bkgnd-weak-2023-06

Bkgnd notes on weak interaction

(1) W, Z decay

(2) ν scattering

(3) β-decay

n-decay

(4) old notes on μ decay (2017)

(5) old notes on n decay (2017)

(6) 2 particle weak decay

π decay

K decay

Λ, Ξ decay

① W decay

$$R = \frac{1}{2m_W\hbar}\int \underbrace{(\text{LIPS})_2}_{\frac{p_f}{(4\pi)^2 E_{cm}}} |A|^2 = \frac{p_f}{2\hbar\,(4\pi)^2 m_W^2}\int d\Omega\, |A|^2$$

$A =$

$p + \sqrt{p^2+m_e^2} = m_W$

$p = \frac{m_W}{2}\left(1 - \frac{m_e^2}{m_W^2}\right)$

$\sqrt{p^2+m_e^2} + p = m_W$

$E = \frac{m_W}{2}\left(1 + \frac{m_e^2}{m_W^2}\right)$

crude approx

$$g\sqrt{(2E_e)(2E_\nu)}\, f$$

$$= g\sqrt{m_W\left(1+\frac{m_e^2}{m_W^2}\right) m_W\left(1-\frac{m_e^2}{m_W^2}\right)}\, f$$

$$= g m_W f \sqrt{1 - \frac{m_e^4}{m_W^4}}$$

$$|A|^2 = G m_W^4 f^2\left(1 - \frac{m_e^4}{m_W^4}\right)$$

Define $G_F = \frac{g^2}{m_W^2}$

$G_F = 1.166\ldots \text{ GeV}^{-2}$

$m_W = 80.4 \text{ GeV}$

$G_F m_W^3 = 6 \text{ GeV}$

$\frac{G_F m_W^3}{16\pi} = 120 \text{ MeV}$

$\hbar = 66 \text{ MeV}(10^{-23}\text{ s})$

$$R = \frac{G_F m_W^3 f^2}{16\pi\hbar}\left(1 - \frac{m_e^2}{m_W^2}\right)\left(1 - \frac{m_e^4}{m_W^4}\right)$$

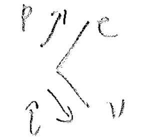
$\left(f^2 = \frac{16}{\sqrt{2}} = 1.8856\right)$

$$\approx \frac{2f^2}{10^{-24}\text{ s}}\left(1 - \frac{m_e^2}{m_W^2}\right)\left(1 - \frac{m_e^4}{m_W^4}\right)$$

$$\frac{g}{4\pi} \Rightarrow G_F = \frac{1}{4\sqrt{2}}\frac{g^2}{m_W^2}$$

W-decay

[Griffiths, p. 97] $\quad \nu e W$ vertex $\quad -i\left(\frac{G_F m_W^2}{\sqrt{2}}\right)^{1/2}\gamma_\mu(1-\gamma_5)$

W  $= -i\left(\frac{G_F m_W^2}{\sqrt{2}}\right)^{1/2} \bar{u}_e\gamma_\mu(1-\gamma_5)v_\nu\,\varepsilon^\mu$

$$|A|^2 = \frac{8G_F m_W^2}{\sqrt{2}}\left(\varepsilon\cdot q\,\varepsilon^*\cdot p + \varepsilon\cdot p\,\varepsilon^*\cdot q - \varepsilon\cdot\varepsilon^*\,p\cdot q + i\epsilon_{\alpha\beta\gamma\delta}\varepsilon^\alpha q^\beta \varepsilon^{*\gamma}p^\delta\right)$$

[eq. 6.7.16]

$(m_e = 0)$

$$|A|^2 = \frac{4}{\sqrt{2}}G_F m_W^4\begin{cases}\sin^2\theta & \varepsilon^\mu = (0,0,0,1)\\ \frac{1}{2}(1+\cos^2\theta) - \cos\theta & \varepsilon^\mu = \frac{1}{\sqrt{2}}(0,-1,-i,0)\\ \frac{1}{2}(1+\cos^2\theta) + \cos\theta & \varepsilon^\mu = \frac{1}{\sqrt{2}}(0,-1,i,0)\end{cases}$$

$$R = \frac{1}{4\hbar(4\pi)^2 m_W}\int d\Omega\,|A|^2$$

$$= \frac{G_F m_W^3}{\sqrt{2}(4\pi)\hbar}\int\frac{d\Omega}{4\pi}\sin^2\theta$$

$$= \frac{G_F m_W^3}{6\pi\sqrt{2}\hbar}$$

$$= \frac{227\ \mathrm{MeV}}{\hbar}$$

$$\int\frac{d\Omega}{4\pi}\sin^2\theta = \int\frac{\sin\theta\,d\theta}{2}\sin^2\theta = \frac{1}{2}\int dx\,(1-x^2)$$

$$\int\frac{d\Omega}{4\pi}\,\frac{1}{2}(1+\cos^2\theta) = \frac{1}{4}\int dx\,(1+x^2)$$

Z · decay

[Quigg p 113]

$W \; \langle^{e}_{\nu} \quad -i\left(\frac{G_F m_W^2}{\sqrt{2}}\right)^{\frac{1}{2}} \gamma_\mu(1-\gamma_5)$

$Z \; \langle^{\nu}_{\bar\nu} \quad -\frac{i}{\sqrt{2}}\left(\frac{G_F m_Z^2}{\sqrt{2}}\right)^{\frac{1}{2}} \gamma_\mu(1-\gamma_5)$

$Z \; \langle^{e^+}_{e^-} \quad -\frac{i}{\sqrt{2}}\left(\frac{G_F m_Z^2}{\sqrt{2}}\right)^{\frac{1}{2}} \gamma_\mu\left[R_e(1+\gamma_5)+L_e(1-\gamma_5)\right]$

$R_e = 2 s_W^2, \quad L_e = 2 s_W^2 - 1$

$m_Z = m_W \sec\theta_W$

— $Z \to \nu\bar\nu$ precisely as in $W \to e\nu$
but $m_W \to m_Z$ and fact of $\frac{1}{2}$

$$K = \frac{G_F m_Z^3}{12\pi\sqrt{2}\,\hbar} = \frac{G_F m_W^3}{6\pi\sqrt{2}\,\hbar}\,\frac{1}{(2\cos^3\theta_W)}$$

— $Z \to e\bar e$ same as $Z \to \nu\bar\nu$ but times $L_e^2 + R_e^2$

$(c_V^2 + c_A^2)$

problem

4 $\langle 0.1 \rangle$

(4-26-17)

$\Gamma^{expt}_{W \to e\bar{\nu}} \sim 220$ MeV

$W^+ \to u\bar{d}$

$$\Gamma = \frac{1}{2M_W} \int d(LIPS)_2 \; |A|^2$$

$$d(LIPS)_2 = \frac{\hat{p}}{16\pi^2 M_W} d\Omega$$

but for massless quarks $\hat{p} = \frac{M_W}{2}$ so $d(LIPS)_2 = \frac{d\Omega}{32\pi^2}$

$$\Gamma = \frac{1}{64\pi^2 M_W} \int d\Omega \; |A|^2$$

angle btw W polarization and direction of qk

Scalar quarks: $A \sim g\epsilon\cdot(p_1 - p_2) \sim 2g\epsilon\cdot p_1 \sim 2g \frac{M_W}{2}\cos\alpha = g M_W \cos\alpha$

$$\boxed{|A|^2 = g^2 M_W^2 \cos^2\alpha} = \underbrace{4\pi\alpha_W M_W^2 \cos^2\alpha}$$

$$\Gamma = \frac{4\pi\alpha_W M_W}{64\pi^2} \underbrace{\int d\Omega \cos^2\alpha}_{2\pi \frac{2}{3}}$$

$$= \frac{\alpha_W M_W}{12} = \frac{G_F M_W^3}{6\sqrt{2}\pi}$$

$$= 227 \text{ MeV}$$

$$\left(G_F = \frac{\pi}{\sqrt{2}} \frac{\alpha_W}{M_W^2}\right)$$

$$\alpha_W = \frac{\sqrt{2} G_F M_W^2}{\pi}$$

② ν Scattering (crude approximation)

$\nu X \to Y l$

$$|A| = G_F \sqrt{(2E_X)(2E_Y)(2E_\nu)(2E_l)}\ |M| \quad \text{(Crude approximation)}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \left(\frac{\hbar}{8\pi E_{cm}}\right)^2 \frac{p_f}{p_i} |A|^2 \quad \text{(isotropic)}$$

$$\sigma = 4\pi \left(\frac{\hbar}{8\pi E_{cm}}\right)^2 \frac{p_f}{p_i} \left(16\, G_F^2\, E_x E_y E_\nu E_l\, M^2\right)$$

$$= \frac{\hbar^2 G_F^2}{\pi} \frac{p_f}{\cancel{p_i}} \frac{E_x E_y \cancel{E_\nu} E_l}{E_{cm}^2} |M|^2$$

$$\left(\text{where } \sqrt{p_f^2 + m_l^2} + \sqrt{p_f^2 + m_Y^2} = E_\nu + \sqrt{E_\nu^2 + m_X^2}\right)$$

$$\frac{\hbar^2 G_F^2}{\pi} \frac{E_X E_Y E_l p_f}{E_{cm}^2} |M|^2$$

$$\sigma \xrightarrow{\text{high energy}} \frac{\hbar^2 G^2}{16\pi} E_{cm}^2 |M|^2 \qquad \frac{\hbar^2 G_F^2 s}{\pi} \frac{|M|^2}{16}$$

$$\sigma_{E_\nu \ll m_X} = \frac{\hbar^2 G_F^2}{\pi} \frac{m_X m_Y\, p_l \sqrt{p_l^2 + m_l^2}}{E_{cm}^2} |M|^2 \approx \frac{\hbar^2 G^2}{\pi} p_l \sqrt{p_l^2 + m_l^2}\, |M|^2$$

$E_{cm} \sim m_X \sim m_Y$

(b) Compute the cross section for this process using the relevant Feynman diagram. Express your final answer in terms of E_ν, m_p, m_n, m_e, and M where M is the matrix element in the amplitude.

(c) Assume that $M = 2$ as we did for neutron decay. Numerically evaluate the cross section (in barns) for $E_\nu = 2.5$ MeV.

63. **23.08.** *Antineutrino cross section.*
Consider a process in which an antineutrino with energy E_ν is absorbed by a stationary nucleus X, converting it into a nucleus Y and a positron: $\bar{\nu}_e + X \to Y + e^+$. Assume that E_ν is low enough that the kinetic energy of Y is negligible. This means that the lab frame and the CM frame are basically the same, so you can calculate everything in the CM frame, ignoring the kinetic energies of the X and Y.

