History of subatomic physics

[Atoms: supposedly smallest constituents of matter
[Atom = uncuttable

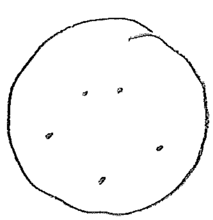
1897 JJ Thomson

[realizes cathode rays are much lighter than H atom
due to large deflection by E + B fields
→ he had discovered first sub atomic particle

Cathode rays = electrons

[electrons must come out of atoms
but atoms are neutral, so what about + charge?
Thomson imagined some sort of jelly in which
electrons were embedded]

plum pudding model of atom



[I skip 3140 of this & we then go on
to explore structure of atom

1896 Henri Becquerel

[studied phosphorescence of uranium:
expose to sunlight, & then goes to radiation
cloudy day, so put uranium in drawer
later decided to develop film anyway
found as much radiation as on sunny days
"uranic rays"

radioactivity (emission of rays by unstable nuclei)

[Also, Th, and Ra, Po, disc. by Marie Curie in pitchblende
due to their high activity

Marie Curie
1 Bq = 1 decay/sec
1 Ci = # decays from 1 g radium/sec

Ernest Rutherford
[New Zealand → McGill → Manchester → Cambridge
[identifies 3 types of radioactivity by
their different ranges
Becquerel observes their curvature in $\vec{B}$ field

Three types of radioactivity

α = positive charge ⇒ see fig 1-2 in Segrè
β = negative charge
γ = neutral

[1909 Royds, Rutherford in Manchester
captured α particles, measured spectrum

α = ^4He nuclei
[radioactivity in earth's crust → He in natural gas deposits
β = e^- [same mass as cathode rays
γ = high energy photons

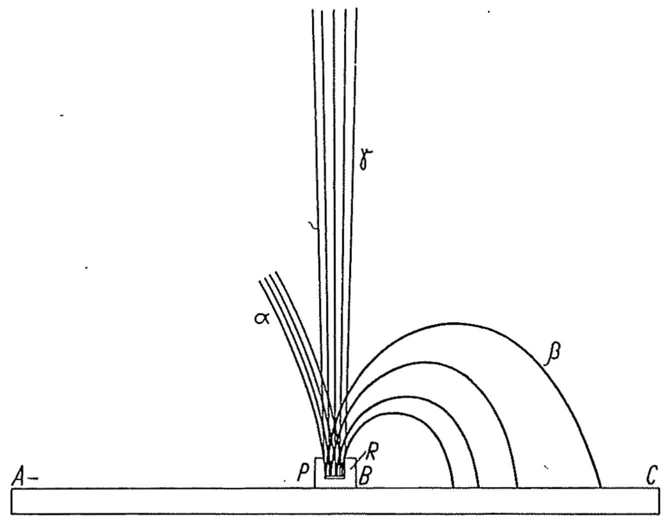


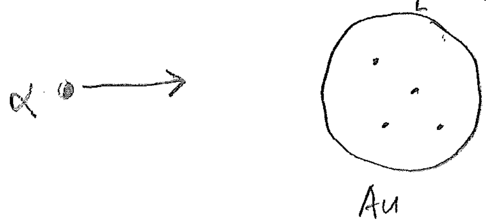
Figure 1-2. Deflection of alpha, beta, and gamma rays in a magnetic field. The nomenclature is due to Rutherford (1899). [Mme Curie, Thesis, 1904.]

