Intergenerational mixing

Force carriers: γ, $W^{\pm}$, Z_0, gluons, gravitons

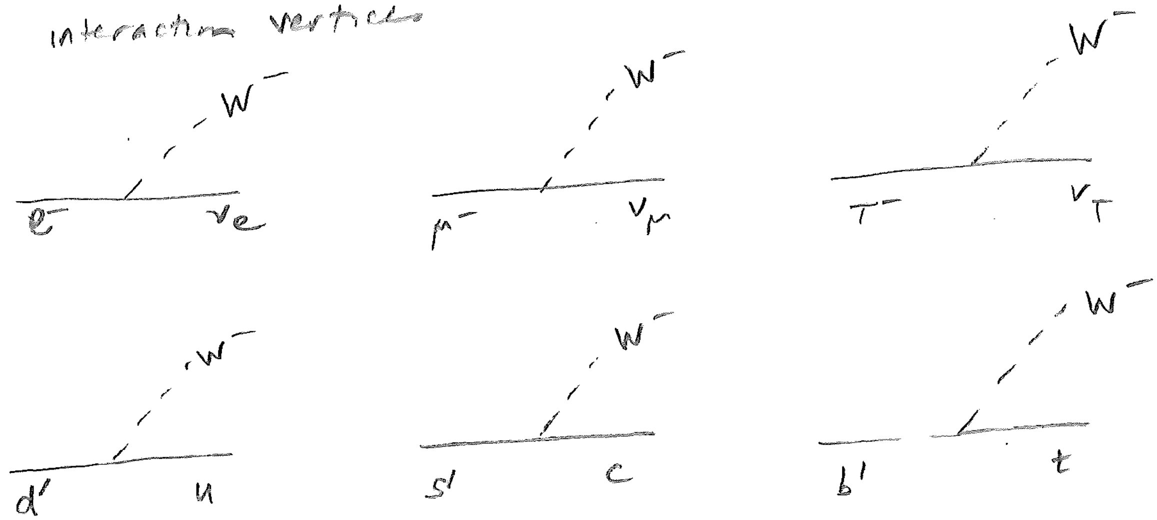
Matter constituents

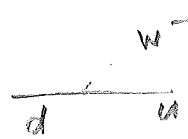
$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} \quad \begin{pmatrix} t \\ b' \end{pmatrix}$$

← electroweak isodoublets

Weak interaction vertices



[Before we wrote 

to discuss neutron decay]

d', s', b' are "almost" same as d, s, b

If they were exactly the same, then the strange + bottom quarks would be absolutely stable.

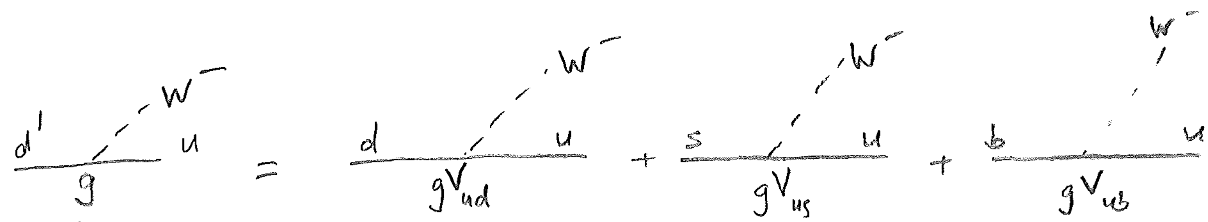
The "weak eigenstates" d', s', b' are related to the "mass eigenstates" d, s, b by a unitary transformation

"CKM matrix", determined experimentally

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

In particular $d' = V_{ud}\, d + V_{us}\, s + V_{ub}\, b$

So



[omit the $\frac{1}{2\sqrt{2}}$ as well as Dirac γ matrices]

[which explains the labels]

where $V_{ud} \sim 0.97$

$V_{us} \sim 0.23$

$V_{ub} \sim 0.004$

Thus the s quark (& b quark) can decay to an up quark.

The CKM matrix can be parametrized by 3 angles $\theta_{12}, \theta_{13}, \theta_{23}$ and one complex phase $e^{i\delta}$ (which allows CP violation)

$$V_{CKM} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

$c_{ij} = \cos\theta_{ij}$

$s_{ij} = \sin\theta_{ij}$

"mixing matrix"

θ_{ij} is roughly the decay amplitude between i & j generations.

Experimentally $\theta_{13} \ll \theta_{23} \ll \theta_{12} \ll 1$.

Let $\begin{cases} \theta_{12} \sim \epsilon \\ \theta_{23} \sim \epsilon^2 \\ \theta_{13} \sim \epsilon^3 \end{cases}$

Very roughly (ignoring CP violating phase) we can approximate

$$V_{CKM} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$$

where ϵ is small.