(a) Determine the minimum energy E_ν required for this process to occur, in terms of $Q = m_X - m_Y - m_e$, where m_X and m_Y are the masses of the nuclei. We assume that $Q < 0$, otherwise the X would be unstable to β^+ decay.

(b) Assume that E_ν is above the threshold calculated in part (a). Determine the energy E_e and momentum p_e of the final-state positron in terms of E_ν, m_e, and Q.

(c) Draw the Feynman diagram for this process and write the amplitude A, making the same assumptions we did in class for the spin and nuclear stuff. Express your result in terms of m_X, E_ν, m_e, Q, and the nuclear matrix element M (as well as fundamental constants).

(d) Compute the differential cross-section $(d\sigma/d\Omega)_{\rm cm}$ for this process, starting from the general $2 \to 2$ formula that we derived in class, expressing your final answer in terms of m_X, E_ν, m_e, Q, and the nuclear matrix element M. Finally, you may assume that E_ν and Q are both much less than m_X, allowing a simplication of your result.

(e) Since the differential cross-section is independent of any angles, the total absorption cross section σ is obtained by simply multiplying it by 4π. Numerically evaluate σ for the absorption of an antineutrino of energy $E_\nu = 2.5$ MeV by a proton, expressing your result in fm^2 and also in barns. Note that since the Feynman diagram involves the same particles that were involved in neutron decay, we can use the same matrix element, namely, $M \approx 2.3$.

64. **23.08. 19.08. Since 2015.** *Cowan-Reines antineutrino detection experiment.*
In the first direct neutrino detection experiment, a beam of antineutrinos from a nuclear reactor traversed a tank of water. A schematic of the experiment can be found on Canvas. The lateral dimensions of the tank of water were 1.83 m $\times$ 1.32 m, and the height of each of the two water-filled sections was 7.6 cm. The beam flux through the tank was $F = 1.2 \times 10^{17}$ antineutrinos per second per meter squared.

(a) The average energy of the reactor antineutrinos was about 2.5 MeV. Show that this is sufficiently high to be absorbed by the hydrogen nuclei but not by the oxygen nuclei in the water.

(b) The cross-section for antineutrino absorption by a proton at the relevant energy scale is about 1.1×10^{-19} barns. This is very small but the flux is very high. Compute the

$\bar{\nu}_e X \to e^+ Y$ (assume $E_\nu \ll m_X$ so cm = lab frame)

$Q = m_X - m_Y - m_e < 0$

$E_\nu + m_X = m_Y + E_e$ (ignoring kinetic energy of X + Y)

$\Rightarrow E_e = E_\nu + m_X - m_Y$

$= m_e + E_\nu + Q \geq m_e \Rightarrow E_\nu > -Q$

$p_e = \sqrt{E_e^2 - m_e^2} = \sqrt{(Q+E_\nu)^2 + 2m_e(Q+E_\nu)}$

$A = G_F \sqrt{(2E_e)(2E_\nu)(2E_X)(2E_Y)}\, M$

$= 4 G_F \sqrt{E_e E_\nu m_X m_Y}\, M$

$= 4 G_F \sqrt{(E_\nu + m_e + Q) E_\nu m_X (m_X - m_e - Q)}\, M$

$E_{cm} = m_X + E_\nu$

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \left(\frac{\hbar}{8\pi E_{cm}}\right)^2 \frac{p_f}{p_i} |A|^2$$

$$= \frac{\hbar^2 G_F^2}{(2\pi)^2} \frac{p_e}{\cancel{E_\nu}} \frac{\cancel{E_\nu}\overbrace{(E_\nu + m_e + Q)}^{E_e} m_X (m_X - m_e - Q)}{(m_X + E_\nu)^2} |M|^2$$

Assume $E_\nu \ll m_X$, $Q + m_e \ll m_X$

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \left(\frac{\hbar G_F}{2\pi}\right)^2 |M|^2 p_e E_e$$

$E_e = E_\nu + \overbrace{m_e + Q}^{m_X - m_Y}$

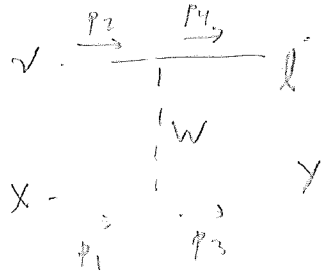
$p_e = \sqrt{E_e^2 - m_e^2}$

$$\sigma_{cm} = \frac{\hbar^2 G_F^2 |M|^2 p_e E_e}{\pi}$$

M ? for fund. [illegible]
Matrix el. of [illegible] (including [illegible])

ν scattering from a fundamental spin-½ particle

$\nu X \to Y \ell^-$



$$A = \frac{1}{8}\left(\frac{g_W}{M_W}\right)^2 \bar{u}_3 \gamma^\mu (1-\gamma^5) u_1 \; \bar{u}_4 \gamma_\mu (1-\gamma^5) u_2 \qquad \left(\frac{g_W}{M_W}\right)^2 = \sqrt{32}\, G_1$$

eq 9.38

[Griffiths, p. 309: $\nu_\mu e \to \nu_e \mu$]
Summing over spins, we get factor ½ ... to average over spin of X
(don't need factor of ½ for ν because only one spin)

$$\langle |A|^2 \rangle = \frac{1}{2}\left(\frac{g_W}{M_W}\right)^4 p_1 \cdot p_2 \; p_3 \cdot p_4 = 64 G_1^2 \; p_1 \cdot p_2 \; p_3 \cdot p_4$$

(Eq. 9.11). Griffiths says he assumes $m_\nu = 0$ in deriving eq (9.7) but in eq (9.11) he says it does not matter (provided G_1 ... of ... 9.8)

$$\langle |A|^2 \rangle = 16 G_1^2 (s - m_Y^2)(s - m_Y^2 - m_\ell^2) \qquad \text{(180° ...)}$$

$$\sigma = 4\pi \left(\frac{\hbar}{8\pi E_{cm}}\right)^2 \frac{p_f}{p_i} |A|^2$$

$$= \frac{\hbar^2 G_1^2}{\pi} \frac{p_f}{p_i} \frac{(s - m_Y^2)(s - m_Y^2 - m_\ell^2)}{s}$$

where in cm frame

$$p_i + \sqrt{p_i^2 + m_i^2} = \sqrt{p_f^2 + m_\ell^2} + \sqrt{p_f^2 + m_Y^2}$$

High ... $p_f \to p_i$

$\sigma \to$ ...

Approximate in limit $E_\nu \ll m_X$

$$p_\nu = (E_\nu, 0, 0, E_\nu)$$

$$p_X : (m_X, 0, 0, -E_\nu)$$

$$p_\ell : (E_\ell, 0, 0, p_f)$$

$$p_Y : (m_Y, 0, 0, -p_f)$$

$$\langle |A|^2 \rangle = 64\, G_f^2 (p_\nu \cdot p_X)(p_\ell \cdot p_Y)$$

$$= 64\, G_f^2\, E_\nu m_X E_\ell m_Y = 4\, G_f^2 (2E_\nu)(2m_X)(2E_\ell)(2m_Y)$$

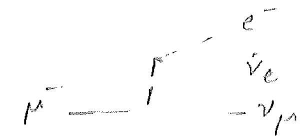
$$\sqrt{\langle |A|^2 \rangle} = G_f \sqrt{(2E_\nu)(2m_X)(2E_\ell)(2m_Y)} \cdot 2$$

my crude approximation, with M: ?

$$\boxed{\sigma = \frac{\hbar^2 G^2}{\pi} p_\ell E_\ell \cdot 4}$$

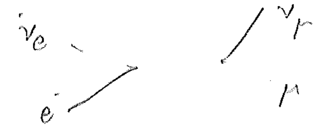
Inverse μ decay [Griffiths, p.309]

Since $\mu^- \to e^- \nu_\mu \bar{\nu}_e$



we can consider either

$$\bar{\nu}_e e^- \to \mu^- \bar{\nu}_\mu$$



(Ans ...)

or

$$\nu_\mu e^- \to \nu_e \mu^-$$

For the latter, we computed

$$\langle |A|^2 \rangle = 16 G_F^2 (s - m_e^2)(s - m_\mu^2)$$

$$\sigma = \frac{\hbar^2 G_F^2}{\pi} \frac{p_f}{p_i} \frac{(s - m_e^2)(s - m_\mu^2)}{s}$$

where $p_i + \sqrt{p_i^2 + m_e^2} = p_f + \sqrt{p_f^2 + m_\mu^2}$

further approximation, $m_e \simeq 0$

$$E_{cm} = 2p_e = p_f + \sqrt{p_f^2 + m_\mu^2}$$

$$\Rightarrow E_{cm}^2 - 2p_f E_{cm} = m_\mu^2$$

$$p_f = \frac{E_{cm}}{2}\left(1 - \frac{m_\mu^2}{E_{cm}^2}\right) = p_i\left(1 - \frac{m_\mu^2}{s}\right)$$

$$\Rightarrow \sigma = \frac{\hbar^2 G_F^2}{\pi} s \left(1 - \frac{m_\mu^2}{s}\right)^2 \quad \text{[Griffiths 9.14]}$$

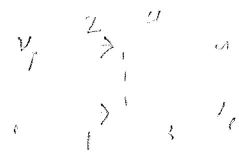
High energy limit: $\sigma \to \frac{\hbar^2 G_F^2 s}{\pi}$

$$\sqrt{\langle |A|^2 \rangle} \to 4 G_F s$$

crude approx $\to$ $A \simeq G_F s M$ as $M \to 4$ in h.e. lim

$\nu_\mu e^- \to \nu_e \mu^-$

2 1 4

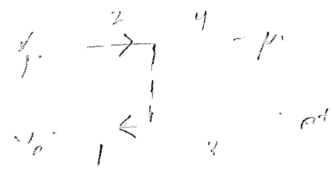


$u_3 \gamma^\mu (1 - \gamma^5) u_1 \quad \bar{u}_4 \gamma_\mu (1-\gamma^5) u_2$

$\langle |A|^2 \rangle \sim p_1 \cdot p_2 \; p_3 \cdot p_4 \sim s^2$

$\nu_\mu \bar{\nu}_e \to e^- \mu^-$

2 1 3 4

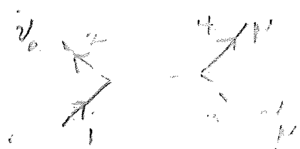


$\bar{v}_1 \gamma^\mu (1 - \gamma^5) v_3 \quad \bar{u}_4 \gamma_\mu (1-\gamma^5) u_2$

$\langle |A|^2 \rangle \sim p_1 \cdot p_4 \; p_2 \cdot p_3 \sim u^2$

$\bar{\nu}_e \; e^+ \to \bar{\nu}_\mu \mu^+$

2 1 3 4



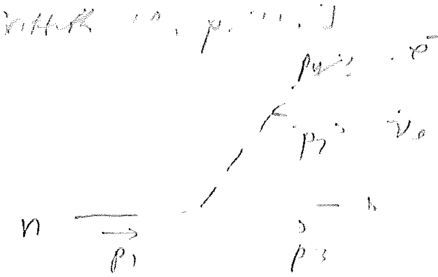
$\bar{v}_2 \gamma^\mu (1-\gamma^5) u_1 \quad \bar{u}_4 \gamma_\mu (1-\gamma^5) v_3$

$\langle |A|^2 \rangle \sim p_1 \cdot p_2 \; p_2 \cdot p_4$

use n decay → to get $\nu p \to n e^+$ cross-section

[mistake in ...]