Segrè, p. 3

Today's discoveries = tomorrow's tools

1909 Rutherford, Geiger, Marsden

[Because α emitted from nuclei
w/ considerable energy
Rutherford et al used them to probe structure of atom.
Used thin gold foil to reduce likelihood of multiple scattering



$m_\alpha = 4 m_H$

m_{Au} $197 m_H$ [distributed]

$m_e \approx \frac{1}{1800} m_H$

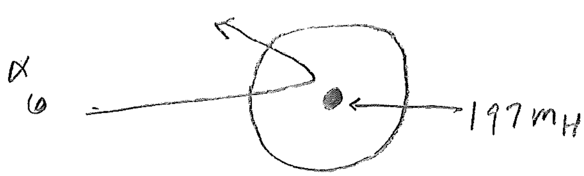
$\frac{m_\alpha}{m_e} \approx 7000$

[$\frac{\text{bowling ball}}{\text{ping pong}}$ $\frac{7\text{ kg}}{2.7\text{ g}} \sim 2500$]

$\hookrightarrow$ not much stopping power

Most α deflected slightly but some experienced large deflections

[Rutherford realized + chg concentrated in heavy nucleus

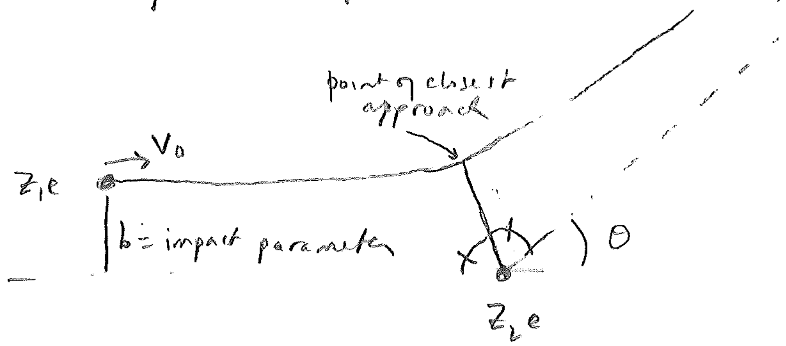


Rutherford model of atom

Rutherford scattering

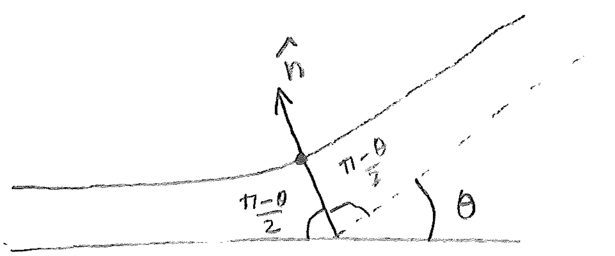
[To validate his model, Rutherford calculated:]

deflection of an α-particle from a fixed, point-like nucleus X by Coulomb force



[inverse square $\Rightarrow$ elliptical, parabolic, hyperbolic]
α moves along hyperbolic orbit symmetric w.r.t. point of closest approach

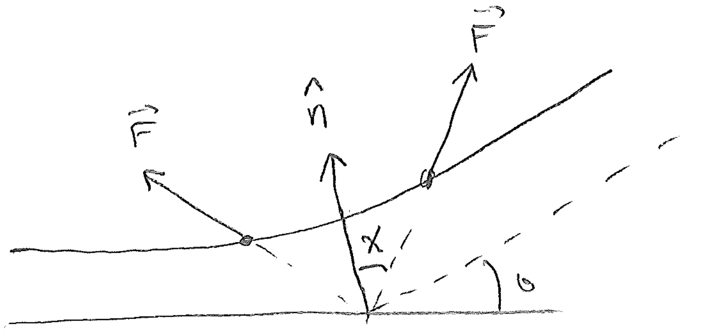
<u>Goal</u>: find θ as a function of v_0 and b



<u>To show</u>: change in momentum $\Delta\vec{p} \parallel \hat{n}$

Energy conservation $\Rightarrow |\vec{p}_f| = |\vec{p}_i|$ [isosceles triangle]

$$\boxed{\Delta p = 2mv_0 \sin\frac{\theta}{2}} \quad (*)$$



[put χ in later]

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\Delta\vec{p} = \int d\vec{p} = \int \vec{F}\, dt = \text{impulse}$$

Net impulse $\perp \hat{n}$ vanishes because orbit symmetric

Net impulse $\parallel \hat{n}$

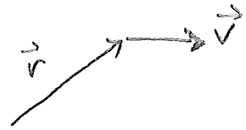
$$|\Delta\vec{p}| = \hat{n}\cdot\Delta\vec{p} = \int \vec{F}\cdot\hat{n}\, dt = \int F\cos\chi\, dt$$

To do this integral, we need to relate dt to $d\chi$.

