Recall continuous β-spectrum + prediction of Pauli

We now calculate the β-spectrum using the QFT describing the weak interaction

Electroweak theory (1968) Glashow, Weinberg, Salam

[In addition to electromagnetic field, ∃ 3 additional fields]

This gave rise to 3 new particles

(vector ⇒ spin-1)

vector bosons

W^+, W^-, Z_0

These are analogous to the photon γ, except massive

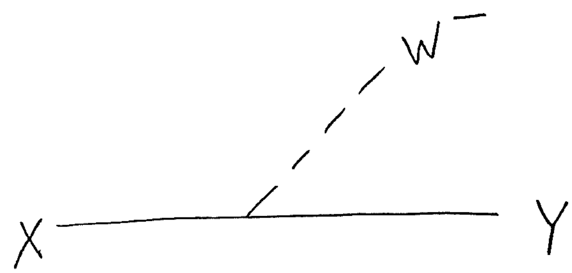
$$m_{W^+} = m_{W^-} = 80.4 \text{ GeV}$$

[PPB: 1st massive entry.

$$m_Z = 91.2 \text{ GeV}$$

β-decay: $_{Z}X \longrightarrow {}_{Z+1}Y + e^- + \bar{\nu}_e$

[One of the vertices in this process is]

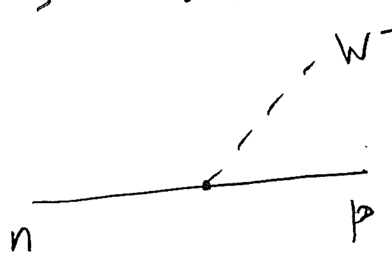


vertex conserves electric charge, so $Q_Y = Q_X + e$

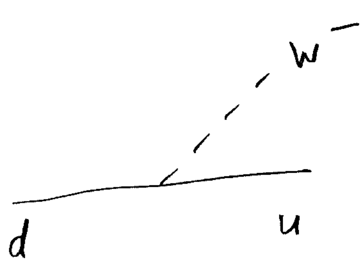
[Electric charge is conserved at the vertex so charge of Y is one greater than charge of X]

Electroweak vertex involves a change of identity for the particle emitting the W^- (unlike QED vertices)

[What is] actually [occuring is] $n \to p + e^- + \bar{\nu}_e$

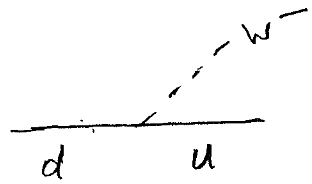


[And on a deeper level] $n = udd$, $p = uud$ so $d \to u + e^- + \bar{\nu}_e$

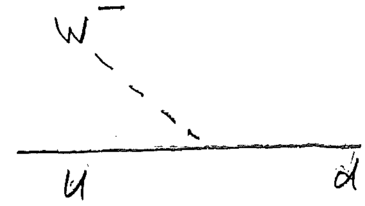


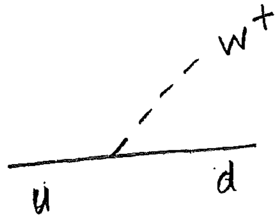
[NB let's not put on arrows]

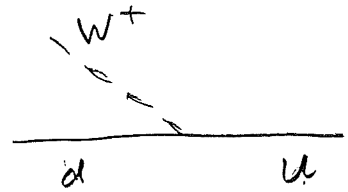
We say $\binom{u}{d}$ is an electroweak doublet



time reverse

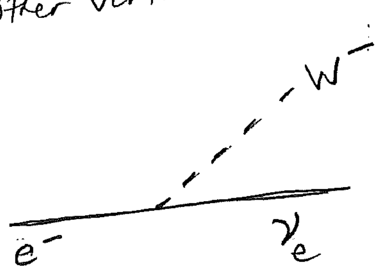


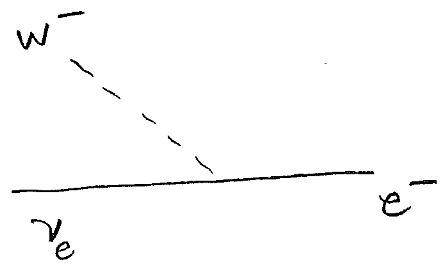


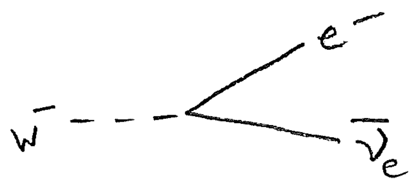


$\begin{pmatrix} \nu_e \\ e \end{pmatrix}$ is also an electroweak doublet

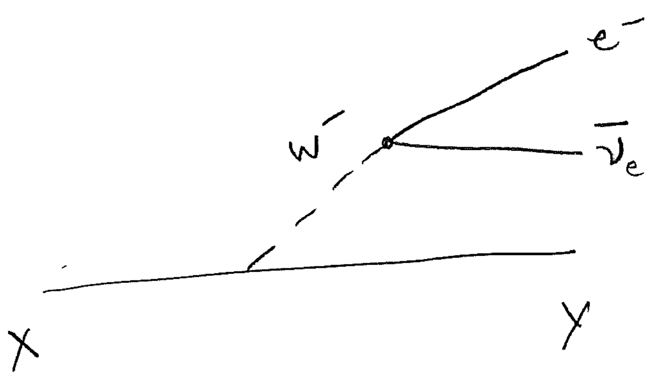
Other vertices



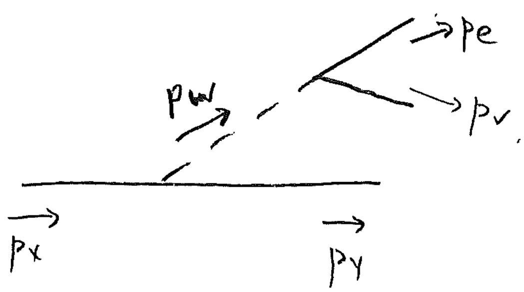




Therefore $X \to Y + e^- + \bar{\nu}_e$ obtained by assembling these



4-momentum is conserved at each vertex and overall



$\cancel{p_X = p_Y + p_e + p_\nu}$

$p_W = p_X - p_Y$

$p_W = p_e + p_\nu$ [be careful w/ arrows]

$\Rightarrow$ overall $p_X = p_Y + p_e + p_\nu$

For typical β-decay, momenta are of order of MeV or 10's of MeV

$\Rightarrow p_W \sim$ MeV

But $m_W \sim 80$ GeV

so $p_W^2 \neq M_W^2$

ie W^- is off-shell (virtual)

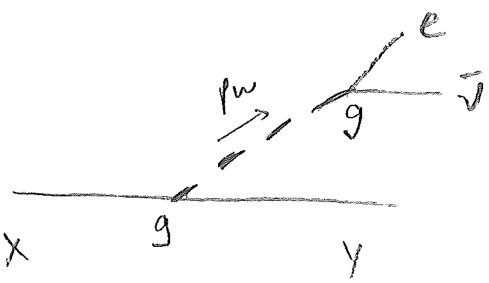
The propagator for the internal line is $\dfrac{1}{p_W^2 - M_W^2}$

(it would $\to \infty$ if W were on-shell)

The strength of a vertex involving $W^\pm$ is proportional to g = "weak charge", analogous to e for vertex involving γ.

[We'll estimate how large it is later.]

we are ignoring spin factors



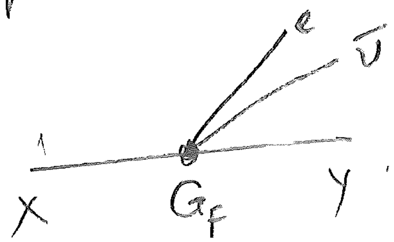
[spin $\Rightarrow \gamma^\mu(1-\gamma^5)$]

$$A = \frac{g^2}{\left|p_W^2 - m_W^2\right|}\left(\begin{array}{c}\text{spin}\\\text{stuff}\end{array}\right)$$

For typical β decay $p_W^2 \ll m_W^2$ so

$$A \approx \frac{g^2}{m_W^2}\left(\begin{array}{c}\text{spin}\\\text{stuff}\end{array}\right)$$

Replace F.d. above w/a 4-ferm. vertex



where $G_F = \dfrac{g^2}{m_W^2}$

$$A = G_F\left(\begin{array}{c}\text{spin}\\\text{stuff}\end{array}\right)$$

1932 Fermi theory of weak interaction.

$G_F = 1.1664\times10^{-5}\ \text{GeV}^{-2}$ [actually $\dfrac{G_F}{(\hbar c)^3} = 1.1664\times10^{-5}\ \text{GeV}^{-2}$]

2019 old notes

~~WI-7~~

The strength of vertices involving $W^\pm$ is proportional to g, analogous to e for the QED vertex.

g = "weak charge"

(set $\epsilon_0 = 1$)

Fine structure constant for QED $\alpha = \frac{e^2}{4\pi\hbar c} \approx \frac{1}{137}$

" " " " weak $\alpha_W = \frac{g^2}{4\pi\hbar c} \approx \frac{1}{30}$ (as we'll see from expt.)