$n \to p \bar{\nu}_e e$



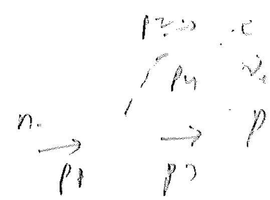
$$A = \frac{1}{8}\left(\frac{g_w}{m_w}\right)^2 \bar{u}_3 \gamma^\mu (c_V - c_A \gamma^5) u_1 \; \bar{u}_4 \gamma_\mu (1-\gamma^5) v_2$$

Griffiths prob 9.8 computes spin averaged amplitude ($\epsilon = \frac{c_A}{c_V}$)

$$\langle |A|^2 \rangle = \frac{1}{2}\left(\frac{g_w}{m_w}\right)^2 \left[(1-\epsilon)^2 p_1\cdot p_2 \; p_3\cdot p_4 + (1+\epsilon)^2 p_1\cdot p_4 \; p_2\cdot p_3 - (1-\epsilon^2) m_p m_n \, p_2\cdot p_4 \right]$$

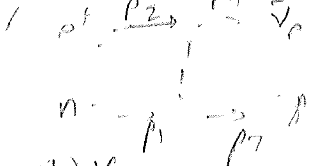
which reduces to $2\left(\frac{g_w}{m_w}\right)^2 p_1\cdot p_2 \; p_3\cdot p_4$ for $\epsilon = 1 \Rightarrow c_A = c_V$

Swap the labels $2 \leftrightarrow 4$



$$A = \frac{1}{8}\left(\frac{g_w}{m_w}\right)^2 \bar{u}_3 \gamma^\mu (c_V - c_A \gamma^5) u_1 \; \bar{u}_2 \gamma_\mu (1-\gamma^5) v_4$$

Now replace outgoing e^- w/ incoming e^+



$$A = \frac{1}{8}\left(\frac{g_w}{m_w}\right)^2 \bar{u}_3 \gamma^\mu (c_V - c_A \gamma^5) u_1 \; \bar{v}_2 \gamma_\mu (1-\gamma^5) v_4$$

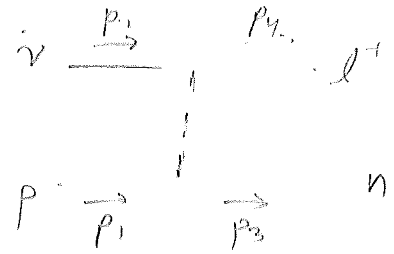
Then (might) involve letting $p_2 \to -p_2$ in $\langle |A|^2 \rangle$, & prob. not be as this must remain positive

Finally we time reverse everything
($p_1 \leftrightarrow p_3$, $p_2 \leftrightarrow p_4$ which has no effect)

$$\langle |A|^2 \rangle = \frac{1}{2}\left(\frac{g_w}{m_w}\right)^2 \left[(1-\epsilon)^2 p_1\cdot p_4 \; p_2\cdot p_3 + (1+\epsilon)^2 p_1\cdot p_2 \; p_3\cdot p_4 - (1-\epsilon^2) m_p m_n \, p_2\cdot p_4 \right]$$

$\bar{\nu}$ scattering from p

$\bar{\nu} p \to n \ell^+$



$$\langle |A|^2 \rangle = \frac{1}{2}\left(\frac{g_w}{m_w}\right)^2 \left[(1-\epsilon)^2 p_1 \cdot p_4 \; p_2 \cdot p_3 + (1+\epsilon)^2 p_1 \cdot p_2 \; p_3 \cdot p_4 - (1-\epsilon^2) m_p m_n \, p_2 \cdot p_4 \right]$$

Evaluate in the limit $E_\nu \ll m_p$

$p_1 = (m_p, 0, 0, 0)$

$p_2 = (E, 0, 0, E)$

$p_3 = (m_n, \ldots)$

$p_4 = (E_\ell, \ldots, p_\ell \cos\theta)$

$p_1 \cdot p_2 = m_p E \qquad p_2 \cdot p_3 = m_n E \qquad p_2 \cdot p_4 = E(E_\ell - p_\ell \cos\theta)$

$p_1 \cdot p_4 = m_p E_\ell \qquad p_3 \cdot p_4 = m_n E_\ell$

$$\langle |A|^2 \rangle = \frac{1}{2}\left(\frac{g_w}{m_w}\right)^2 \left[(1-\epsilon)^2 m_p m_n E E_\ell + (1+\epsilon)^2 m_p m_n E E_\ell - (1-\epsilon^2) m_p m_n E (E_\ell - p_\ell \cos\theta) \right]$$

This term integrates to zero in the cross-section

$$= \frac{1}{2}\left(\frac{g_w}{m_w}\right)^2 m_p m_n E E_\ell \left[\underbrace{(1-\epsilon)^2 + (1+\epsilon)^2 - (1-\epsilon^2)}_{1+3\epsilon^2} \right]$$

$$= 16 G_F^2 m_p m_n E E_\ell (1 + 3\epsilon^2)$$

so $M = \sqrt{1+3\epsilon^2}$

$$= 16 G_F^2 m_p m_n E E_\ell \; M^2$$

(crude approx)

If $c_V = 1$ and $c_A = 1.27$ (eq. 9.64)

then $M = \sqrt{1+3(1.27)^2} = 2.42$

$\theta_c = 13.1°$ $\quad M \cos\theta_c = 2.36$

$\bar{\nu} p \to n \ell^{+}$ in ... 1 GeV

$\bar{\nu}$ — ... p

$p_2 \cdot p_4$ ... $p_1 \cdot p_3$

$\langle |M|^2 \rangle = \frac{1}{2}\left(\frac{g_W}{m_W}\right)^2 \left[(1 - \epsilon)^2 \ldots + (1+\epsilon)^2 \ldots \right]$

$= \frac{1}{2}\left(\frac{g_W}{m_W}\right)^4 \left[(1 - \epsilon)^2 E^4 (1 + \cos\theta)^2 \ldots \right]$

$s^2 = 16E^4$

$4 G_F^2 \ldots \left[(1-\epsilon)^2 \left(\frac{1+\cos\theta}{2}\right)^2 + (1+\epsilon)^2 \right]$

Now $\left\langle \left(\frac{1+\cos\theta}{2}\right)^2 \right\rangle = \frac{1}{2}\int_{-1}^{1} dx \left(\frac{1+x}{2}\right)^2 = \frac{1}{8}\left(x + x^2 + \ldots\right) = \frac{1}{3}$

$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \left(\frac{\hbar}{8\pi \ldots}\right)^2 \ldots \langle |M|^2 \rangle = \frac{\hbar^2 G_F^2 \ldots}{(4\pi)^2} \left[(1-\epsilon)^2 \left(\frac{1+\cos\theta}{2}\right)^2 + (1+\epsilon)^2 \right]$

$\sigma = \frac{\hbar^2 G_F^2 \ldots}{4\pi} \left[(1-\epsilon)^2 \frac{1}{3} + (1+\epsilon)^2 \right]$

... $\frac{\hbar \ldots s}{\pi} \frac{\ldots}{16} \Rightarrow |M|^2 = 4\left[(1-\epsilon)^2 \frac{1}{3} + (1 \ldots \right]$

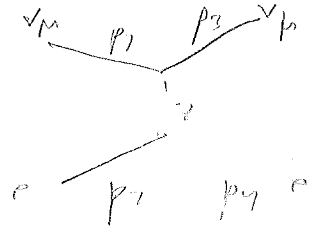
If $\epsilon = 1.27$ ... $|M|^2 = 1.16 \Rightarrow \ldots 2.68$

$\ldots \frac{\hbar^2 G_F^2 s}{\pi} \cdot (0.48) = 1.694 \ldots s\,(0.48)$

$8.1 \ldots \left(\frac{E}{\text{MeV}}\right)^2$

$\boxed{\nu_\mu e \to \nu_\mu e}$

[Griffiths^{2e} p. 332]



$c_V = -\frac{1}{2} + 2\sin^2\theta_W$, $c_A = -\frac{1}{2}$ } electron

$g_Z = \frac{g_W}{\cos\theta_W}$

$m_Z = \frac{m_W}{\cos\theta_W}$

$$A = \frac{g_Z^2}{8 m_Z^2} \bar{u}_\nu \gamma^\mu (1-\gamma^5) u_\nu \; \bar{u}_e \gamma_\mu (c_V - c_A \gamma^5) u_e$$

$\left(\frac{g_Z^2}{m_Z^2} = \frac{g_W^2}{m_W^2} \text{ ...}\right)$

$$\langle |A|^2 \rangle = \frac{1}{2}\left(\frac{g_Z}{m_Z}\right)^4 \Big[(c_V + c_A)^2 (p_1 \cdot p_2)(p_3 \cdot p_4) + (c_V - c_A)^2 (p_1 \cdot p_4)(p_2 \cdot p_3) - m^2 (c_V^2 - c_A^2)(p_1 \cdot p_3) \Big]$$

$\frac{g_W}{m_W} = (\sqrt{32}\, G_F)^{\frac{1}{2}}$

cm frame and $m_e \to 0$

$$\langle |A|^2 \rangle = 2 \left(\frac{g_Z}{m_Z}\right)^4 \Big[(c_V + c_A)^2 + (c_V - c_A)^2 \cos^4\frac{\theta}{2} \Big]$$

$$= 64\, G_F^2 E^4 \Big[(c_V + c_A)^2 + (c_V - c_A)^2 \cos^4\frac{\theta}{2} \Big]$$