Use cons. of angular moment

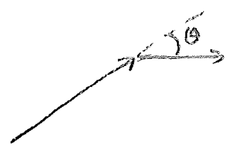
Orbital angular momentum

$\vec{l} = \vec{r} \times m\vec{v}$



[which direction? into page]

$l = rmv \sin\theta$

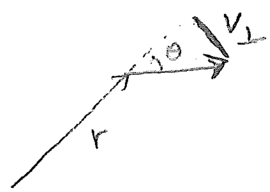


θ = angle between $\vec{r}$ and $\vec{v}$

Define $r_\perp$ = component of $\vec{r}$ $\perp$ to $\vec{v}$ $= r\sin\theta$

$\Rightarrow l = mvr_\perp$

Define $v_\perp$ component of $\vec{v}$ $\perp$ to $\vec{r}$ $= v\sin\theta$

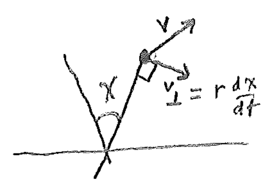


$\Rightarrow l = mv_\perp r$

Initially

$l = r_\perp mv \quad bmv_0$

At arbitrary point in orbit



$$v_\perp = r\frac{d\chi}{dt}$$

$$l = rmv_\perp = r^2 m \frac{d\chi}{dt}$$

Conservation of angular momentum in a central force (radial)

$$l = bmv_0 = r^2 m \frac{d\chi}{dt}$$

$$\frac{d\chi}{dt} = \frac{bv_0}{r^2}$$

$$dt = \frac{r^2}{bv_0} d\chi$$

Coulomb force

$$|\vec{F}| = \frac{k}{r^2} \qquad \text{where} \quad k = K(Ze_1)(Ze_2)$$

Then

$$|\Delta\vec{p}| = \int F \cos\chi \, dt$$

$$= \int \frac{k}{r^2} \cos\chi \, \frac{r^2}{bv_0} \, d\chi$$

$$= \frac{k}{bv_0} \int \cos\chi \, d\chi$$

$$= \frac{k}{bv_0} (\sin\chi) \Big|_{-\left(\frac{\pi-\theta}{2}\right)}^{\frac{\pi-\theta}{2}}$$

$$= \frac{2k}{bv_0} \sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

$$\Delta p = \frac{2k}{bv_0} \cos\left(\frac{\theta}{2}\right) \qquad (**)$$

Equating two expressions for Δp: (✱) + (✱✱)

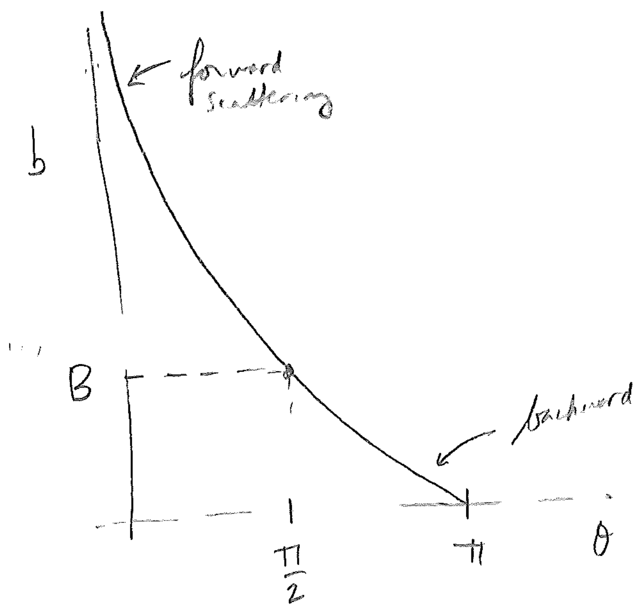
$$2mv_0 \sin\frac{\theta}{2} = \frac{2k}{v_0 b}\cos\frac{\theta}{2}$$

