron can't fall apart]

$D \rightarrow p + n$ kinematically forbidden [$\Delta = -B_d < 0$]

[but what about]

$D \xrightarrow{?} p p e^- \bar{\nu}_e$ [Can n inside D decay?]

$$Q = m_D - 2m_p - m_e - \cancel{m_\nu}^{\,0}$$

$$= (m(^2H) - m_e) - 2(m(^1H) - m_e)) - m_e$$

$$= m(^2H) - 2m(^1H)$$

$$= [2u + \Delta(^2H)] - 2[1u + \Delta(^1H)]$$

$$= \Delta(^2H) - 2\Delta(^1H)$$

$$= -1.442 \text{ MeV}$$ still kinematically forbidden

Free neutron is unstable but
neutron bound to proton is stable [due to strong force which reduces it's energy]

Tritium ^3H (pnn)

$$(pnn) \longrightarrow (pnp) \quad e^- \quad \bar{\nu}_e$$

$$Q = [m(^3H) - m_e] - [m(^3He) - 2m_e] - m_e - m_\nu$$

$$= [3u + \Delta(^3H)] - [3u + \Delta(^3He)] - m_\nu$$

$$= \Delta(^3H) - \Delta(^3He) - m_\nu$$

$$= 0.019\ MeV - m_\nu$$ kinematically allowed (barely)

$\hookrightarrow T_{\frac{1}{2}} \sim 12$ years

$m(^3H) = 2809.4496$ MeV
$m(^3He) = 2809.4310$ MeV
0.0186 MeV

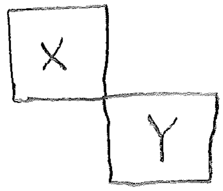
- tritium important in setting upper bound on $\bar{\nu}$ mass

eg: $m_\nu \lesssim 19$ keV for decay to occur

$$Q \quad T_e + \cancel{T_{^3H}} + T_\nu$$
$\downarrow 0$

by mean of $(T_e)^{max}$ can put more stringent bound on m_ν

β^- decay



β-decay always connects isobars
[same # baryons]

$${}^{A}_{Z}X \xrightarrow{?} {}^{A}_{Z+1}Y \quad e^- \quad \bar{\nu}_e$$

[baryon #, lepton #, charge]

one of neutrons converted to a proton

$$Q = m_{nucl}(X) - m_{nucl}(Y) - m_e$$

$$= [Au + \Delta(X) - Zm_e] - [Au + \Delta(Y) - (Z+1)m_e] - m_e$$

$$= \Delta(X) - \Delta(Y)$$

If $\Delta(X) > \Delta(Y)$, then X is unstable to β^--decay

What if $\Delta(X) < \Delta(Y)$?

Can a proton convert to a neutron ?

$e^- \;\; p \rightarrow n \;\; \nu_e$ — kinematically forbidden for e^-, p at rest (but formation of a neutron star)

[conservation laws obeyed]

$$e^- \;\; {}^{A}_{Z+1}Y \xrightarrow{?} {}^{A}_{Z}X \;\; \nu_e \quad \text{(electron capture)}$$

one of the atomic electrons — one of protons converted to neutron.

$$Q = m_e + m_{nucl}(Y) - m_{nucl}(X)$$

$$= m_e + [\,Au + \Delta(Y) - (Z+1)m_e\,] - [\,Au + \Delta(X) - Z m_e\,]$$

$$= \Delta(Y) - \Delta(X)$$

If $\Delta(Y) > \Delta(X)$, then Y is unstable to EC (electron capture)

EC is followed by X-ray emission, as one of outer shell electrons fills the vacancy created by the s-shell electron

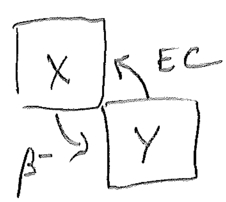
[first observed 1938 Alvarez : cf Krane, p. 272]

Given two adjacent isobars, one will always be unstable (but next to adjacent isobars can exist)

[See next pg for Δ's] [Segrè chart]

${}^{113}_{48}Cd \xrightarrow{\beta^-} {}^{113}_{49}In \quad 9.3 \times 10^{15}$ yrs

${}^{123}_{52}Te \xrightarrow{EC} {}^{123}_{51}Sb \quad 1.3 \times 10^{13}$ yrs



(LBNL shows as stable on Segrè chart).