$\to 4 G_F^2 s^2 \implies |A| \sim \underline{\underline{2 G_F s}}$

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi (2E)}\right)^2 |A|^2$$

$$= \frac{\hbar^2 E^2}{2^7 \pi^2} \underbrace{\left(\frac{g_Z}{m_Z}\right)^4}_{32 G_F^2} \Big[(c_V + c_A)^2 + (c_V - c_A)^2 \cos^4\frac{\theta}{2} \Big]$$

$$= \frac{\hbar^2 G_F^2 E^2}{4\pi^2}$$

$$= \frac{\hbar^2 G_F^2 s}{16\pi^2}$$

$$\sigma = \frac{4\hbar^2}{3\pi} \underbrace{\left(\frac{g_Z}{2 m_Z}\right)^4}_{4 G_F} E^2 (c_V^2 + c_A^2 + c_V c_A)$$

$$= \frac{8 G_F^2 E^2}{3\pi}$$

down by a factor of 4 from CC process because of $\frac{1}{2}$ in vertex factor

(3) β decay (3 pcl final state)

$$R = \frac{1}{2m_x \hbar} \int (\text{LIPS})_3 \, |A|^2$$

nucleus mtx elt

crude approx

$A \sim$  $= \dfrac{g^2}{M_W^2} \sqrt{(2E_x)(2E_y)(2E_e)(2E_\nu)} \; M$

$\rightarrow G_F$

$$R = \frac{G_F^2}{\hbar (2\pi)^5} \int d^3p_e \, d^3p_\nu \, \delta(T_e + E_\nu - Q) \, |M|^2, \qquad Q = m_x - m_y - m_e$$

(do not include m_ν)

Assume M insensitive to $\theta_{e\nu}$

$$R = \frac{G_F^2}{2\pi^3 \hbar} \int p_e^2 \, dp_e \int p_\nu^2 \, dp_\nu \, \delta(T_e + E_\nu - Q) \, |M|^2$$

Assume ν massless

$$= \int dp_e \, \frac{G^2}{2\pi^3 \hbar} \, p_e^2 \, (Q - T_e)^2 \, |M|^2$$

If M insensitive to E_e $\qquad E_e \, dE_e = p_e \, dp_e$

$$= \frac{G^2 |M|^2}{2\pi^3 \hbar} \int dp_e \, p_e^2 \, (Q - T_e)^2$$

$$\int dE_e \, E_e \sqrt{E_e^2 - m_e^2} \, (m_x - m_y - E_e)^2$$

neutron decay very crude approx

$m_n - m_p$

m_e E_e $\sqrt{E_e^2 - m_e^2}$ $(m_n - m_p - E_e)^2$ = 0.057 MeV^5

If n & p same ~~[illegible]~~ spin 1/2, $|M| \approx 1$.

$$R = \frac{G_F^2}{2\pi^3 \hbar} |M|^2 (0.057 \text{ MeV}^5)$$

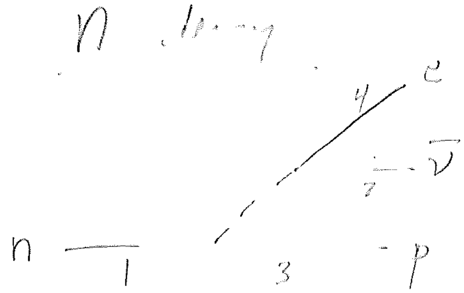
$$= (1.9 \times 10^{-4} \text{ s}^{-1}) |M|^2$$

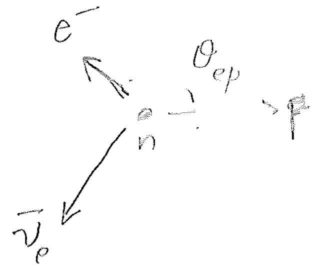
$$\tau = \frac{1}{R} = \frac{5300 \text{ sec}}{|M|^2}$$

expt ⇒ τ = 880 s → M^2 = 6.02

M = 2.45

n decay





Griffiths (p [illegible])

$$\langle |A|^2 \rangle = \frac{1}{2}\left(\frac{[illegible]}{M_W^2}\right)^2 \left[(1-\epsilon)^2 \, p_1 \cdot p_2 \; p_3 \cdot p_4 + (1+\epsilon)^2 \, p_1 \cdot p_4 \; p_2 \cdot p_3 - (1-\epsilon^2) m_p m_n \, p_2 \cdot p_4 \right]$$

$p_1 = (m_n, 0, 0, 0)$

$p_2 = (E_\nu, E_\nu \sin\theta', 0, E_\nu \cos\theta')$ — θ' = angle for $p + \nu$

$p_3 = (E_p, 0, 0, p_p)$

$p_4 = (E_e, p_e \sin\theta, 0, p_e \cos\theta)$ — θ = angle for $p + e$

$p_1 \cdot p_2 = m_n E_\nu$

$p_3 \cdot p_4 = E_e E_p \left(1 - \frac{p_p p_e}{E_p E_e} \cos\theta\right) = E_e E_p (1 - v_p v_e \cos\theta)$

$p_1 \cdot p_4 = m_n E_e$

$p_2 \cdot p_3 = E_\nu E_p \left(1 - \frac{p_p}{E_p} \cos\theta'\right) = E_\nu E_p (1 - v_p \cos\theta')$

$p_2 \cdot p_4 = E_\nu E_e \left(1 - \frac{p_e}{E_e} [illegible] \right) = [illegible] \, v_e \cos(\theta - \theta'))$.

angle for $e + \nu$

We [illegible] that θ, θ' dependence [illegible] [illegible] but not when you do the [illegible] [illegible] dependence [illegible] [illegible] expect because v_n, v_e [illegible]

Also $E_p = m_p$ [illegible]

$p_1 \cdot p_2 \; p_3 \cdot p_4 = [illegible] = m_n E_p E_e E_\nu$

[illegible]

$$[illegible] = \frac{1}{166\pi^{?}} \left([illegible] \right)^2 m_n \, [illegible] \left[(1 [illegible])^2 + (1 [illegible])^2 (1 [illegible]) \right]$$

$$= [illegible] \left[(1 + 3 [illegible]) \right]$$

But [illegible]

$$= [illegible] \; M^{[illegible]}$$

so we can just set

$$M = [illegible] / 1 + 3 [illegible]$$

[illegible] M [illegible]

[illegible] $M = 2.4[illegible]$

$$T = \frac{5300 \text{ s}}{|M|^2} \qquad M [illegible] \Rightarrow [illegible] \; 1300 \text{ s} \quad \text{(Gottfried [illegible])}$$

$$M = 2.4[illegible] \Rightarrow [illegible] \; 900 \text{ s} \quad \text{(Gottfried [illegible])}$$

expt $\Rightarrow$ [illegible] $\approx$ 880 sec   Cabibbo [illegible] $\Rightarrow M = [illegible] \underbrace{[illegible]}_{0.974} [illegible] \cdot 36$

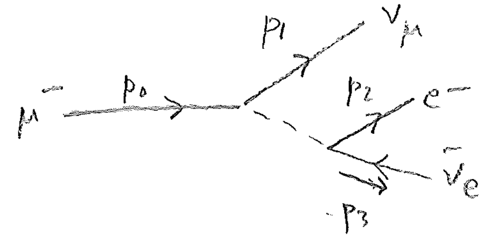
[illegible] 950 s [illegible]

(4-22-17) (4) | Old notes (2017) | (1)

μ decay

$$\mu^- \longrightarrow \nu_\mu \; e^- \; \bar{\nu}_e$$

$$p_0 = p_1 + p_2 + p_3$$



$$A \sim [\bar{u}(2)\gamma_\lambda(1-\gamma^5)v(3)][\bar{u}(1)\gamma^\lambda(1-\gamma^5)u(0)]$$

After squaring, summing over final spins and averaging over initial

$$\overline{|A|^2} = 64\,G_F^2\,(p_0\cdot p_3)(p_1\cdot p_2)$$

$$p_0\cdot p_3 = m_\mu E_3$$

$$p_1\cdot p_2 = \tfrac{1}{2}\left[(p_1+p_2)^2 - p_1^2 - p_2^2\right]$$

$$= \tfrac{1}{2}\left[(p_0-p_3)^2 - p_1^2 \; p_2^2\right]$$

$$= -m_\mu E_3 + \tfrac{1}{2}m_\mu^2 - \tfrac{1}{2}m_e^2 \qquad \text{for massless neutrinos}$$

energy of $\bar{\nu}_e$

$$\Rightarrow \boxed{\overline{|A|^2} = 64\,G_F^2\,m_\mu E_3\left(\tfrac{1}{2}m_\mu^2 - m_\mu E_3 - \tfrac{1}{2}m_e^2\right)} \leftarrow \text{assume massless neutrinos}$$

Alternatively $p_1\cdot p_2 = E_1E_2\left(1 - \frac{\vec{p}_1\cdot\vec{p}_2}{E_1E_2}\right)$

$$= E_1E_2\left(1 - \frac{p_1p_2}{E_1E_2}\cos\theta_{12}\right)$$

$$\overline{|A|^2} = 64\,G_F^2\,m_\mu E_1E_2E_3\left(1 - \frac{p_1p_2}{E_1E_2}\cos\theta_{12}\right) \leftarrow \text{no assumptions}$$

angle between electron and ν_μ

Width of μ^-

$$\Gamma = \frac{1}{2m_\mu}\int d(LIPS)_3 \, \overline{|A|^2}$$

massless neutrinos

$$= \frac{1}{2m_\mu}\frac{1}{(2\pi)^5}\frac{1}{8}\int \underbrace{d\Omega_3}_{4\pi}\,\underbrace{d\phi_{23}}_{2\pi}\, dE_2\, dE_3\, \underbrace{\overline{|A|^2}}_{64 G_F^2\, m_\mu E_3\left(\frac{1}{2}m_\mu^2 - m_\mu E_3 - \frac{1}{2}m_e^2\right)}$$

$$\boxed{\Gamma = \frac{4G_F^2}{(2\pi)^3}\int dE_2\, dE_3\; E_3(m_\mu^2 - 2m_\mu E_3 - m_e^2)}$$

Limits are easier if $m_e = 0$. Then $\int_0^{m_\mu/2} dE_3 \int_{m_\mu/2 - E_3}^{m_\mu/2} dE_2 = \int_0^{m_\mu/2} dE_3\, E_3$

$$\Gamma = \frac{4G_F^2}{(2\pi)^3}\int_0^{\frac{m_\mu}{2}} dE_3\; E_3^2\, m_\mu(m_\mu - 2E_3) \qquad x = \frac{2E_3}{m_\mu}$$

$$= \frac{G_F^2 m_\mu^5}{2(2\pi)^3}\underbrace{\int_0^1 dx\; x^2(1-x)}_{1/12}$$

$$\Gamma_{exp} = 2.99598\times 10^{-16}$$

$$\boxed{\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3} = 3.00918E{-16} = 1.0044\,\Gamma_{exp}}$$

0.4% discrepancy

Corrections to this !!