$\alpha_W > \alpha$! Why is weak force so much weaker than EM?

Because the $W^\pm$ is so massive !

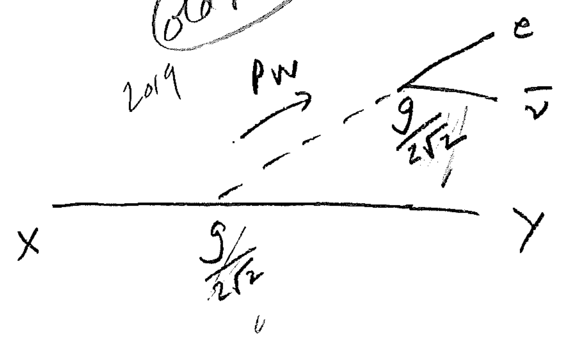
$$g = \frac{e}{\sin\theta_W}$$

$$\sin^2\theta_W = \frac{e^2}{g^2} = \frac{\alpha}{\alpha_W} = 0.215$$

old notes

~~WI-8~~

2019



[really $\frac{g}{2\sqrt{2}}(\gamma^\mu - \gamma^\mu\gamma^5)$]

Amplitude for decay $\quad A \sim \dfrac{\frac{1}{8}g^2}{p_W^2 - m_W^2} M$

For typical β-decay, $p_W^2 \ll m_W^2$ so $\qquad$ [HW]

$$A \sim \frac{g^2}{8 m_W^2} M$$

[minus sign irrelevant because $|A|^2$ to get rate.]

Large $m_W \longrightarrow$ small $A \longrightarrow$ small rate $\rightarrow$ long mean life

Determine M using unit analysis (as before)

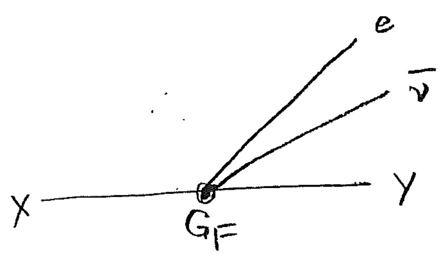
$$A \sim \frac{g^2}{8(m_W c)^2}\left(\frac{\hbar^2}{L^3}\right) = \frac{1}{L^3}\,\frac{1}{8}\left(\frac{g\hbar}{m_W c}\right)^2$$

2019 notes

~~WI-9~~

1932 Fermi proposed the 1st theory of weak interactions

Knowing nothing of the W boson, he proposed the vertex



[N.B. 2 new particles <u>created</u> in the interaction

He labelled the strength of the interaction G_F

determined from experiment to be [PPB]

$$\frac{G_F}{(\hbar c)^3} = 1.1664 \times 10^{-5}\ \mathrm{GeV}^{-2}, \qquad G_F = 8.9620\ \mathrm{E}{-5}\ \mathrm{MeV}\cdot\mathrm{fm}^3$$

$$A \sim \frac{G_F}{L^3}$$

compare w/ previous

The exact relationship between G_F and our previous constants turns out to be

$$G_F = \frac{1}{4\sqrt{2}}\left(\frac{g\hbar}{m_W c}\right)^2$$

[differs by $\sqrt{2}$ from previous $\frac{1}{8}\left(\frac{g\hbar}{m_W c}\right)^2$]

From this we can calculate the weak fine structure const

$$\alpha_W = \frac{g^2}{4\pi\hbar c} = \frac{4\sqrt{2}\,G_F}{4\pi\hbar c}\left(\frac{m_W c}{\hbar}\right)^2 = \frac{\sqrt{2}}{\pi}\,\frac{G_F}{(\hbar c)^3}\,(m_W c^2)^2$$

$$= \frac{\sqrt{2}}{\pi}(1.1664\times 10^{-5})(80.38)^2 = 0.03392 = \frac{1}{29.48}$$

Decay $\quad X \to 1 + 2 + \dots + n_f$

QFT → Decay rate $\quad R = \frac{1}{2m_X \hbar} \int (LIPS)_{n_f} |A|^2$

$$(LIPS)_{n_f} = \prod_{j=1}^{n_f} \frac{d^3 p_j}{(2\pi)^3 (2E_j)} (2\pi)^4 \delta^{(4)}(\Delta p^\mu)$$

Dimensional analysis:

$(LIPS)_{n_f}$ has dimension $E^{2n_f} E^{-4}$

$R\, m_X \hbar$ has dimension $t^{-1} \cdot E \cdot (E \cdot t) = E^2$

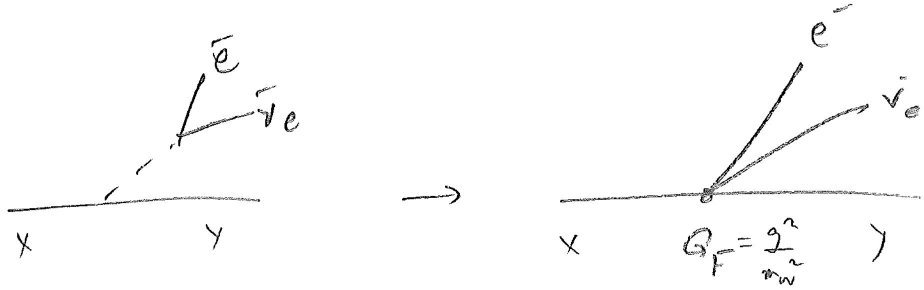
$\Rightarrow |A|^2$ has dimension $E^{-2n_f + 6}$

A has dimension $E^{3 - n_f}$

[in general E^{4-n},
n = # of initial + final states]

β-decay

$X \to e\ \bar{\nu}_e\ Y$



$A = G_F$ (spin & nuclear stuff)

$n_f = 3 \Rightarrow A$ is dimensionless

G_F has units E^2 "nuclear matrix element"

We will write:

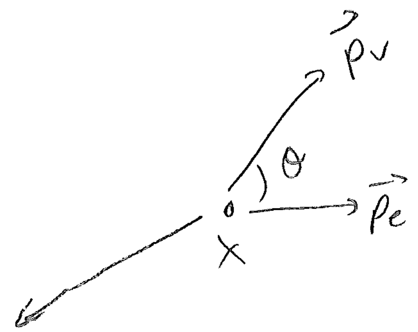
$$A = G_F \sqrt{(2E_x)(2E_y)(2E_e)(2E_\nu)}\ M$$

$$\Rightarrow R = \frac{1}{2m_x\hbar}\int (\mathrm{LIPS})_3\ |A|^2$$

$$= \frac{1}{2m_x\hbar}\int \frac{d^3p_e}{(2\pi)^3\,\cancel{2E_e}}\ \frac{d^3p_\nu}{(2\pi)^3\,\cancel{2E_\nu}}\ \frac{d^3p_y}{(2\pi)^3\,\cancel{2E_y}}\ (2\pi)^4\,\delta_4(p_e+p_\nu+p_y-p_x)\ G_F^2\,(2E_x)\cancel{(2E_y)}\cancel{(2E_e)}\cancel{(2E_\nu)}|M|^2$$

cm frame $\Rightarrow$ X at rest $\Rightarrow$ $E_x = m_x$, $\vec{p}_x = 0$

$$= \frac{G_F^2}{\hbar\,(2\pi)^5}\int d^3p_e\, d^3p_\nu\, d^3p_y\ \delta(\vec{p}_e+\vec{p}_\nu+\vec{p}_y)\ \delta(E_e+E_\nu+E_y-m_x)\ |M|^2$$