$^{113}_{48}Cd$, $\Delta = -89.051$, $T_{\frac{1}{2}} = 9.3E15$ yrs β^- decay, 12.22% $J = \frac{1}{2}+$

$^{113}_{49}In$ $\Delta = -89.367$ 4.3% $J = \frac{9}{2}+$

$^{123}_{51}Sb$ $\Delta = -89.223$ 42.7% $J = \frac{7}{2}+$

$^{123}_{52}Te$ $\Delta = -89.172$ 1.3×10^{13} yrs EC, 0.908% $J = \frac{1}{2}+$

β^+ decay

[baryon #, lepton #, charge]

$p \rightarrow n \quad e^+ \quad \nu_e$ not kinematically possible

$${}^{A}_{Z+1}Y \xrightarrow{?} {}^{A}_{Z}X \quad e^+ \quad \nu_e$$

proton converted to neutron

$$Q = m_{nucl}(Y) - m_{nucl}(X) - m_e$$

$$= [Au + \Delta(Y) - (Z+1)m_e] - [Au + \Delta(X) - Z m_e] - m_e$$

$$= \Delta(Y) - \Delta(X) - 2m_e$$

If $\Delta(Y) - \Delta(X) > 2m_e = 1.022$ MeV

then Y is unstable to β^+ decay (also EC)

β^+ decay is followed by

$$e^+ \quad e^- \rightarrow \gamma\gamma \qquad \text{(pair annihilation)}$$

$$Q = m_{e^+} + m_{e^-} = 1.022 \text{ MeV}$$

Each γ carries of 0.511 MeV (tell-tale sign of e^+e^- annihilation)

[first observed 1934 Joliot-Curie; cf Krane p.272]

Recall neutral atom

$$m(X) : \quad A\underbrace{u}_{931.494\,\text{MeV}} + \underbrace{\Delta(X)}_{\text{mass excess}} \qquad \text{experimental}$$

Also

$$m(X) = Z m_p + Z m_e + N m_n - \underbrace{B}_{\substack{\text{binding energy of nucleus} \\ (+\text{ atom}) \\ \text{negligible}}}$$

$$= Z\, m({}^1H) + N m_n - B$$

(ignore 13.6 eV)

$$= Z(1u + \Delta({}^1H)) + N(1u + \Delta(n)) - B$$

$$= A\,u + Z\,\Delta({}^1H) + N\Delta(n) - B$$

Compare

$$B = Z\underbrace{\Delta({}^1H)}_{7.289} + N\underbrace{\Delta(n)}_{8.071} - \Delta(X)$$

[check: B for ¹H and n]

Rewrite in terms of A and N − Z

$$B = \underbrace{(N+Z)}_{A}(7.680\,\text{MeV}) + (N-Z)(0.391\,\text{MeV}) - \Delta(X)$$

For ^{12}C, $A = 12$, $N = Z$, $\delta = 0$ so

$B(^{12}C) = 12\ (7.680\ \text{MeV})$

For nuclei similar to ^{12}C, $N \approx Z$ and $\delta \approx 0$

$B = (7.7\ \text{MeV})\, A$

ie binding energy per nucleon is approx const.

[see curve of binding energy: fig 3.16

$m(X) = Z\,m(^1H) + N\,m_n - B$

$\approx A\ (939\ \text{MeV}) - A\ (8\ \text{MeV})$

$\approx (931\ \text{MeV})\ A$

Nuclei are $\sim 1\%$ less massive than sum of constituents

[deuteron only 0.1% $\Rightarrow$ weak binding]

$\downarrow$

see curve!