$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}\underbrace{\left(1 - 8\frac{m_e^2}{m_\mu^2} + \dots\right)}_{0.999813}$$

$\rightarrow$ 0.02% not enough to fix

Returning to $\Gamma = \frac{4G_F^2}{(2\pi)^3}\int dE_2\, dE_3\; E_3\,(m_\mu^2 - 2m_\mu E_3 \cdot m_e^2)$

we again set $m_e = 0$ but now take the integrations in the reverse order

$$\Gamma = \frac{4G_F^2}{(2\pi)^3}\int_0^{\frac{m_\mu}{2}} dE_2 \underbrace{\int_{\frac{m_\mu}{2}-E_2}^{\frac{m_\mu}{2}} dE_3\, m_\mu E_3\,(m_\mu - 2E_3)}$$

$$\frac{m_\mu^2}{2}E_3^2 - \frac{2m_\mu}{3}E_3^3 \Big|_{\frac{m_\mu}{2}-E_2}^{\frac{m_\mu}{2}}$$

$$= \frac{m_\mu^2}{2}\left(m_\mu E_2 - E_2^2\right) - \frac{2m_\mu}{3}\left(\frac{3m_\mu^2}{4}E_2 - \frac{3m_\mu}{2}E_2^2 + E_2^3\right)$$

$$= -\frac{2m_\mu}{3}E_2^3 + \frac{m_\mu^2}{2}E_2^2$$

$$\Gamma = \frac{4G_F^2}{(2\pi)^3}\int_0^{\frac{m_\mu}{2}} dE_2\; E_2^2\, m_\mu\left(\frac{m_\mu}{2} - \frac{2E_2}{3}\right)$$

$$= \int_0^{\frac{m_\mu}{2}} dE_2 \underbrace{\frac{2G_F^2 m_\mu^2}{(2\pi)^3}E_2^2\left(1 - \frac{4E}{3m_\mu}\right)}$$

$\frac{d\Gamma}{dE_2}$ = electron energy distribution

agrees w/ Griffiths (10.35), 1e

Now let's make contact w/ pre-relativistic approach.

Width of μ^- (redux)

$$\Gamma = \frac{1}{2m_\mu}\int d(LIPS)_3 \overline{|A|}^2$$

$$= \underbrace{\int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3}(2\pi)^4\delta(\Sigma p^\mu)}_{\text{(non Lorentz invariant) phase space}}\underbrace{\left[\frac{\overline{|A|}^2}{(2m_\mu)(2E_1)(2E_2)(2E_3)}\right]}_{|\text{ampl.}|^2}$$

After averaging and summing

$$|\text{amplitude}|^2 = \frac{64G_F^2(p_0\cdot p_3)(p_1\cdot p_2)}{(2m_\mu)(2E_1)(2E_2)(2E_3)} \qquad 4G_F^2\left(1-\frac{p_1p_2}{E_1E_2}\cos\theta_{12}\right)$$

(corrected)

$$\boxed{\Gamma = \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3}(2\pi)^4\delta(\Sigma p^\mu)\left[4G_F^2\left(1-\frac{p_1p_2}{E_1E_2}\cos\theta_{12}\right)\right]}$$

We could integrate out $\vec{p}_3$ to get ← electron neutrino

$$\Gamma = \int \frac{d\Omega_1\, p_1^2\, dp_1}{(2\pi)^3}\frac{d\Omega_{12}\, p_2^2\, dp_2}{(2\pi)^3}(2\pi)\,\delta(m_\mu-E_1-E_2-E_3)\,4G_F^2\left(1-\frac{p_1p_2}{E_1E_2}\cos\theta_{12}\right)$$

Then integrate over $d\Omega_1 \to 4\pi$ and $d\phi_{12}\to 2\pi$

$$\Gamma = \frac{2}{(2\pi)^3}\int p_1^2\,dp_1\; p_2^2\,dp_2\; d(\cos\theta_{12})\,\delta(m_\mu-E_1-E_2-E_3)\left[4G_F^2\left(1-\frac{p_1p_2}{E_1E_2}\cos\theta_{12}\right)\right]$$

where $E_3: \sqrt{\vec{p}_3^{\,2}} = \sqrt{(\vec{p}_1+\vec{p}_2)^2} = \sqrt{p_1^2+2\vec{p}_1\cdot\vec{p}_2+p_2^2} = \sqrt{p_1^2+p_2^2+2p_1p_2\cos\theta_{12}}$

$$\boxed{\Gamma = \frac{8G_F^2}{(2\pi)^3}\int dp_1\,dp_2\,d(\cos\theta_{12})\,p_1^2p_2^2\left[1-\frac{p_1p_2}{E_1E_2}\cos\theta_{12}\right]\delta\left(m_\mu-\underset{\nu_\mu}{E_1}-\underset{\text{electron}}{E_2}-\sqrt{p_1^2+p_2^2+2p_1p_2\cos\theta_{12}}\right)}$$

Hard to do in this form.
Dimensional considerations tell us

$$\Gamma = \frac{G_F^2 m_\mu^5}{\pi^3}\cdot(\quad)$$ and the alternative calculation gives $\frac{1}{192}$.

If we want to eliminate E_3 ($\to E_1$?)

argument of δ fns vanishes if

$$E_2^2 + E_3^2 + 2E_2E_3\cos\theta_{23} = (E_2 + E_3 - m_\mu)^2$$

$$\Rightarrow 2E_2E_3\cos\theta_{23} = 2E_2E_3 - 2E_2 m_\mu - 2E_3 m_\mu + m_\mu^2$$

$$\hat{E}_3 = \frac{m_\mu(m_\mu - 2E_2)}{2(m_\mu - E_2 + E_2\cos\theta_{23})}$$

$$\frac{\partial}{\partial E_3}\left(\sqrt{E_2^2+E_3^2+2E_2E_3\cos\theta_{23}} + E_2 + E_3 - m_\mu\right)\Big|_{E_3=\hat{E}_3}$$

$$= \left(\frac{E_3 + E_2\cos\theta_{23}}{\sqrt{E_2^2+E_3^2+2E_2E_3\cos\theta_{23}}} + 1\right)\Big|_{E_3=\hat{E}_3}$$

$$= \frac{E_3 + E_2\cos\theta_{23} + (m_\mu - E_2 - E_3)}{m_\mu - E_2 - E_3}\Big|_{E_3=\hat{E}_3}$$

$$= \frac{m_\mu - E_2 + E_2\cos\theta_{23}}{m_\mu - E_2 - \frac{m_\mu(m_\mu - 2E_2)}{2(m_\mu - E_2 + E_2\cos\theta_{23})}}$$

$$= \frac{2(m_\mu - E_2 + E_2\cos\theta_{23})^2}{2(m_\mu - E_2)(m_\mu - E_2 + E_2\cos\theta_{23}) - m_\mu(m_\mu - 2E_2)}$$

$$\text{let } A = \frac{2(m_\mu - E_2 + E_2\cos\theta_{23})^2}{m_\mu^2 - 2m_\mu E_2 + 2E_2^2 + 2(m_\mu - E_2)E_2\cos\theta_{23}}$$

$$\delta(\sqrt{} + E_2 + E_3 - m) = \delta(A[E_3 - \hat{E}_3]) = \frac{1}{A}\delta(E_3 - \hat{E}_3)$$

Here's how to summarize this for class

2019

$$\Gamma = \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \frac{d^3p_3}{(2\pi)^3} (2\pi)^4 \delta(\Sigma p) \; |amp|^2$$

$$= \frac{2}{(2\pi)^3} \int p_1^2 \, dp_1 \int p_2^2 \, dp_2 \; d(\cos\theta_{12}) \; \delta(m_\mu - E_1 - E_2 - E_3) \; |amp|^2$$

$$E_3 = \sqrt{p_1^2 + p_2^2 + 2p_1 p_2 \cos\theta_{12}}$$

massless $e^-, \bar{\nu}_e, \nu_\mu$

$$= \frac{2}{(2\pi)^3} \int_0^{m_\mu/2} dE_1 \int_{\frac{m_\mu}{2} - E_1}^{m_\mu/2} dE_2 \int_{-1}^{1} d(\cos\theta_{12}) \; E_1^2 E_2^2 \; \delta\left(m_\mu - E_1 - E_2 + \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\theta_{12}}\right) \; |amp|^2$$

This can be done but hard.

Now dim'l this goes as m_μ^5 & $|amp|^2 \sim G_F^2$

so

$$\frac{G_F^2 m_\mu^5}{(2\pi)^3} [\#] \quad + \text{calc} \Rightarrow \# = \frac{1}{24}$$

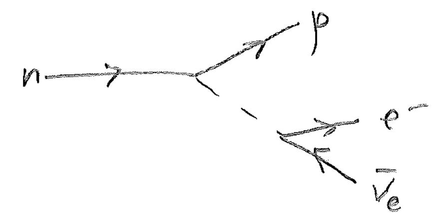
so $\Gamma = \dfrac{G_F^2 m_\mu^5}{192\pi^3}$

(4-25-17) (5) (OLD Notes (7/17)) (1)

n decay

$n \to p \; e^- \; \bar{\nu}_e$

$p_0 = p_1 + p_2 + p_3$



Pretend n + p are fundamental spin-$\frac{1}{2}$ particles

$$\overline{|A|^2} = 64\, G_F^2\, (p_0\cdot p_3)(p_1\cdot p_2)$$

$$p_0\cdot p_3 = m_n E_3$$

$$p_1\cdot p_2 = \tfrac{1}{2}\left[(p_1+p_2)^2 - p_1^2 - p_2^2\right]$$

$$= \tfrac{1}{2}\left[(p_0-p_3)^2 - p_1^2 - p_2^2\right]$$

energy of ν_e

$$p_1\cdot p_2 = -m_n E_3 + \tfrac{1}{2}m_n^2 - \tfrac{1}{2}m_p^2 - \tfrac{1}{2}m_e^2$$

$$\boxed{\Rightarrow \overline{|A|^2} = 64\, G_F^2\, m_n E_3 \left(\tfrac{1}{2}m_n^2 - \tfrac{1}{2}m_p^2 - \tfrac{1}{2}m_e^2 - m_n E_3\right)}$$

Alternatively $p_1\cdot p_2 = E_1E_2\left(1 - \frac{\vec{p}_1\cdot\vec{p}_2}{E_1E_2}\right)$

$$= E_1E_2\left(1 - \frac{p_1 p_2}{E_1E_2}\cos\theta_{12}\right)$$

$$\overline{|A|^2} = 64\, G_F^2\, m_n E_1 E_2 E_3 \left(1 - \frac{p_1 p_2}{E_1E_2}\cos\theta_{12}\right)$$

angle between p and e^-

$$= G_F^2\, |2|^2\, (2m_n)(2E_1)(2E_2)(2E_3)$$

cancels some factors in denom

so $M=2$ if $A = GM$

Width gn

$$\Gamma = \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3}(2\pi)^4\,\delta(\Sigma p^\mu)\left[\frac{\overline{|A|^2}}{(2m_n)(2E_1)(2E_2)(2E_3)}\right]$$

Now $\dfrac{\overline{|A|^2}}{(2m_n)(2E_1)(2E_2)(2E_3)} = \begin{cases} 4G_F^2\left[\dfrac{\frac{1}{2}m_n^2 - \frac{1}{2}m_p^2 - \frac{1}{2}m_e^2 - m_nE_3}{E_1E_2}\right] \\[2ex] 4G_F^2\left[1 - \dfrac{p_1p_2}{E_1E_2}\cos\theta_{12}\right] \end{cases}$

angle between proton and electron

We'd like to integrate over $\vec{p}_1$, the proton momentum.