$\vec{p}_Y = -\vec{p}_e - \vec{p}_\nu$

Do $d^3\vec{p}_Y$ using momentum δ-function

Energy δ-fn: $\delta(E_e + E_\nu + E_Y - m_X)$

$$= \delta(m_e + T_e + E_\nu + m_Y + T_Y - m_X)$$

<u>Define</u> $Q = m_X - m_Y - m_e$ (N.B. we do not include m_ν)

$$\delta(T_e + E_\nu + T_Y - Q)$$

typically

$$T_Y = \frac{(\vec{p}_e + \vec{p}_\nu)^2}{2m_Y} \ll T_e + E_\nu \le Q \ll m_Y$$

which allows non rel. formula for T_Y

so ignore T_Y in this eqn:

$$\Rightarrow \boxed{R = \frac{G_F^2}{\hbar (2\pi)^5} \int d^3p_e \, d^3p_\nu \, \delta(T_e + E_\nu - Q) \, |M|^2}$$

$d^3p_e \, d^3p_\nu = p_e^2 \, dp_e \, d\Omega_e \;\; p_\nu^2 \, dp_\nu \, d\Omega_\nu$

$p_e \cdot |\vec{p}_e|$
$p_\nu \cdot |\vec{p}_\nu|$

Assume M is relatively insensitive to θ ($\measuredangle$ between $\vec{p}_e$ & $\vec{p}_\nu$)

Integrate over angles $\int d\Omega_e \cdot \int d\Omega_\nu = 4\pi$

$$\frac{(4\pi)^2}{(2\pi)^5} = \frac{1}{2\pi^3}$$

$$\Rightarrow R = \frac{G_F^2}{2\pi^3 \hbar} \int p_e^2 \, dp_e \int p_\nu^2 \, dp_\nu \, \delta(T_e + E_\nu - Q) \, |M|^2$$

[Segre 9-5.14
p. 352]

Next, assume the ν is strictly massless.

$$\Rightarrow \quad E_\nu = p_\nu$$
$$dE_\nu = dp_\nu$$

$$R = \frac{G_F^2}{2\pi^3\hbar}\int p_e^2\, dp_e \underbrace{\int E_\nu^2\, dE_\nu\, \delta(T_e + E_\nu - Q)\, |M|^2}_{(Q-T_e)^2\, |M|^2}$$

evaluated at $E_\nu = Q - T_e$

$$= \int dp_e \underbrace{\frac{G_F^2}{2\pi^3\hbar}\, p_e^2 (Q - T_e)^2 |M|^2}_{\text{call this } \frac{dR}{dp_e}}$$

$dp_e \frac{dR}{dp_e}$ is the decay rate to electrons w/ momenta in the range p_e to $p_e + dp_e$ and can be experimentally measured.

One measures $\frac{dR}{dp_e}$ and then plots

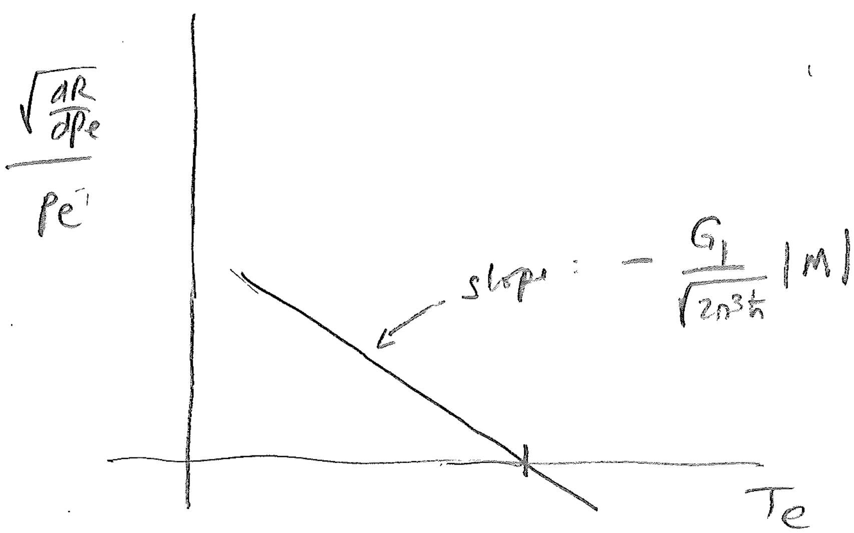
$$\frac{\sqrt{\frac{dR}{dp_e}}}{p_e} = \frac{G_F}{\sqrt{2\pi^3\hbar}}(Q - T_e)\,|M|$$

as a function of T_e

(Kurie plot)

If $|M|$ is insensitive to T_e (as well as to θ)

this plot will be linear



(Indeed this seems to be the case)

If the neutrino has mass, then expect [HW]

$$\frac{\sqrt{\frac{dR}{dp_e}}}{p_e} \sim (Q - T_e)\Big[\underbrace{1 - \left(\frac{m_\nu}{Q - T_e}\right)^2}_{\text{vanishes if } T_e = Q - m_\nu}\Big]^{\frac{1}{4}} |M|$$

only important when $Q - T_e$ is small

(see plots)

$m_\nu < 0.3\ \mathrm{eV}$

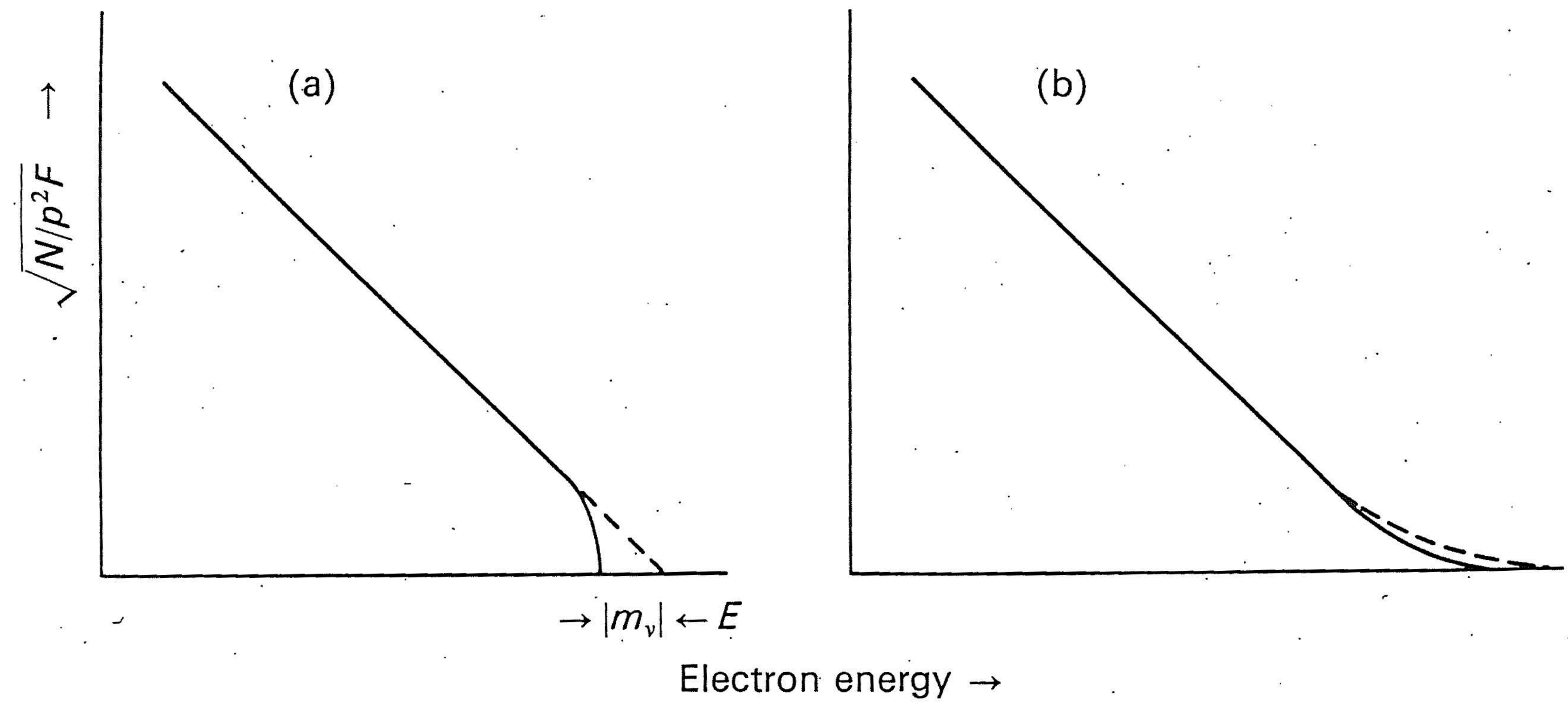


Fig. 6.2 Kurie plot of allowed transition for zero and finite neutrino mass: (a) per
resolution, (b) finite resolution.