$$\Rightarrow \quad b = \frac{k}{mv_0^2}\cot\frac{\theta}{2}$$

$$\boxed{b = \frac{Z_1 Z_2 K e^2}{m v_0^2}\cot\frac{\theta}{2}}$$

For convenience define $B = \dfrac{Z_1 Z_2 K e^2}{m v_0^2}$

so $b = B\cot\frac{\theta}{2}$

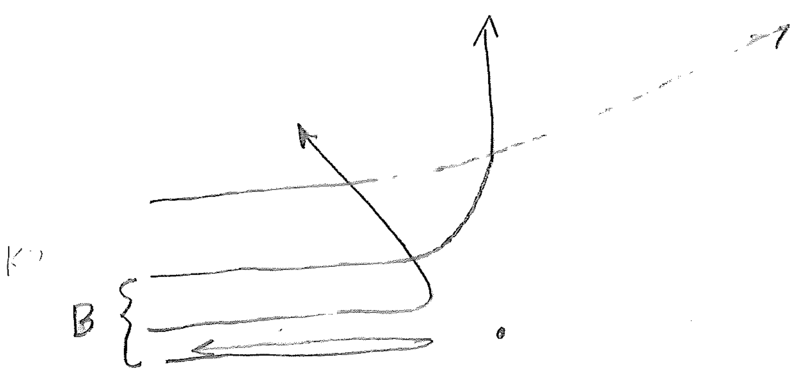


$\cot\frac{\theta}{2} = \dfrac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$

$\theta = 0 \Rightarrow \dfrac{\cos(0)}{\sin(0)} = \infty$

$\theta = \frac{\pi}{2} \Rightarrow \dfrac{\cos(\frac{\pi}{4})}{\sin(\frac{\pi}{4})} = 1$

$\theta = \pi \Rightarrow \dfrac{\cos(\frac{\pi}{2})}{\sin(\frac{\pi}{2})} = 0$



B = impact parameter for 90° scatter.

Scattering cross section

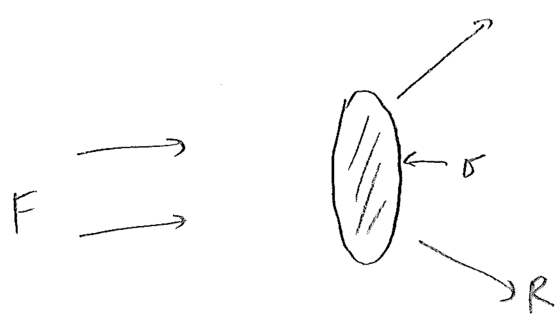
Let F = incident flux of α-particles = $\dfrac{\text{\# incident ptls}}{\text{sec} \cdot \text{area}}$

Let R = scattering rate of α-particles $\dfrac{\text{\# ptls scattered}}{\text{sec}}$

Expect R to be proportional to F [more incident ptls $\Rightarrow$ more ptls scattered]