We can either use the first expression above, which doesn't depend on $\vec{p}_1$, or we can use the second, and make crude approximation that $\cos\theta_{12}$ integrates to ~~zero~~. The first gives

$$\Gamma = \int \frac{d^3p_2}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3}(2\pi)\,\delta(m_n - E_1 - E_2 - E_3)\,4G_F^2\left[\frac{\frac{1}{2}m_n^2 - \frac{1}{2}m_p^2 - \frac{1}{2}m_e^2 - m_nE_3}{E_1E_2}\right]$$

while in the crude approximation we omit the term in brackets.
↳ i.e. it is set to unity

Next we integrate over $d\Omega_2$ and $d\phi_{23}$ to obtain

$$\boxed{\Gamma = \frac{2}{(2\pi)^3}\int p_2^2\,dp_2\;p_3^2\,dp_3\;d(\cos\theta_{23})\;\delta(m_n-E_1-E_2-E_3)\left(4G_F^2\right)[\quad]}$$

Here $E_1 = \sqrt{m_1^2 + \vec{p}_1^{\,2}} = \sqrt{m_1^2 + (\vec{p}_2+\vec{p}_3)^2} = \sqrt{m_p^2 + p_2^2 + p_3^2 + 2p_2p_3\cos\theta_{23}}$

But later we'll see that this crude approx. is almost correct (within < 0.1% !)

There are now two routes that can be followed:
(1) nonrelativistic proton approx.
(2) exact approach

Massive proton approximation

$$\Gamma = \frac{2}{(2\pi)^3}\int p_2^2\, dp_2\, p_3^2\, dp_3\, d(\cos\theta_{23})\, \delta(m_n - E_1 - E_2 - E_3)\, 4G^2 \left[\quad\right]$$

Since proton is much more massive than the electron or neutrino, let's assume that it can absorb any amount of momentum w/ $E_1 = m_p$. Then θ_{23} is arbitrary and $E_2 + E_3 = m_n - m_p$..

Also we can set $p_3 = E_3$ and $p_2\, dp_2 = E_2\, dE_2$..

$$\Gamma = \frac{2}{(2\pi)^3}\int dE_2\, dE_3\, p_2 E_2 E_3^2\, \underbrace{d(\cos\theta_{23})}_{2}\, \delta(m_n - m_p - E_2 - E_3)\, 4G_F^2 \left[\quad\right]$$

$$= \frac{4(4G)^2}{(2\pi)^3}\int dE_2\, p_2 E_2 (m_n - m_p - E_2)^2 \left[\frac{\frac{1}{2}m_n^2 - \frac{1}{2}m_p^2 - \frac{1}{2}m_e^2 - m_n(m_n - m_p - E_2)}{m_p E_2}\right]$$

$$= \frac{4(4G_F^2)}{(2\pi)^3}\int dE_2\, p_2 E_2 (m_n - m_p - E_2)^2 \left[\frac{m_n E_2 - \frac{1}{2}(m_n - m_p)^2 - \frac{1}{2}m_e^2}{m_p E_2}\right]$$

Now E_2 goes from m_e to $m_n - m_p$

$$\boxed{\Gamma = \frac{4(4G_F^2)}{(2\pi)^3}\int_{m_e}^{m_n - m_p} dE_2\, p_2 E_2 (m_n - m_p - E_2)^2 \left[\frac{m_n E_2 - \frac{1}{2}(m_n - m_p)^2 - \frac{1}{2}m_e^2}{m_p E_2}\right]}$$ (c)

7. Plotting [ ] shows that it ranges from 0.999362 to 1.00058

so it is not such a crude approximation to neglect it

$$\boxed{\frac{d\Gamma}{dE_2} = p_2^2 (Q - E_2)^2}$$

In the "crude" approximation

$$\Gamma = \frac{4(4G_F^2)}{(2\pi)^3}\int_{m_e}^{Q} dE_2 \sqrt{E_2^2 - m_e^2}\, E_2 (Q - E_2)^2$$

This is almost perfect

The massless electron approximation gives

$$\Gamma = \frac{4(4G_F^2)}{(2\pi)^3}\underbrace{\int_0^Q dE_2\, E_2^2 (Q - E_2)^2}_{\frac{Q^5}{30} = 0.1205 (\text{meV})^5} = \frac{(4G_F^2)\, Q^5}{60\pi^3}$$

agrees w/ Griffiths eq. (10.6?) in the $Q \to \infty$ limit

But this approx is in no way justified

Then $\tau = \frac{\hbar}{\Gamma} = \frac{60\pi^3 \hbar}{(4G^2)Q^5} = 623$ sec (wow! Almost spot on... Just lucky)

If the electron is not massless, a numerical evaluation gives

$$\int_{m_e}^{Q} dE_2 \sqrt{E_2^2 - m_e^2}\, E_2 (Q - E_2)^2 \approx 0.056909 (\text{meV})^5$$

Then increases the τ to

$\tau = 1320$ sec ← this is actually the prediction of Griffiths

(Griffiths then explains why this is off by so much)

Keeping the correction factor in the massless electron approx.

$$\Gamma = \frac{16G^2}{(2\pi)^3}\underbrace{\int_0^Q dE_2\, E_2^2 (Q - E_2)^2 \left[1 + \frac{Q}{m_p} - \frac{Q^2}{2m_p E_2}\right]}_{\frac{Q^5}{30}\underbrace{\left(1 - \frac{Q}{4m_p}\right)}_{0.99965,\ \text{small change}}}$$

Keeping correction fact. in the massive electron case gives numerically

0.056912 (MeV)^5

hardly any change

so ⊙ gives $\tau \approx 1320$ sec

Return to p ② to do this integral the more traditional (exact) way

(5)

$$\Gamma = \frac{2(4G_F^2)}{(2\pi)^3} \int p_2^2\,dp_2\, p_3^2\,dp_3 \int_{-1}^{1} d(\cos\theta_{23})\; \delta\Big(m_n - E_2 - E_3 - \underbrace{\sqrt{m_p^2 + p_2^2 + p_3^2 + 2p_2p_3\cos\theta_{23}}}_{E_1}\Big)\,[\;]$$

indep of $\cos\theta_{23}$

Now instead of setting $E_1 = m_p$ and doing integral over θ_{23}, we will change variables from $\cos\theta_{23}$ to E_1 by

$$dE_1 = \frac{\partial E_1}{\partial(\cos\theta_{23})}\, d(\cos\theta_{23}) = \frac{p_2 p_3}{E_1}\, d(\cos\theta_{23}) \quad \text{so}$$

$$\Gamma = \frac{2(4G_F^2)}{(2\pi)^3} \int p_2^2\,dp_2\, p_3^2\,dp_3 \int_{E_1^-}^{E_1^+} \frac{E_1}{p_2 p_3}\, \delta(m_n - E_2 - E_3 - E_1)\; [\;]$$

where $E_1^{\pm} = \sqrt{m_p^2 + (p_2 \pm p_3)^2}$

$$= \frac{2(4G_F^2)}{(2\pi)^3} \int \underbrace{p_2\,dp_2}_{E_2 dE_2}\; \underbrace{p_3\,dp_3}_{E_3 dE_3}\; E_1 \;[\;] \quad \text{provided } E_1^- < m_n - E_2 - E_3 < E_1^+$$

otherwise zero

This provides limits on the E_3 integration.

Griffiths (10.55) i.e. states $\dfrac{\frac{1}{2}(m_n^2 - m_p^2 - m_e^2) - m_n E_2}{m_n - E_2 - p_2} \le E_3 \le \dfrac{\frac{1}{2}(m_n^2 - m_p^2 - m_e^2) - m_n E_2}{m_n - E_2 + p_2}$

$$\Gamma = \frac{2(4G_F^2)}{(2\pi)^3} \int dE_2 \int_{E_3^-}^{E_3^+} dE_3\, (E_1 E_2 E_3)\;[\;]$$

These cancel the usual normalization $\frac{1}{2E_1}\frac{1}{2E_2}\frac{1}{2E_3}$

Now since $[\;] = \dfrac{\frac{1}{2}m_n^2 - \frac{1}{2}m_p^2 - \frac{1}{2}m_e^2 - m_n E_3}{E_1 E_2}$ we have

$$\Gamma = \frac{2(4G_F^2)}{(2\pi)^3} \int dE_2 \underbrace{\int_{E_3^-}^{E_3^+} dE_3\; E_3\left(\tfrac{1}{2}m_n^2 - \tfrac{1}{2}m_p^2 - \tfrac{1}{2}m_e^2 - m_n E_3\right)}_{\text{This is just } \frac{1}{2} J(E_3) \text{ of Griffiths eq. (10.56)}}$$

agrees w/ Griffiths (10.58) and generalizes for calc.

$$\Gamma = \frac{2(4G_F^2)}{(2\pi)^3}\int dE_2 \underbrace{\left[\frac{1}{4}(m_n^2 - m_p^2 - m_e^2)(E_3^{+2} - E_3^{-2}) - \frac{m_n}{3}(E_3^{+3} - E_3^{-3})\right]}_{\text{Griffiths shows this approximates to}}$$

$$2E_2\sqrt{E_2^2 - m_e^2}\,\left[(m_n - m_p) - E_2\right]^2$$

$$= \frac{4(4G_F^2)}{(2\pi)^3}\int_{m_e}^{Q} dE_2 \; E_2\sqrt{E_2^2 - m_e^2}\,(Q - E_2)^2$$

But hey! this is exactly the integral we found before when we did the "crude" (actually quite accurate) approximation; ie neglecting the $\cos\theta_{12}$ term and letting $E_1 \approx m_1$

Thus it is perfectly reasonable to use the earlier approx in class

(7)

Finally what if we started w/ the (very accurate) crude approximation by letting

$$\frac{|\bar{A}|^2}{(2m_n)(2E_1)(2E_2)(2E_3)} \approx 4 G_F^2$$

then

$$\Gamma = \frac{2(4G_F^2)}{(2\pi)^3} \int dE_2 \int_{E_3^-}^{E_3^+} dE_3 \; \underbrace{E_1 E_2 E_3}_{E_2 E_3 (m_n - E_2 - E_3)}$$

$$\underbrace{}_{E_2 \left[\frac{E_3^2}{2}(m_n - E_2) - \frac{E_3^3}{3} \right] \Big|_{E_3^-}^{E_3^+}}$$

OK, we could evaluate this # probably & would find that it approximates to the same integral as before. But not clear it is worth the effort at this point.