Perkins

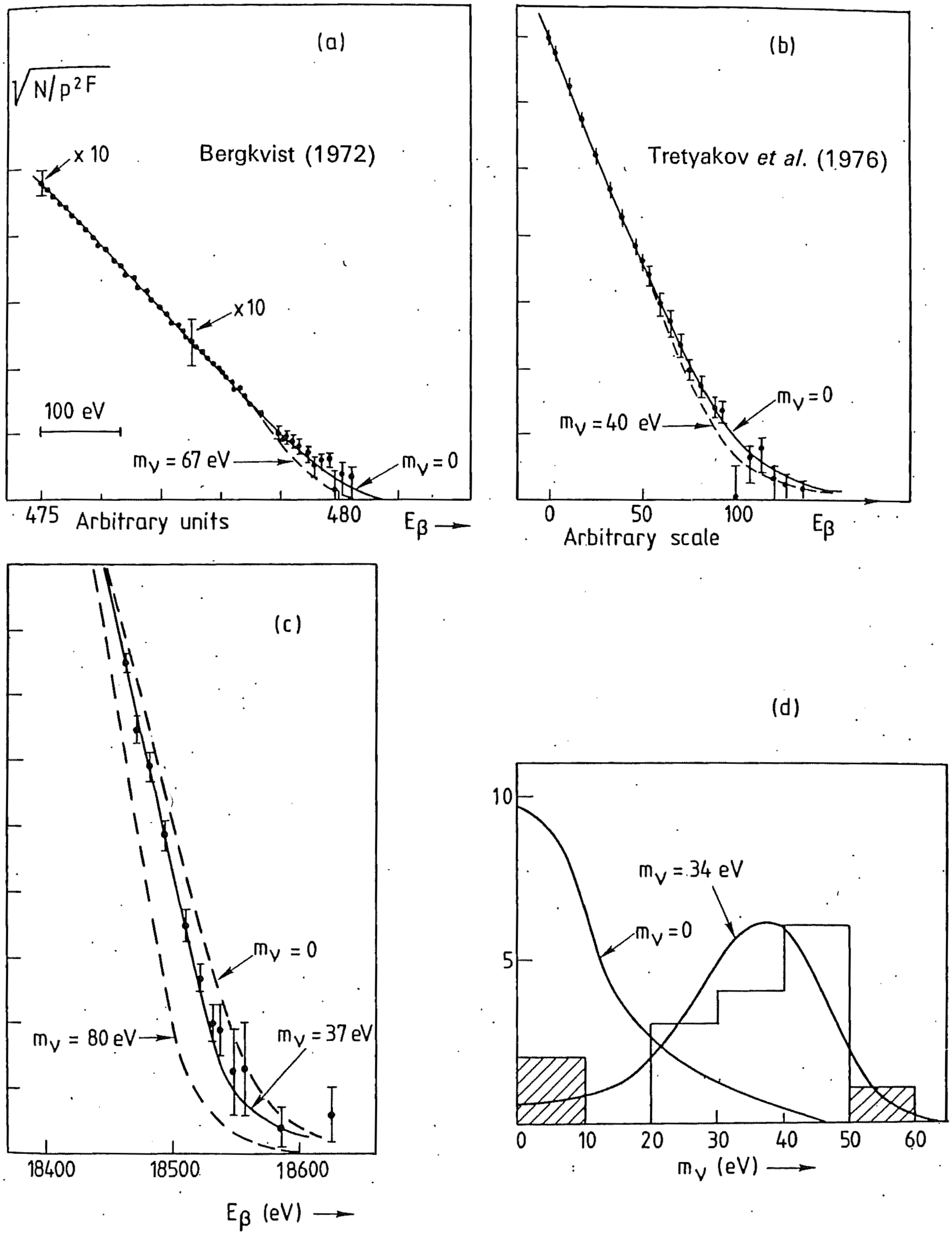


Fig. 6.3 Kurie plots from recent measurements of tritium β-decay. (a) Bergkvist (1972); (b) Tretyakov *et al.* (1976); (c) and (d) Lyubimov *et al.* (1980). (c) shows the average of several runs, and (d) the "best" mass estimate for m_ν from each of 16 runs. The expected distributions for $m_\nu = 0$ and $m_\nu = 35$ eV/c^2 are indicated.

Perkins

Skip this since we are not going to try to do neutron decay accurately, need not do this...

~~WI - 14~~

Estimate decay rate, assuming $|M|$ is insensitive to E_e

$$R = \frac{G_F^2 \, |M|^2}{2\pi^3 \, \cancel{\hbar^7 c^3}} \int dp_e \, |\vec{p}_e|^2 \, (Q - T_e)^2$$

Recall $E_e^2 = (\cancel{c}\,p_e)^2 - (m_e \cancel{c^2})^2$

$$E_e \, dE_e = \cancel{c^2} |\vec{p}_e| \, dp_e$$

$$Q - T_e = m_x - m_y - m_e - T_e = m_x - m_y - E_e$$

$$R = \frac{1}{2\pi^3 \cancel{\hbar}} \left[\frac{G_F}{\cancel{(\hbar c)^3}} \right]^2 |M|^2 \int_{m_e}^{m_x - m_y} dE_e \, E_e \sqrt{E_e^2 - (m_e \cancel{c^2})^2} \, (m_x - m_y - E_e)^2$$

mean life $\tau = \frac{1}{R}$

Perhaps unnecessary to do this, since even though phase space integral is more accurate, still need to assume $M=2$, & even so, the result is 50% off

~~WI-15~~

Neutron decay

$$n \to p + e^- + \bar{\nu}_e$$

Numerically integrate

$$\int_{m_e}^{m_n - m_p} dE_e \, E_e \sqrt{E_e^2 - m_e^2} \, (m_n - m_p - E_e)^2 = 0.057 \text{ MeV}^5$$

If n were a fundamental spin-$\frac{1}{2}$ particle, then $|M| = 2$.

$$R = \frac{\Gamma}{\hbar} = \frac{(1.166\text{E}{-11} \text{ MeV}^{-2})^2}{2\pi^3 \, (6.6\text{E}{-22} \text{ MeV}\cdot\text{s})} (2)^2 (0.057 \text{ MeV}^5)$$

$$= 7.6 \times 10^{-4} \text{ s}^{-1}$$

$$\tau = 1300 \text{ s} \approx 22 \text{ min}$$

$$\tau_{expt} \approx 15 \text{ min}$$ [close!]