Define $\boxed{\sigma = \dfrac{R}{F}}$

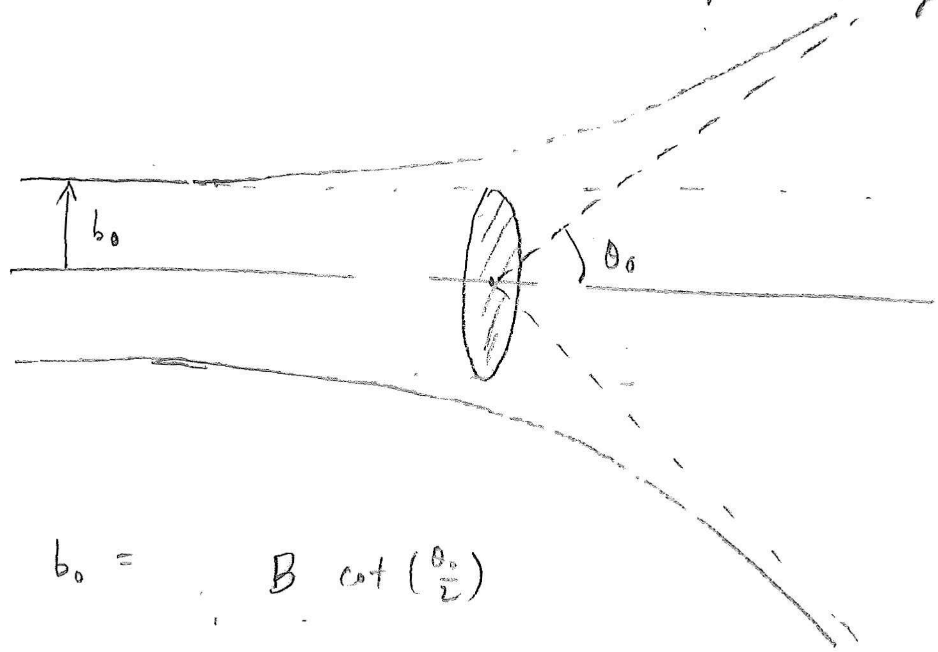
σ has units of area; it is the area of the scatterer intercepted by the incident flux



$R = F\sigma$

σ is called the scattering cross-section

Define $\sigma(\theta > \theta_0)$ = cross-section for scattering thru angle $> \theta_0$



$b_0 = B \cot\left(\frac{\theta_0}{2}\right)$

If $b < b_0$ then $\theta > \theta_0$

Any α particle that would have struck a disk of radius b_0 [had it not been deflected] will be scattered through $\underline{\theta > \theta_0}$

$\sigma(\theta > \theta_0) = \pi b_0^2$

Total cross-section for Rutherford scattering $\sigma_{tot} = \sigma(\theta > 0)$

is technically infinite because all incident particles are scattered (deflected) to some extent.

Practically speaking, α particles that pass beyond the electron shells are unscattered (atom is neutral) so effective total cross-sect is no larger than πR_{atom}^2.

Griffiths, Quantum Mechanics, 2nd ed.

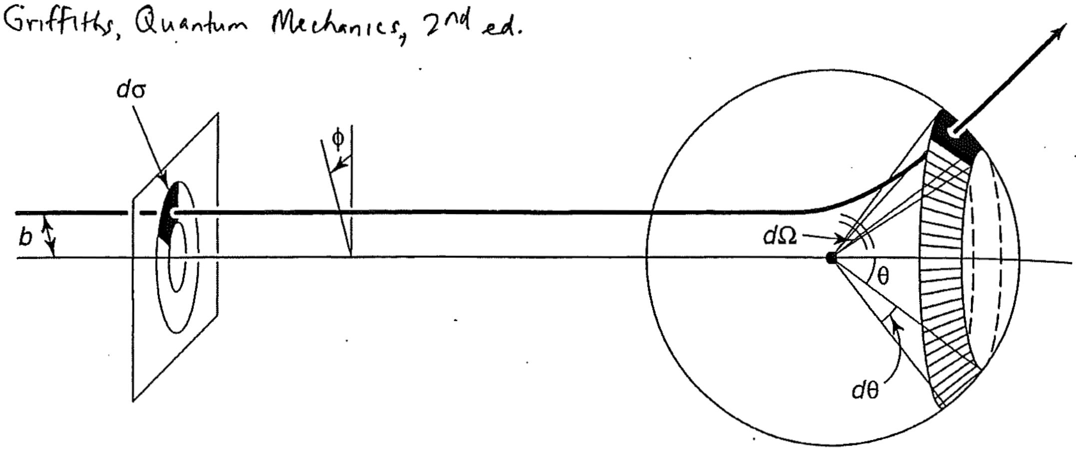
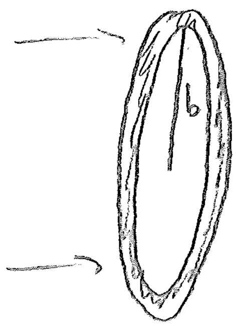


FIGURE 11.3: Particles incident in the area $d\sigma$ scatter into the solid angle $d\Omega$.