Since the nonrelativistic approx. ~~works for the~~ gives accurate results, let's use that for the proton

Here's how to present for class 2019

$$\Gamma = \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3}(2\pi)^4\,\delta(\Sigma p^\mu)\,|amp|^2$$

$$= \frac{2}{(2\pi)^3}\int p_2^2\,dp_2\; p_3^2\,dp_3\; d(\cos\theta_{23})\,\delta(m_n - E_1 - E_2 - E_3)\,|amp|^2$$

exactly as for μ decay

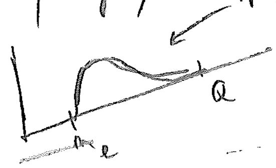
$$E_1 = \sqrt{m_p^2 + p_2^2 + p_3^2 + 2p_2p_3\cos\theta_{23}}$$

- Now approximate $|amp|^2 \approx \underline{\text{const}} = 4G_F^2$ [this is pretty accurate w/in 0.1%] (why physically?)
- Since ~~...~~ $m_p \gg p_2, p_3$ we approx $E_1 \approx m_p$
- also $p_3 = E_3$ since $m_\nu = 0$

$$= \frac{2}{(2\pi)^3}\int p_2^2\,dp_2\; E_3^2\,dE_3\; \underbrace{d(\cos\theta)}_{2}\,\delta(m_n - m_p - E_2 - E_3)\,|amp|^2$$

$$\approx \frac{4}{(2\pi)^3}\underbrace{\int p_2^2\,dp_2}\,(m_n - m_p - E_2)^2\,|amp|^2$$

fig in Griffiths



$$\int_{m_e}^{m_n-m_p} E_2\sqrt{E_2^2 - m_e^2}\,dE_2$$

Now massless electron $\Rightarrow \Gamma = \frac{4}{(2\pi)^3}\frac{Q^5}{30}|amp|^2 = \frac{Q^5(4G_F^2)}{60\pi^3}$

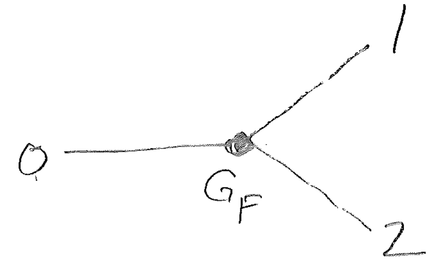
$0.1205\,(\text{MeV})^5$

4

46 numerical eval for $m_e \neq 0 \Rightarrow \frac{4}{(2\pi)^3}(0.0569)(\text{MeV})^5$

$\tau = 1320$ sec (actually ~ 880 s)

(b) $\boxed{\text{2-particle weak decay}}$



A has dimensions of energy for 1→2 decay

$$A : \quad G_f\, m_0^3\, M$$

[crude approximation]

[previously we used $\sqrt{(2E_0)(2E_1)(2E_2)}$ to cancel corresponding factors in the phase space but the dimensions don't add up.]

$$R : \quad \frac{p_f}{2\hbar\,(4\pi m_0)^2} \int d\Omega\, |A|^2$$

$$: \quad \frac{p_f}{8\pi\hbar\, m_0^2}\, G_F^2\, m_0^6\, M^2$$

$$m_0 = E_1 + E_2 \qquad E_1 : \frac{m_0^2 + m_1^2 - m_2^2}{2m_0}$$

$$= \sqrt{p_f^2 + m_1^2} + \sqrt{p_f^2 + m_2^2}$$

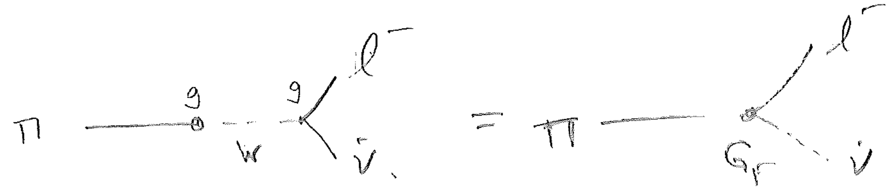
[Griffiths 2e prob 3.19]

$$p_f : \frac{1}{2m_0}\sqrt{(m_0+m_1+m_2)(m_0-m_1-m_2)(m_0+m_1-m_2)(m_0-m_1+m_2)}$$

$$R : \frac{G_F^2\, m_0^3}{16\pi\hbar}\sqrt{(m_0^2 - [m_1+m_2]^2)(m_0^2 - (m_1-m_2)^2)}\,|M|^2$$

$$: \frac{G_F^2\, m_0^5}{16\pi\hbar}\,\lambda\,|M|^2, \qquad \lambda = \sqrt{\left(1-\left(\frac{m_1+m_2}{m_0}\right)^2\right)\left(1-\left(\frac{m_1-m_2}{m_0}\right)^2\right)}$$

$\pi \to \mu \bar{\nu}_\mu$ (crude approx)



Crude approx: $A = G_F m_\pi^3 M$

$$R = \frac{G_F^2 m_\pi^5}{16\pi\hbar} \lambda |M|^2, \qquad \lambda = 1 - \left(\frac{m_\ell}{m_\pi}\right)^2$$

$G_F = 1.1664E-5 \text{ GeV}^{-2}$
$\hbar = 6.626E-25 \text{ GeV s}$
$m_\pi = 0.13957 \text{ GeV}$

$\Rightarrow \frac{G_F^2 m_\pi^5}{16\pi\hbar} = 2.163E8 \text{ s}^{-1}$

$\lambda = 0.4269$ for $\pi \to \mu\bar{\nu}$

$R = (9.235E7 \text{ s}^{-1}) |M|^2$ for $\pi \to \mu\bar{\nu}$

But $\tau_{exp} = 2.603E-8$ (BR only changes 0.1%)

$R = 3.841E7 \text{ s}^{-1}$

$\Rightarrow |M|^2 = 0.416$

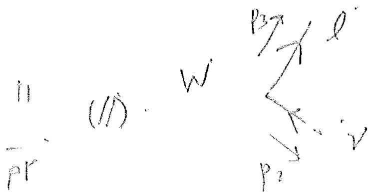
$|M| = 0.645$

$$M = \sqrt{2}\, \frac{f_\pi}{m_\pi} \frac{m_\ell}{m_\pi} \sqrt{1 - \frac{m_\ell^2}{m_\pi^2}}$$ from Griffiths calculation

$$= \frac{f_\pi}{m_\pi} \underbrace{\sqrt{2}(0.7570)(0.6534)}_{0.70} \quad \text{for } \ell = \mu$$

$\Rightarrow f_\pi = 0.922\, m_\pi$.

π decay $\left(\begin{smallmatrix}\text{done}\\ \text{right}\end{smallmatrix}\right)$ [Griffiths, p. 3..]



$$A = \frac{1}{8}\left(\frac{g_w}{M_W}\right)^2 F^\mu \ \bar{u}_\ell \gamma_\mu (1 - \gamma^5) v_\nu$$

F^μ form factor coupling W to ud

$= \tilde{f}_\pi \cos\theta_c\, p^\mu = f_\pi p^\mu$

$$\langle |A|^2 \rangle = \frac{1}{8} f_\pi^2 \left(\frac{g_w}{M}\right)^4 [\,2(p \cdot p_2)(p \cdot p_3) - p^2 (p_2 \cdot p_3)\,]$$

$p = p_2 + p_3 \Rightarrow p \cdot p_2 = \ldots$

$p \cdot p_3 = \ldots$

$p^2 = 2 p_2 p_3 + m_\ell^2 \ldots$

$$\Rightarrow 2 p_2 p_3 = m_\pi^2 - m_\ell^2$$

$$\langle |A|^2 \rangle = \frac{f_\pi^2}{8} \ldots \frac{1}{2}\left[\underbrace{(m_\pi^2 - m_\ell^2)(\ldots + m_\ell^2)}_{m_\ell^2 (m_\pi^2 - m_\ell^2)} - m_\pi^2 (m_\pi^2 - m_\ell^2)\right]$$

$$= \ldots G_F^2 f_\pi^2 m_\ell^2 (m_\pi^2 - m_\ell^2) = G_F^2 m_\pi^4 \underbrace{\left(2 \frac{f_\pi^2}{m_\pi^2} \frac{m_\ell^2}{m_\pi^2}\left(1 - \frac{m_\ell^2}{m_\pi^2}\right)\right)}_{M^2}$$

NB. ... $m_\ell \to 0$ ... ...