(n is <u>not</u> a fund. spin $\frac{1}{2}$ particle)

cf p 320 Griffiths for refinements

FermiIntegral.nb

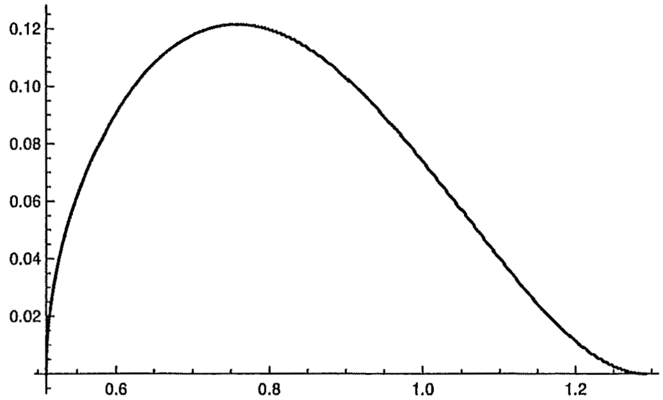
```
In[30]:= P[me_, del_] := Plot[e Sqrt[e^2 - me^2] (del - e)^2, {e, me, del}]

In[31]:= F[me_, del_] := Integrate[ e Sqrt[e^2 - me^2] (del - e)^2, {e, me, del}]

In[32]:= me = 0.511; del = 939.565 - 938.272;

In[33]:= P[me, del]
```

Out[33]=

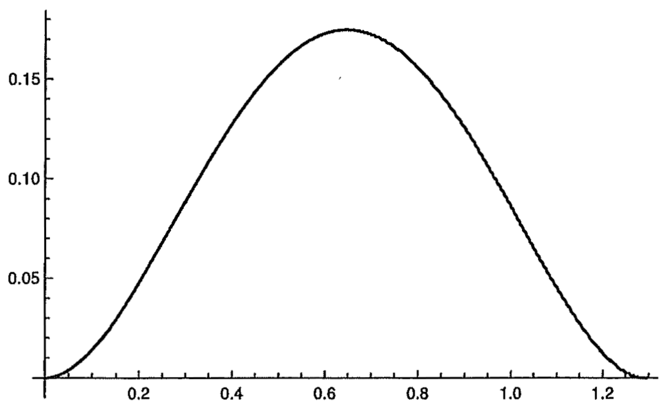



```
In[34]:= F[.511, del]

Out[34]= 0.0569086

In[35]:= P[0, del]
```

Out[35]=




```
In[36]:= F[0, del]

Out[36]= 0.120468
```

massless electron approximation

Suppose $Q \gg m_e$

Then generally $T_e \gg m_e$ (except near endpoint)

so electron is ultrarelativistic & ∴ effectively massless

so $p_e \approx E_e$

$T_e \approx E_e$

so

$$R = \frac{G_F^2}{2\pi^3 \hbar} \underbrace{\int_0^Q dE_e \; E_e^2 \, (Q - E_e)^2}_{\frac{1}{30} Q^5} \; |M|^2$$

again assume $|M|$ is relatively independent of E_e

$$= \underbrace{\frac{G_F^2}{60\pi^3 \hbar}}_{1.111\times 10^{-4}\ \text{MeV}^{-5}\cdot\text{s}^{-1}} |M|^2 \, Q^5$$

Sargent's rule ⇒ decay rate grows rapidly w/ Q due to increased phase space

[Use experimental data to measure $|M|^2$ & ∴ explore nuclear structure]

G_F . 1.1664×10^{-11} MeV^{-2}

$\hbar$. 6.58212×10^{-22} MeV · s

Neutron decay rate

$$n \to p\, e^- \bar{\nu}_e$$

expt: $\tau = 880\ s$ $(\sim 15\ \text{min})$

$R = \frac{1}{\tau} = 1.1 \times 10^{-3}\ s^{-1}$

Sargent formula: $R = (1.11 \times 10^{-4}\ \text{MeV}^{-5}\ s^{-1})\ Q^5\ |M|^2$

$Q = m_n - m_p - m_e \quad 0.8\ \text{MeV}$

but massless electron approx $Q = m_n - m_p = 1.3\ \text{MeV}$

choose $1.1\ \text{MeV}$, which is consistent w/ exact calculation not assuming m_e.

$$R = (1.9 \times 10^{-4}\ s^{-1})\ |M|^2$$

Suggests $M = 2.3$ for neutron decay matrix element

[Frauenfelder + Henley]

$\bar{\nu}$ absorption (see problem)

Frauenfelder + Henley

p. 279, table 11.1 $\quad n \to p \quad ft_{\frac{1}{2}} = 1100$ sec

p. 307 eq 11.74 : $|GM|^2 = |\langle n e | H_w | p \bar{\nu} \rangle|^2$

p. 278 eq 11.11 $\quad = \dfrac{2\pi^3}{f\tau} \dfrac{\hbar^7}{m_e^5 c^4}$

$$= \underbrace{\frac{2\pi^3 \hbar^7 \ln 2}{m_e^5 c^4}} \frac{1}{ft_{\frac{1}{2}}}$$

$$= \frac{2\pi^3 \ln 2 \, (\hbar c)^7}{c \, (m_e c^2)^5}$$

strangely disagrees by eq 11.12

$$= \frac{2\pi^3 \ln 2 \, (197.327 \text{ MeV fm})^7}{(3E23 \frac{fm}{s})(0.511 \text{ MeV})^5} = (4.8E-5) \text{ MeV}^2 \text{ fm}^6 \text{ s}$$

$$|GM|^2 = 4.4 E-8 \text{ MeV}^2 \text{ fm}^6$$

$$GM = 2.1E-4 \text{ MeV fm}^3 \quad \leftarrow (11.13)$$

$\dfrac{G}{(\hbar c)^3} = 1.1664E-5 \text{ GeV}^{-2}$

$= 1.1664E-11 \text{ MeV}^{-2}$

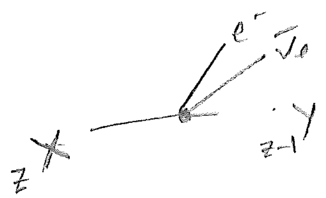
$G = (1.1664E-11)(197)^3$

$= 8.9E-5 \text{ MeV} \cdot \text{fm}^3$

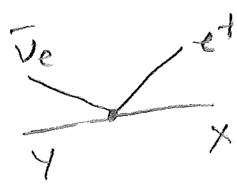
$M = 2.3$

Continuous spectrum of β-decay
gives indirect evidence for neutrinos

Can one detect them directly?



ν emission



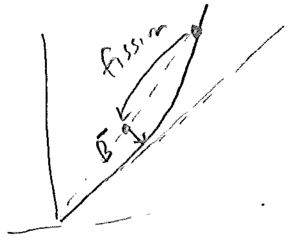
ν detection

Hw: calc. $\sigma(Y\bar{\nu}_e \to Xe^+)$, find ~ 10^{-19} barns
[a very small barn]

mean free path of $\bar{\nu}$ ~ light years

[Hw: $l \cdot \frac{1}{n\sigma} \sim 10^{18}$ m ~ 100 ly]

Need a lot of $\bar{\nu}$. ⇒ fission reactors

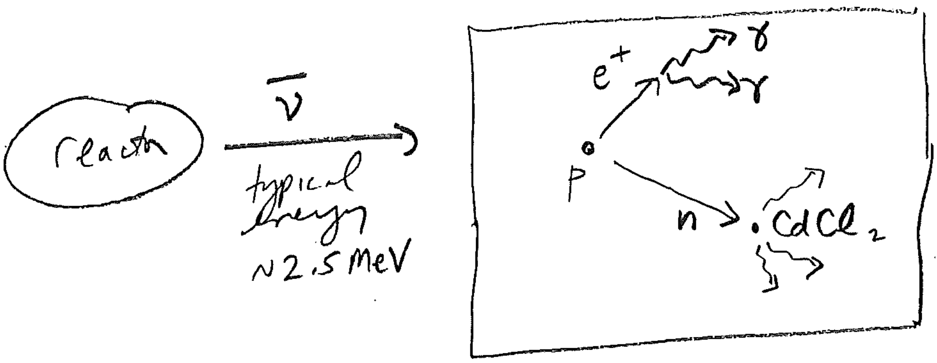


$n \to p\, e^- \bar{\nu}$

$\bar{\nu}\, p \to n\, e^+$
provided E_ν is large enough

1956 Cowan-Reines antineutrino detection experiment at Savannah River commercial reactor (S. Carolina)

[show picture]



[recall cadmium good at absorbing neutrons]

experimental signature: 2 0.511 MeV γ

followed by 3 to 4 γ w/ $E_{tot} \sim 9$ MeV after $\sim 10^{-6}$ s.