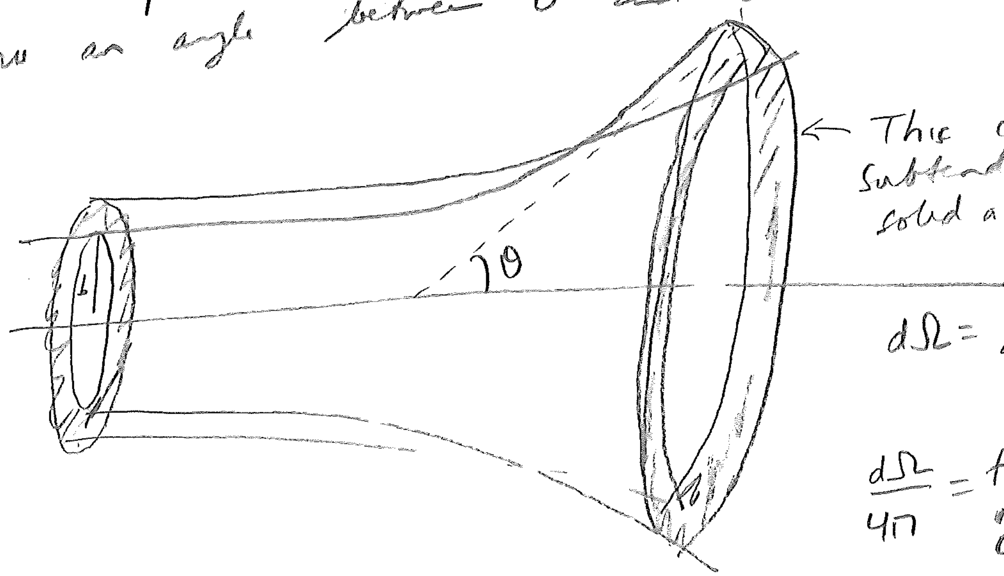
Differential cross section



Consider incident particles of impact parameter between b and $b + db$

$d\sigma =$ area of annulus $= 2\pi b \, db$

All particles passing through this annulus are scattered thru an angle between θ and $\theta + d\theta$



This collar subtends solid angle $d\Omega$

$d\Omega = 2\pi \sin\theta \, d\theta$

$\frac{d\Omega}{4\pi}$ = fraction of area of unit sphere

Define differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{2\pi b \, db}{2\pi \sin\theta \, d\theta}$$

[absolute value because $\theta \downarrow$ as $b \uparrow$]

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta} \left| \frac{db}{d\theta} \right|}$$

← classical differential cross section

Then $\sigma(\theta > \theta_0) = \int d\sigma = \int \frac{d\sigma}{d\Omega} \, d\Omega$

$$= \int_{\theta=\theta_0}^{\theta=\pi} \frac{d\sigma}{d\Omega} \, 2\pi \sin\theta \, d\theta$$

Total cross section $\sigma = \sigma(\theta > 0) = \int_0^{\pi} \frac{d\sigma}{d\Omega} \, 2\pi \sin\theta \, d\theta$