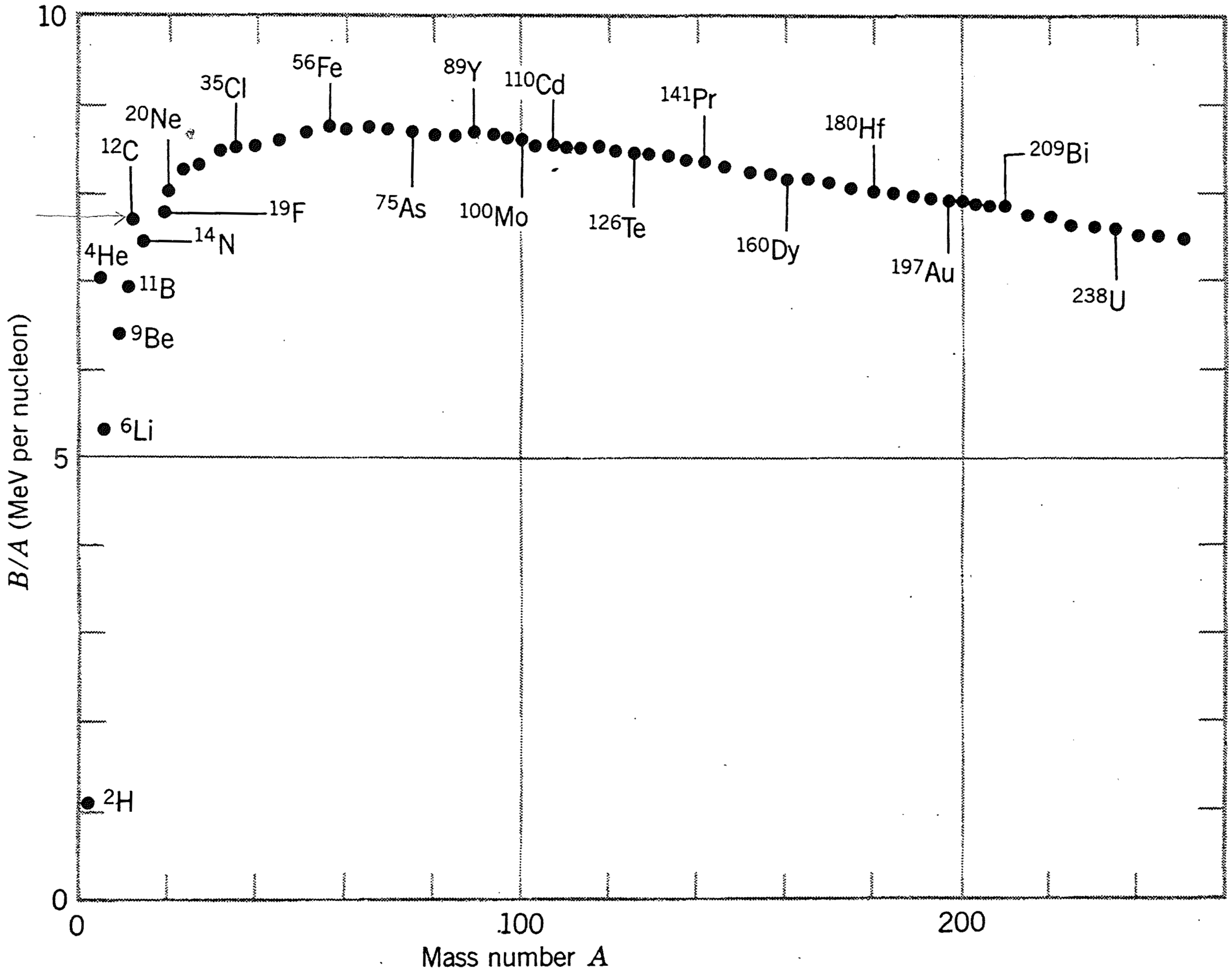


Figure 3.16 The binding energy per nucleon.