given in HW problem (no need to write) → [reactor flux $F \sim 1.2 \times 10^{17} \frac{\bar{\nu}}{m^2\text{-}s}$

detector efficiency $\epsilon \sim 0.025$

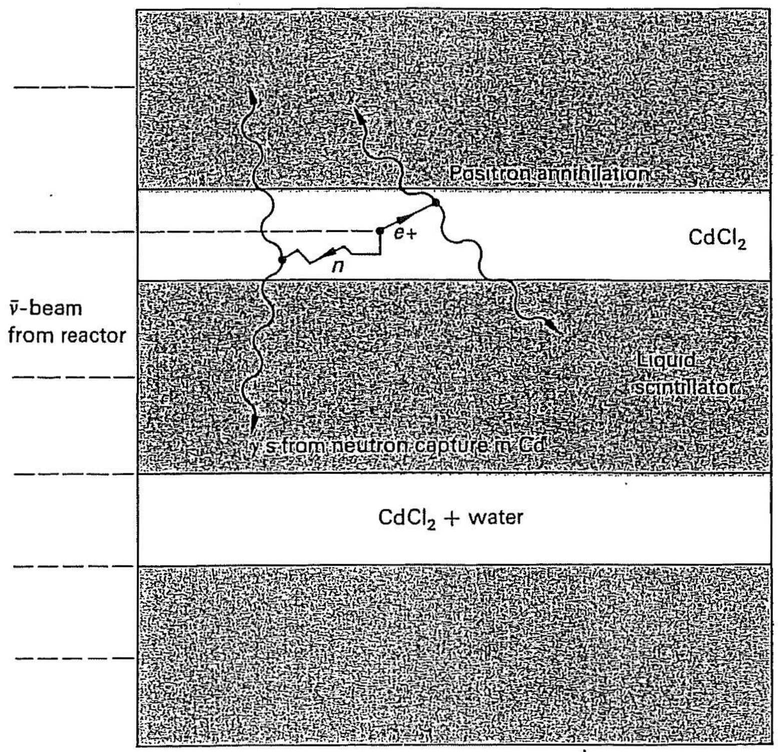
[HW: calculate # events per day]

Recall σ · $\frac{P}{I}$

scattering rate $R = \sigma F$ per target

targets $N = nV$ n = target density $\left(\frac{\#}{\text{vol}}\right)$

total rate $RN = nV\sigma F$



Positron annihilation
e+
n
CdCl₂
ν̄-beam
from reactor
Liquid
scintillator
γ's from neutron capture in Cd
CdCl₂ + water

183 cm × 132 cm

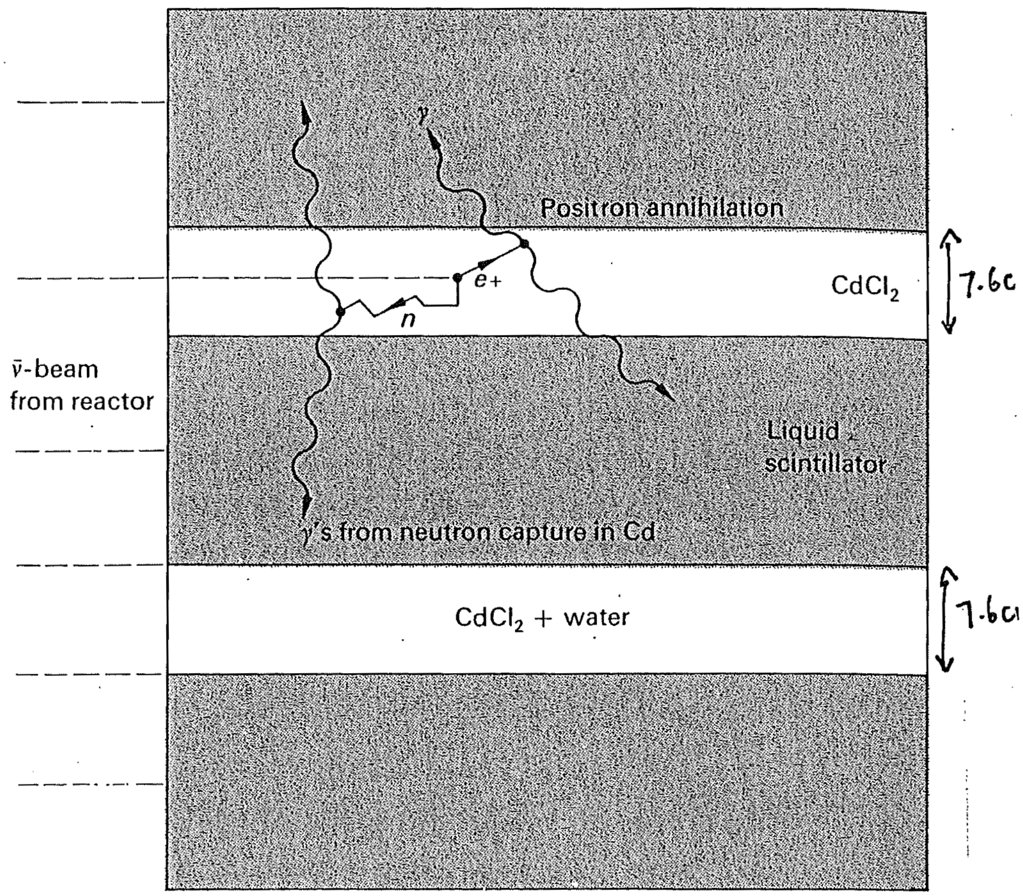


Fig. 6.4 Schematic diagram of the experiment by Reines and Cowan (1959), interactions of free antineutrinos from a reactor.

Perkins

water volume ~ 400,000 cm^3

6′ 4 $\frac{1}{3}$′

183 cm × 132 cm × 2(7.6 cm) = 367,000 cm^3

Asimov: 6 $\frac{1}{4}$′ × 4 $\frac{1}{2}$′ × 6″

What about ν detection?

$$\nu + n \rightarrow p + e^-$$

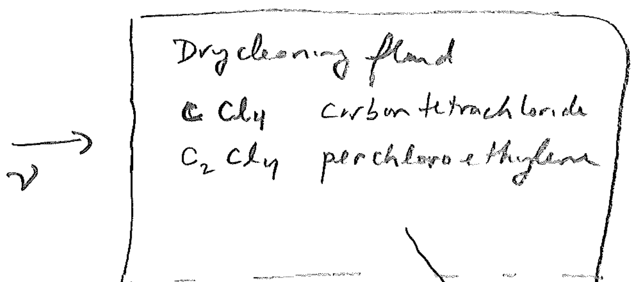
but no free neutrons. Neutrons must be in nuclei

$$\nu + {}^{A}_{Z}Y \rightarrow {}^{A}_{Z+1}X + e^-$$

e.g. $\nu + {}^{37}_{17}Cl \rightarrow {}^{37}_{18}Ar + e^-$ $\qquad Q = -0.814\,\text{MeV}$

Ray Davis experiment (1955) [F+H, p. 187]

[Davis + Harmon, 1959]



${}^{37}Ar$ [flush out + concentrate]

(unstable w/ $T_{\frac{1}{2}} = 35$ days)

$36,38,40Ar$ stable

EC: $e^- + {}^{37}_{18}Ar \longrightarrow {}^{37}_{17}Cl + \nu$ $\qquad (Q = 0.814\,\text{MeV})$

(Auger electrons)

Absence of such reactions demonstrated ~~that~~, ν and $\bar{\nu}$ are distinct

Lepton # conservation. [Konopinski, Mahmoud 1953]

$L(e^-) = L(\nu) = 1$

$L(e^+) = L(\bar{\nu}) = -1$

→ Solar neutrino problem!

WI-19

μ^- decay

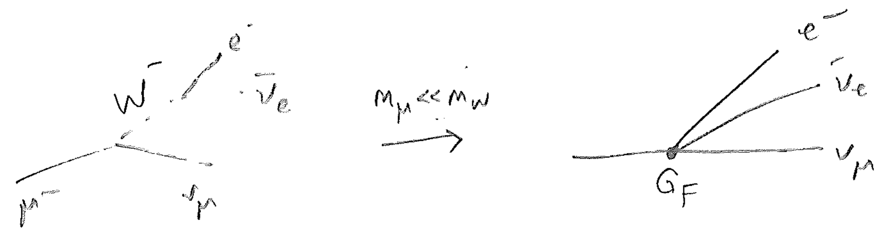
[→ In problem they work out maximum ($\frac{(m_\mu - m_e)^2}{2m_\mu} = 52.32$) and minimum (0) kinetic energies for the electron]

[PPB] $\mu^- \to e^- \bar{\nu}_e \nu_\mu$

$m_\mu = 105.6$ MeV
m_e 0.5 MeV

$\tau = 2.197 \times 10^{-6}$ s

$R_{exp} = \frac{1}{\tau} = 4.552 \times 10^5 \, s^{-1}$ (much faster than neutrons)