For Rutherford scattering, $b = B \cot\left(\frac{\theta}{2}\right)$

$$B = \frac{Z_1 Z_2 k e^2}{m v_0^2}$$

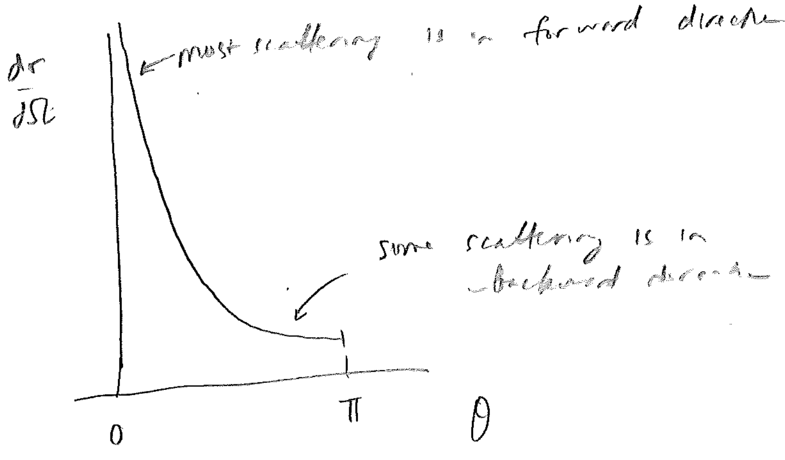
$$\left|\frac{db}{d\theta}\right| = \frac{1}{2} B \csc^2\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \frac{d\sigma}{d\Omega} \; \frac{1}{2} B^2 \frac{\cot\left(\frac{\theta}{2}\right)}{\sin\theta} \csc^2\left(\frac{\theta}{2}\right)$$

$$\sin\theta = 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$$

$$\frac{d\sigma}{d\Omega} \; \frac{1}{4} B^2 \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

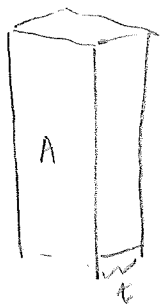
$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 k e^2}{2 m v_0^2}\right)^2 \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}}$$ Rutherford differential cross section



[HW: show $\sigma(\theta > \theta_0) = \int \frac{d\sigma}{d\Omega} d\Omega = \pi b_0^2$]

In Rutherford's experiment,
What <u>fraction</u> of all incident α particles will be scattered through an angle $> \theta_0$?

Consider a thin foil of thickness t and area A.

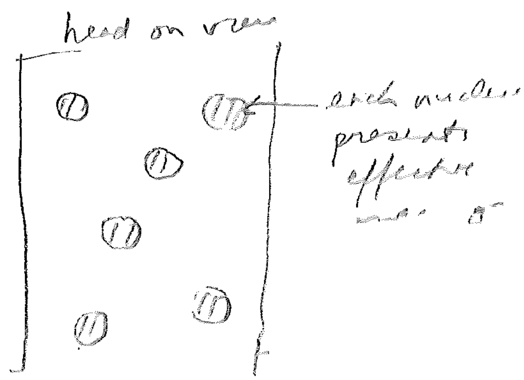


Let n = number density of nuclei
Total # of nuclei in foil $N = nAt$

Each nucleus presents a target of size $\sigma(\theta > \theta_0)$.

Total target area $= N\sigma$
Total area $= A$
Fraction occupied by targets

$$f = \frac{N\sigma}{A} = nt\sigma$$



[This is the fraction of α-particles scattered.

$\boxed{\text{fraction scattered } f = nt\sigma}$]

[Note: advantage of <u>thin</u> foil is to eliminate possibility of multiple interactions, so each α particle scatters only once]

Fraction of α-particles scattered through angle > θ_0

$$f = nt\,\sigma(\theta > \theta_0) = nt\pi b_0^2 = nt\pi B \cot\left(\frac{\theta_0}{2}\right)$$

Fraction of backscattered α-particles (ie $\theta \geq \frac{\pi}{2}$)

↳ $\cot\left(\frac{\pi}{4}\right) = 1$

$$f = nt\pi B, \text{ where } B = \frac{Z_1 Z_2 Ke^2}{mv_0^2}$$

Consider α-particles of kinetic energy $\frac{1}{2}mv_0^2 = 5$ MeV [is this nonrel?]
incident on gold foil 0.5 μm thick.
What fraction is back scattered?