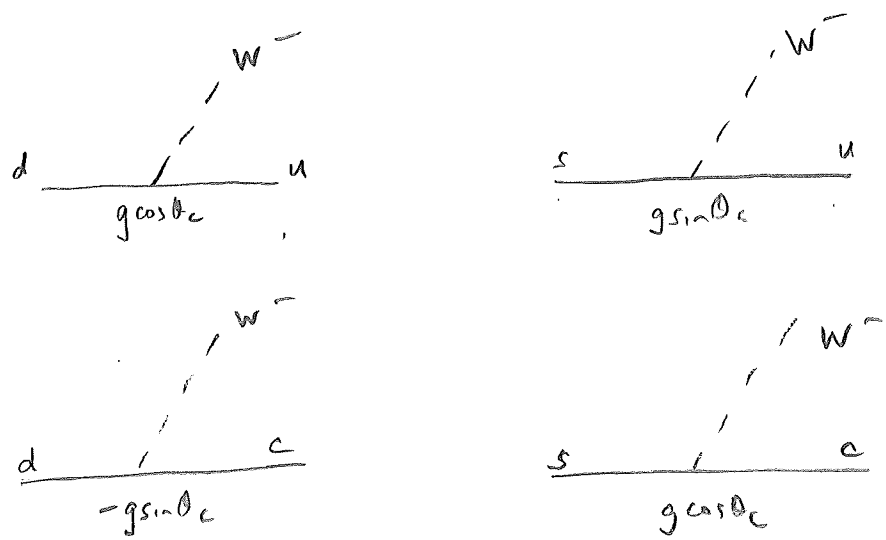
Hence top quark decays in stages:

$$t \xrightarrow[1]{} b \xrightarrow[\epsilon^2]{} c \xrightarrow[1]{} s \xrightarrow[\epsilon]{} u$$

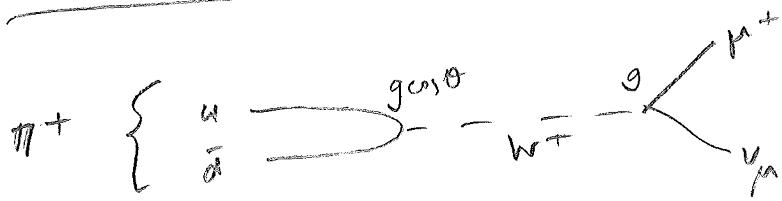
If we set $\theta_{13} = \theta_{23} = 0$ and $\theta_{12} = \theta_c$ = Cabbibo angle

$$V_{ckm} = \begin{bmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{aligned} &\theta_c \approx 12^\circ \\ &\cos\theta_c \simeq 0.98 \\ &\sin\theta_c \simeq 0.23 \end{aligned}$$

we only have mixing between first two generations.



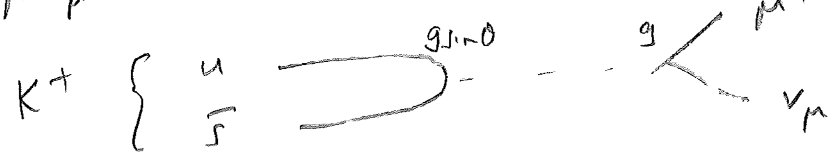
Meson decay



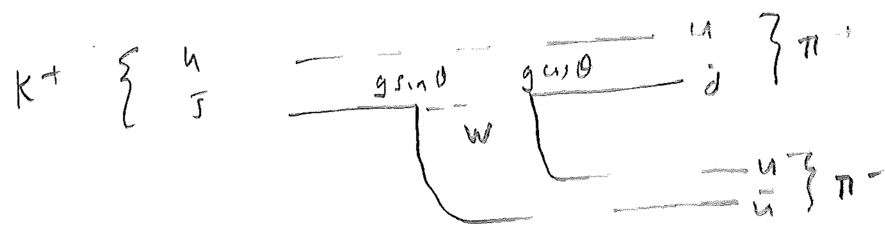
Cabibbo suppression

strange particles are more "stable" than expected

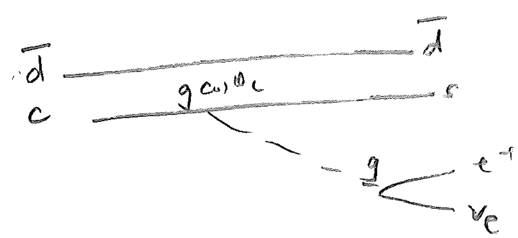
$K^+ \to \mu^+\nu_\mu$ [64%]



$K^+ \to \pi^+\pi^0$ [21%]

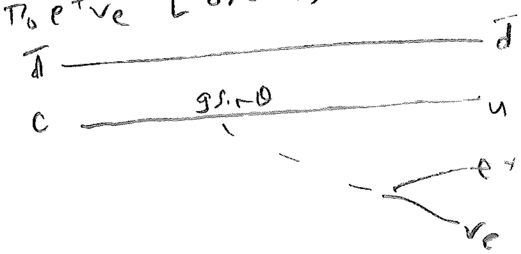


$D^+ \to K_0 e^+ \nu_e$ [0.088]



Ratio of amplitudes $= \frac{\sin\theta_c}{\cos\theta_c} = \tan\theta_c$

$D^+ \to \pi_0 e^+ \nu_e$ [0.004]



Ratio of BR $= \tan^2\theta_c = 0.045$

Prob. 2.5: $D^0 = c\bar{u} \to K^-\pi^+$ [0.034]

$\to \pi^-\pi^+$ [0.00140]

$\to K^+\pi^-$ [0.00015]