Sargent: $R = \frac{G_F^2}{60\pi^3\hbar} |M|^2 Q^5$ $Q = m_\mu - m_e \simeq m_\mu$

not valid because assumes $X \to Y e^- \bar{\nu}_e$ w/ $M_Y \gg m_e$ (so $e, \bar{\nu}$ get all the energy)
whereas $m_{\nu_\mu} \ll m_e$ (so split 3 ways, cf problem above)

An accurate calculation gives

$$R = \frac{G_F^2}{192\pi^3\hbar} m_\mu^5$$ in massless electron approximation (+ doing spin correctly)

$$= (1.111 \times 10^{-4} \text{ MeV}^{-5} s^{-1}) \frac{60}{192} (105.66 \text{ MeV})^5$$

$$R_{th} = 4.572 \times 10^5 \, s^{-1}$$

R_{th} & R_e very close! ($\frac{1}{2}$%)

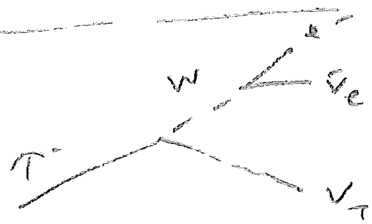
[including $m_e \neq 0$ does not improve]

τ^- decay

$$\tau^- \to e^- \bar{\nu}_e \nu_\tau$$

expt $\tau = 2.90 \times 10^{-13}$ s

$R = 3.45 \times 10^{12}\ s^{-1}$



$$R = \frac{G_F^2}{192\pi^3\hbar^3} m_\tau^5 = 6.15 \times 10^{11}\ s^{-1}$$

about 6 times too slow

τ decay channels

PPB:

$\tau \to e^- \bar{\nu}_e \nu_\tau$ 17.8%

$\to \mu^- \bar{\nu}_\mu \nu_\tau$ 17.4%

$\to \pi^- \nu_\tau$ 10.8%

more decay channels $\Rightarrow$ faster rate

$R_{tot} = \sum_i R_i$ $R_i \equiv$ "partial rates"

Branching ratio $(BR)_i = \frac{R_i}{R_{tot}}$ $\sum (BR)_i = 1$

$$\Rightarrow R_{tot} = \frac{R_i}{(BR)_i} = \frac{6.15 \times 10^{11}\ s^{-1}}{0.178} = 3.45 \times 10^{12}\ s^{-1}\ ✓$$

W^- decay

W, Z first directly detected 1983 at CERN

[PPB] $m_W = 80.4$ GeV

$W^+ \rightarrow e^+ \nu_e$ (10.7%)

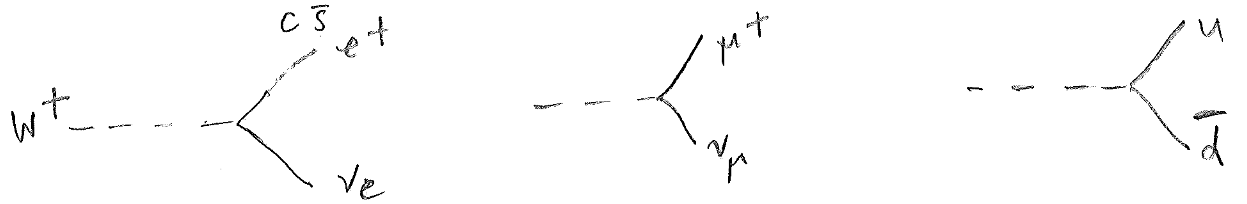
$\mu^+ \nu_\mu$

$\tau^+ \nu_\tau$

$u \bar{d}$

$c \bar{s}$

- might expect 20%
- but quarks have color so ∃ multiply by # colors



kinematics: $p_W = (m_W, 0, 0, 0)$

$p_e = (E_e, 0, 0, p)$

$p_\nu = (E_\nu, 0, 0, -p)$

$\nu \longleftarrow \overset{W}{\bullet} \longrightarrow e$

$$m_W = E_e + E_\nu = \sqrt{p^2 + m_e^2} + \sqrt{p^2 + m_\nu^2} \approx 2p$$

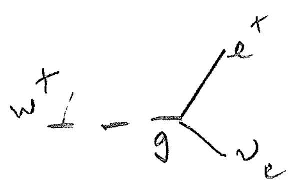
$m_\nu, m_e \ll m_W$

$$R_W = \frac{1}{2 m_W \hbar} \int (LIPS)_2 \, |A|^2$$

$$(LIPS)_2 = \frac{p_f}{(4\pi)^2 E_{cm}} \, d\Omega \approx \frac{1}{2(4\pi)^2}$$

$$R_W = \frac{1}{4(4\pi)^2 m_W \hbar} \underbrace{\int d\Omega}_{4\pi} \, |A|^2 = \frac{|A|^2}{16\pi m_W \hbar}$$

$\Rightarrow$ A has dimensions of energy $\qquad (E^{4-n} = E^1)$



$A = g$ (spin stuff)

g is dimensionless

Let us write $\qquad$ (spin stuff) $= \sqrt{(2E_e)(2E_\nu)} \, f$

$2E_e \simeq 2p = m_W$
$2E_\nu \simeq 2p = m_W$ $\quad \Rightarrow \quad A = g \, m_W f$

$$R(W \to e^+ \bar{\nu}_e) = \frac{g^2 m_W}{16\pi\hbar} = \frac{G_F m_W^3}{16\pi\hbar} f^2$$

Recall $\frac{g^2}{m_W^2} = G_F \Rightarrow g^2 = G_F m_W^2$

$$\left[\begin{array}{ll} G_F = 1.16 \times 10^{-5} \text{ GeV}^{-2} & \frac{G_F m_W^3}{16\pi} = 0.12 \text{ GeV} \\ m_W = 80.4 \text{ GeV} & \end{array} \right]$$

$$R(W \to e^- \bar{\nu}_e) = \frac{G_F m_W^3}{16\pi\hbar} f^2$$

This is a partial rate.

[Hw: use full rate and branching ratio to estimate f]

[Ans: $f^2 = \frac{16}{6\sqrt{2}} = 1.8856$, $f = 1.373$]

$$R_W = \sum_i R_i$$

PPB $\Rightarrow$ $\tau \sim 3 \times 10^{-25}\ s$

$\Gamma = 2.085$ GeV $\qquad c\tau \sim 0.1$ fm $\Rightarrow$ How do you measure this?

Define $\underbrace{\text{width } \Gamma}_{\text{units of energy}}$ of unstable particle as $\hbar R = \frac{\hbar}{\tau}$

Γ may be interpreted as imag part of mass

Free particle: $\psi(x,t) = e^{\frac{ipx}{\hbar}} e^{-\frac{iEt}{\hbar}}$

At rest: $\psi = e^{-\frac{imt}{\hbar}}$

Let $m = (\mathrm{Re}\, m) - i\frac{\Gamma}{2}$

$$\psi = e^{-\frac{i}{\hbar}(\mathrm{Re}\, m - \frac{i\Gamma}{2})t}$$

$$= e^{-\frac{i(\mathrm{Re}\, m)t}{\hbar}} e^{-\frac{\Gamma}{2\hbar}t}$$

$\mathrm{Prob} = |\psi|^2 = e^{-\frac{\Gamma}{\hbar}t}$ but $\mathrm{Prob} = e^{-Rt}$

so $\Gamma = \hbar R$

$\Gamma \ll \mathrm{Re}\, m$ for fairly stable particle ($\hbar = 6.6 \times 10^{-22}$ MeV s)

eg. muon $\tau = 2.2 \times 10^{-6}$ s $\Rightarrow \Gamma = \frac{6.6 \times 10^{-22}}{2.2 \times 10^{-6}} \frac{\text{MeV s}}{\text{s}} = 3 \times 10^{-16}$ MeV

$= 3 \times 10^{-10}$ eV

but W^-: $\Gamma = 2$ GeV $\Rightarrow \tau = \frac{\hbar}{\Gamma} = 3.3 \times 10^{-25}$ sec

---

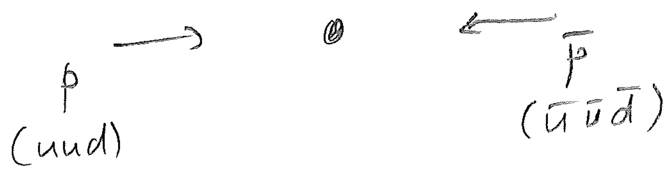
Partial width. $\Gamma_i = \hbar R_i$ (R_i = partial rate)

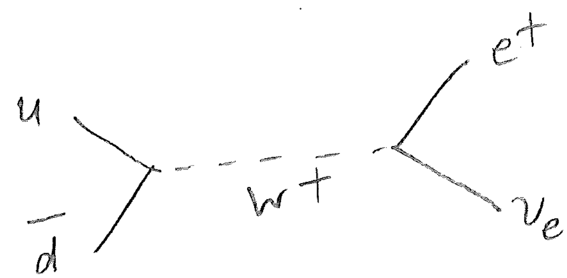
Full width $\Gamma = \Sigma \Gamma_i$

$(BR)_i = \frac{\Gamma_i}{\Gamma}$

How does one detect the existence of a very short-lived particle like the W?