$\pi \to \overset{\longleftarrow}{\bar\nu} \quad \overset{\longrightarrow}{\ell}$ for spin 1 or 0

r.h. l.h.

$$R = \frac{p_f}{2 (4\pi m_\pi)^2} \int d\Omega \, |A|^2$$

$$= \frac{p_f}{8\pi \hbar m_\pi^2} \cdot 2 G_F^2 f_\pi^2 (m_\pi^2 - m_\ell^2) m_\ell^2$$

$$\text{Since } p_f + \sqrt{p_f^2 + m_\ell^2} = m_\pi \Rightarrow p_f = \frac{m_\pi^2 - m_\ell^2}{2 m_\pi}$$

$$R = \frac{G_F^2}{8\pi\hbar} \frac{f_\pi^2 (m_\pi^2 - m_\ell^2)^2 m_\ell^2}{m_\pi^3} = \frac{G_F^2}{8\pi\hbar} \left(\frac{f_\pi}{m_\pi}\right)^2 \left(\frac{m_\ell}{m_\pi}\right)^2 \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2 m_\pi^5$$

$$\Gamma = \frac{R_\pi}{\hbar} = \frac{G_F^2 f_\pi^2 (m_\pi^2 - m_\ell^2)^2 m_\ell^2}{8\pi m_\pi^3} \underset{m_\ell \ll m_\pi}{\approx} \frac{G_F^2 f_\pi^2 m_\pi m_\ell^2}{8\pi}$$

$$\frac{\Gamma(\pi \to e \bar{\nu}_e)}{\Gamma(\pi \to \mu \bar{\nu}_\mu)} = \frac{m_e^2}{m_\mu^2} \frac{(m_\pi^2 - m_e^2)^2}{(m_\pi^2 - m_\mu^2)^2} = 1.283 \times 10^{-4}$$

(expt. 1.230×10^{-4})

$$R = \underbrace{\frac{G_F^2\, m_\pi^5}{8\pi\hbar}}_{2(2.163E8)} \left(\frac{f_\pi}{m_\pi}\right)^2 \underbrace{\left(\frac{m_\ell}{m_\pi}\right)^2}_{0.5731} \underbrace{\left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2}_{(0.4269)^2}$$

$$= \left(4.518E7\ s^{-1}\right)\left(\frac{f_\pi}{m_\pi}\right)^2$$

---

expt R_π 3.841 [illegible] $\Rightarrow \left(\frac{f_\pi}{m_\pi}\right)^2 = 0.850$

$$f_\pi = 0.922\, m_\pi = \tilde{f}_\pi \underbrace{\cos\theta_c}_{0.974}$$

$$\tilde{f}_\pi = 0.947 m_\pi$$

(see Griffiths, [illegible])

Griffiths [illegible] knows of no reason why $\tilde{f}_\pi \approx m_\pi$!

Does anyone?

$\pi \to \mu \bar{\nu}_\mu$

π at rest $\Rightarrow$ $E_\nu = \dfrac{m_\pi^2 - m_\nu^2}{2m_\pi} = 30\,\text{MeV}$

π in flight w/ $\gamma = \dfrac{E_\pi}{m_\pi}$

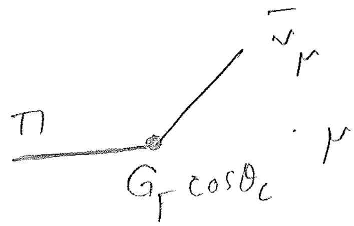
ν spectrum flat (?) w/ endpts

$\dfrac{E_\nu}{E_\pi} < \dfrac{m_\pi^2 - m_\mu^2}{2m_\pi^2}$ (2.13 Bargo)

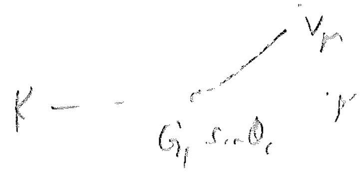
↑ derive

$K \to \mu \nu_\mu$

$\pi \to \mu \nu_\mu$



$K \to \mu \nu$



crude approx

$$\frac{R_K}{R_\pi} = \frac{\sin^2\theta_c}{\cos^2\theta_c}\left(\frac{m_K}{m_\pi}\right)^5 \frac{\lambda_K}{\lambda_\pi} \qquad (0.0546)(553)\left(\frac{0.954}{0.4769}\right) = 67.$$

$$\left(\frac{m_K}{m_\pi}\right)^5 = 553 \qquad \tan^2\theta_c = 0.0546$$

$m_K = 493.7$

$m_\pi = 139.6$

$m_\mu = 105.7$

$$\lambda_\mu \quad 1 - \left(\frac{m_\mu}{m_K}\right)^2 = 0.954$$

$$\lambda_\pi \quad 1 - \left(\frac{m_\mu}{m_\pi}\right)^2 = 0.4769$$

not very good!

$R_{exp}(\pi \to \mu \nu_\mu) = 3.841E7\ s^{-1}$

$R_{exp}(K \to \mu \bar{\nu}_\mu) = 5.134E7\ s^{-1}$

$$\left(\frac{R_K}{R_\pi}\right)_{exp} = 1.34$$

$\tau_K = 1.238E-8\ s$

$R = 8.078E7\ s^{-1}$

$BR = 0.6356$

$K \to \mu^- \bar{\nu}_\mu$ (Griffiths 2e, p 325)

$$R(\pi \to \mu \bar{\nu}_\mu) = \frac{G_F^2}{8\pi\hbar} \frac{\tilde{f}_\pi^2 (m_\pi^2 - m_\mu^2)^2 m_\mu^2}{m_\pi^3} \cos^2\theta_c$$

$$R(K \to \mu \bar{\nu}_\mu) = \frac{G_F^2}{8\pi\hbar} \frac{\tilde{f}_K^2 (m_K^2 - m_\mu^2)^2 m_\mu^2}{m_K^3} \sin^2\theta_c$$

$$\frac{R(K \to \mu \bar{\nu}_\mu)}{R(\pi \to \mu \bar{\nu}_\mu)} = \frac{\tilde{f}_K^2}{\tilde{f}_\pi^2} \frac{\left(1 - \frac{m_\mu^2}{m_K^2}\right)^2}{\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2} \left(\frac{m_K}{m_\pi}\right) \tan^2\theta_c$$

$$= \frac{\tilde{f}_K^2}{\tilde{f}_\pi^2} \frac{(0.954)^2}{(0.4269)^2} (3.54)(0.0546)$$

$$= (0.965) \frac{\tilde{f}_K^2}{\tilde{f}_\pi^2}$$

[NB. my crude approx gave a factor of $\left(\frac{m_K}{m_\pi}\right)^5$, rather than $\left(\frac{m_K}{m_\pi}\right)$!]

approx equality of $\tilde{f}_K$, $\tilde{f}_\pi$

see prev. page $\left(\frac{R(K \to \mu \bar{\nu}_\mu)}{R(\pi \to \mu \bar{\nu})}\right)_{exp} = 1.34$

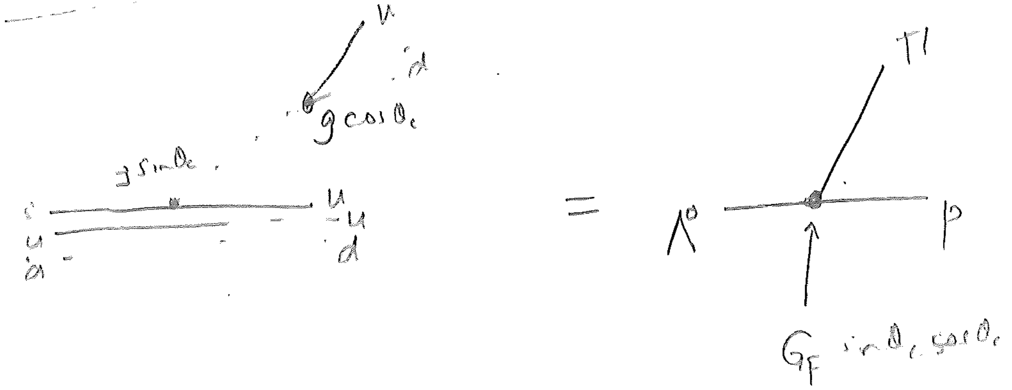
(not $f_K = m_K$, $f_\pi = m_\pi$ which would give a ratio of 12!)

---

of course, $K \to e^- \bar{\nu}_e$ will be down from $K \to \mu^- \bar{\nu}_\mu$ by a factor of $\left(\frac{m_e}{m_\mu}\right)^2 = 2.3E-5$

PDB $\frac{1.582E-5}{0.6356} = 2.5E-5$

$\Lambda^0 \to p\pi$ (crude approx)



$$A = G_F (\sin\theta_c \cos\theta_c)\, m_\Lambda^3\, M$$

$\theta_c : 13.15$

$\sin\theta_c \cos\theta_c \quad 0.221$

$$R = \frac{G_F^2 m_\Lambda^5}{16\pi\hbar}\, \lambda\, (\sin\theta_c \cos\theta_c)^2\, |M|^2$$

$$\lambda \cdot \sqrt{\left(1-\left(\frac{m_p+m_\pi}{m_\Lambda}\right)^2\right)\left(1-\left(\frac{m_p-m_\pi}{m_\Lambda}\right)^2\right)} \quad 0.180$$

$m_\Lambda = 1115.7$
$m_p = 938.3$
$m_\pi = 139.6$

$$\frac{G_F^2 m_\Lambda^5}{16\pi\hbar} \quad (2.163E8\ s^{-1})\left(\frac{m_\Lambda}{m_\pi}\right)^5 \quad 7.05 \times 10^{12}\ s^{-1}$$

$$R - (1.27E12\ s^{-1})(0.049)\,|M|^2 = (6.22E10\ s^{-1})|M|^2$$

$\tau_{exp} = 2.63E-10\ s$

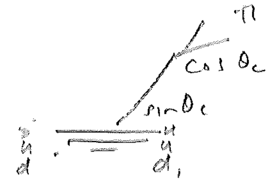
$R = 3.80E9\ s^{-1}$

$BR = 0.64$

$R(\Lambda \to p\pi) : 2.43E9\ s^{-1} \Rightarrow |M|^2 \quad 0.039 \Rightarrow M = 0.20$

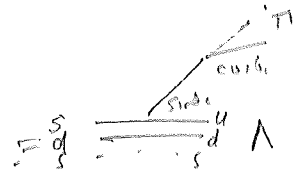
Not too bad

$\Lambda^0 \to p\pi^-$ $\quad R = 2.43E9\ s^{-1}$



$\Xi \to \Lambda\pi^-$ $\quad R = 6.10E9\ s^{-1}$

$\tau = 1.64E{-10}$



$$\left(\frac{R_\Xi}{R_\Lambda}\right)_{expt} = 2.51$$

Crude approx: $\dfrac{R_\Xi}{R_\Lambda} = \left(\dfrac{m_\Xi}{m_\Lambda}\right)^5 \dfrac{\lambda_\Xi}{\lambda_\Lambda} \dfrac{|M_\Xi|^2}{|M_\Lambda|^2} = 2.73 \dfrac{|M_\Xi|^2}{|M_\Lambda|^2}$

not bad!

$m_\Lambda = 1115.7$, $\lambda_\Lambda = 0.180$,

$m_\Xi = 1321.7$, $\lambda_\Xi = 0.211$,

$$\left(\frac{m_\Xi}{m_\Lambda}\right)^5 = 2.33$$

but just lucky because $m_\Xi \approx m_\Lambda$