$n = 5.9 \times 10^{28}\ m^{-3}$ for gold

$t = 5 \times 10^{-7}$ m

$Z_1 = 2$

$Z_2 = 79$

$mv_0^2 = 10$ MeV

Strength of EM force is characterized by dimensionless fine structure constant

$$\boxed{\alpha = \frac{Ke^2}{\hbar c} \approx \frac{1}{137}}$$

A very useful constant to know is $\left[\frac{197.327}{137.036} = 1.43996\right]$

$$\boxed{\hbar c = 197 \text{ MeV}\cdot\text{fm}}$$

$\Rightarrow Ke^2 = \alpha\hbar c = \frac{197}{137}$ MeV fm = 1.44 MeV fm

$$B = \frac{2\cdot(79)(1.44 \text{ MeV fm})}{(10 \text{ MeV})} = 22.7 \text{ fm} \quad [22.75]$$

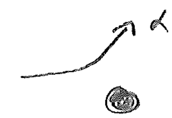
$f = 5 \times 10^{-5} \approx \frac{1}{20{,}000}$, small but measurable

[Geiger + Marsden: 1 in 8000 (Evans, p. 2)]

Rutherford & colleague's observations agreed w/ predictions of Rutherford scattering (w.r.t angular dependence, Z_2, initial v_0)

suggesting that the positive charge of an atom

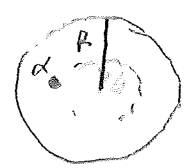
↳ is concentrated in a tiny, point-like nucleus



Rutherford scattering formula accurate for heavy nuclei (w/ $T_\alpha \lesssim 10$ MeV), but deviations observed for light nuclei

Suppose nucleus has a radius R. ⇒ finite size of nucleus

Then if incident particle comes closer than R, one expects deviations from the Rutherford prediction ~~because~~:



(1) nucleus no longer a point charge

(2) if incident particle is a hadron, strong-force acts between it & nucleus

No deviations from Rutherford prediction for heavy nuclei (w/ $T_\alpha \lesssim 10$ MeV).

but Deviations observed for light nuclei.

Also sometimes nuclear transmutation occurs:

$$\alpha + {}^{14}_{7}N \longrightarrow {}^{17}_{8}O + p$$

nuclear transmutation

[Use deviations from Rutherford scattering to estimate size of nuclei

Various experiments suggested that (e^- scattering)

nuclear radius $R \sim A^{1/3}$

Specifically $R = A^{1/3} r_0$

$r_0 \simeq 1.2 \times 10^{-15}\ m = 1.2\ fm$

Thus most nuclei between 1 – 10 fm

[Atomic radii are ~ 1 Å $\simeq 10^{-10}\ m$, so 4 to 5 orders of magnitude

If hydrogen atom were size of this room [$\sim 10\ m$]

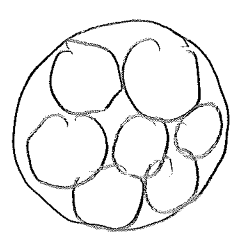
then size of nucleus is $\sim \underbrace{0.1\ mm}_{}$ and 1 mm

human hair width.

Volume of nucleus $= \frac{4}{3}\pi R^3 = \left(\frac{4}{3}\pi r_0^3\right) A$

proportional to # nucleons

$\Rightarrow$ nuclei act as a collection of close-packed incompressible nucleons



Bethe + Morrison

size of nuclei determined by

① neutron cross-section ~20 MeV (for fast but not too fast neutrons)

treat neutrons geometrically

nucleus becomes transparent

$$\lambda = \frac{\hbar}{p} = \frac{\hbar c}{\sqrt{2mc^2 T}} = \frac{200}{\sqrt{2\cdot(20)(940)}} \cdot 1 \text{ fm}$$

② α-decay lifetime

③ nuclear σ for chgd parts (must traverse barrier)

④ mirror nuclei

$^{3}H,\ ^{3}He$
$^{7}Li,\ ^{7}Be$
$^{11}C,\ ^{11}B$
$^{13}C,\ ^{13}N$
$^{15}N,\ ^{15}O$
$^{17}O,\ ^{17}F$
$^{29}Si,\ ^{29}P$

⑤ semi-empirical

⑥ e^- scattering

⑦ μ-mesic systems