$p\bar{p}$ collider (super proton synchrotron at CERN)





$$\underbrace{p_u + p_{\bar{d}}}_{\substack{\text{in cm frame} \\ =(E_{cm},\, 0)}} = p_W$$

so $p_W^2 = E_{cm}^2$

If $E_{cm} \ll m_W$ then off-shell (virtual)

Amplitude for the process above

$$A \sim \frac{g^2}{p_W^2 - m_W^2} \sim \frac{g^2}{E_{cm}^2 - m_W^2}$$

scattering cross section $\sigma \sim |A|^2$

If $E_{cm} \sim m_W$, then $A \to \infty \Rightarrow \sigma \to \infty$

But if W is unstable, m_W has an imag. component

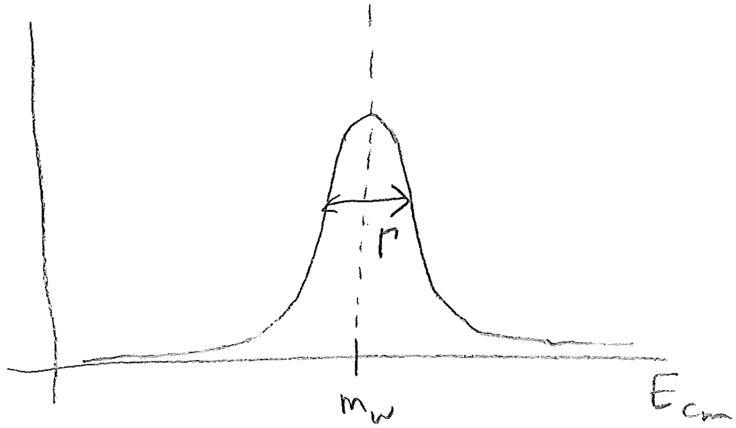
$$A \sim \frac{1}{E_{cm}^2 - (m_W - \frac{i\Gamma}{2})^2}$$

$$\sim \frac{1}{(E_{cm} - m_W + \frac{i\Gamma}{2})(E_{cm} + m_W - i\frac{\Gamma}{2})}$$

For $E_{cm} \sim m_W$, 2nd term $\sim 2m_W - \frac{i\Gamma}{2} \approx 2m_W$

$$A \sim \frac{1}{2m_W(E_{cm} - m_W + \frac{i\Gamma}{2})}$$

$$\sigma \sim |A|^2 \sim \frac{1}{(2m)^2\left[(E_{cm} - m_W)^2 + \frac{\Gamma^2}{4}\right]}$$ (Breit-Wigner curve)



Analogous to resonance of a damped driven harmonic osc.

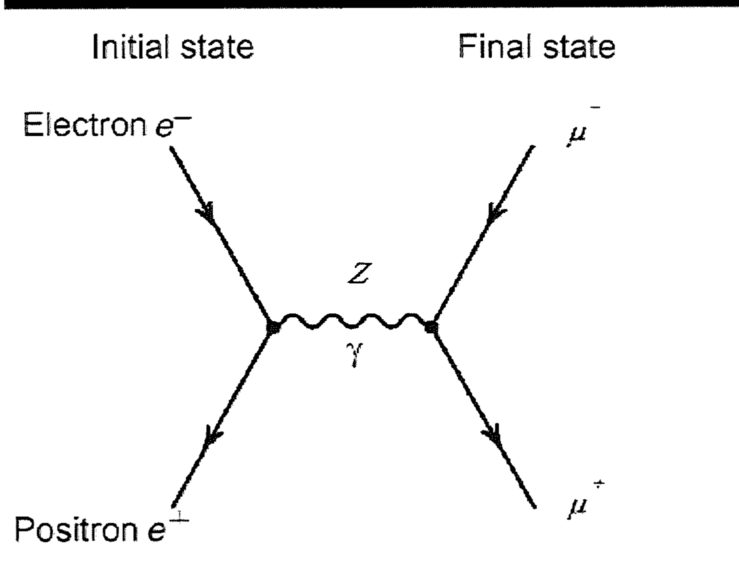
→ resonance curve

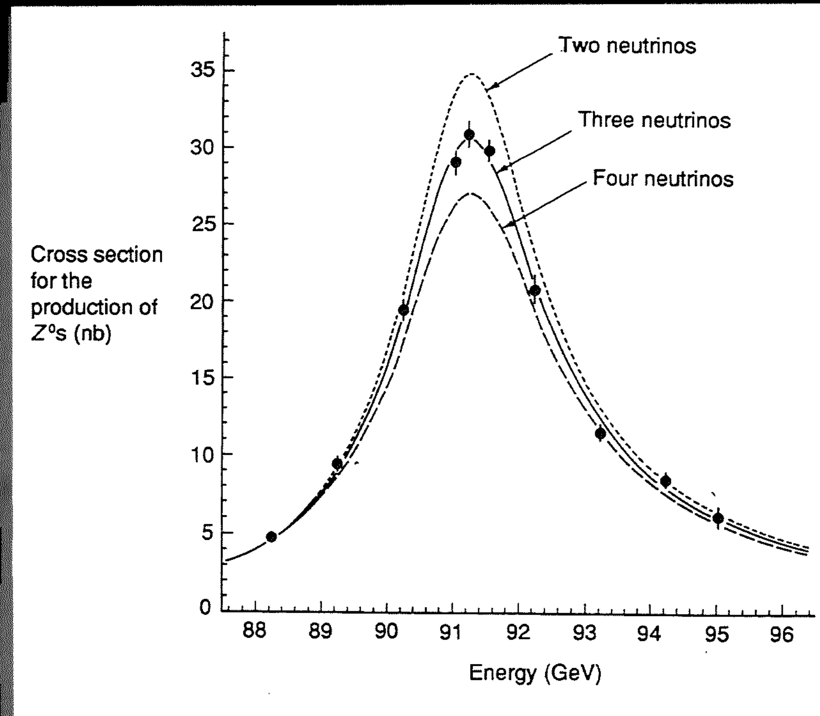
Γ is the width of the resonance

(σ = half maximum for $E_{cm} - m_W = \frac{\Gamma}{2}$
(Γ = full width half-maximum

unstable particles are detected through bumps in the scattering cross-section as a fcn of E_{cm}

How Many Neutrinos?





$Z^0 \to q\bar{q}(u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}, b\bar{b})$

$Z^0 \to l\bar{l}\,(e^-e^+, \mu^-\mu^+, \tau^-\tau^+)$

$Z^0 \to \nu\bar{\nu}(\nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau)$

$\nu\bar{\nu}$

Γ_Z measured

$\Gamma_{had}, \Gamma_l, \Gamma_\nu$ calculated

$\Gamma_Z = \Gamma_{had} + 3\Gamma_l + N_\nu$

$N = 2.99 + 0.02$

otal width Γ ~ decay probability (~1/lifetime)

artial Γ_i ~ branching rate (channel i)

Heisenberg uncertainty relation

$$\Delta p \, \Delta x \geq \frac{\hbar}{2}$$

$$\Rightarrow \Delta E \, \Delta t \geq \frac{\hbar}{2}$$

What does Δt mean? The time it takes to measure the energy

But unstable particle only lives τ so $\Delta t < \tau$

$$\Rightarrow \Delta E \geq \frac{\hbar}{2\tau} = \frac{\Gamma}{2}$$ so $\frac{\Gamma}{2}$ = uncertainty in mass of a particle