To ~~describe~~ understand scattering and decay processes, we use quantum field theory (QFT)

In this framework, fields are the fundamental entities of the universe

one field for each type of fundamental particle.

Particles are understood as quantum excitations of the field

For example

electromagnetic field (spin 1) $\longrightarrow$ photons

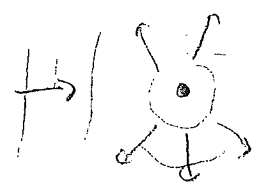
Dirac fields (spin $\frac{1}{2}$) $\longrightarrow$ electrons, neutrinos, quarks

Higgs field (spin 0) $\longrightarrow$ Higgs boson

classical physics: deterministic
eg. Rutherford scattering

In a classical process,
the final state is completely determined by
the initial state + the laws of physics.

eg. in a collision of α-particle w/ nucleus,
given p_A and P_B (and impact parameter) one can
determine p_f and P_2



quantum physics: probabilistic

In a quantum process,
the final state is not completely determined.
one can compute only probabilities of various final states
that are allowed by the laws of physics

QFT gives a recipe for computing probabilities.
The probability of a given final state is given by the square of modulus of the amplitude $|A|^2$

The rate of a process (eg. scattering, decays) is then obtained by adding up the probabilities of all possible final states.

$$\text{Rate} = \sum_{\substack{\text{final}\\\text{states}}} |A|^2$$

A is represented by a sum of Feynman diagrams; these depend on the strength of the interaction among other things

↳ depends on 1) # of channels
2) final state phase space

[~~How fast a process proceeds~~ The rate] depends on 2 things:

① Amplitude A, represented by a sum of Feynman diagrams

– [depends on strength of the interactions and other factors]

② Final state phase space [how many final states there are]

– [In an inelastic process, this depends on amt. of kinetic energy released, Q]

Generally, larger Q ⇒ more phase space (more final states available)

⇒ process is more likely

(larger cross-section, shorter lifetime)

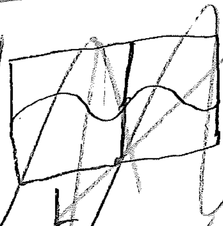
Phase space of a single free particle

free particles of momentum $\vec{p}$ are described by $e^{\frac{i\vec{p}\cdot\vec{x}}{\hbar}}$

(normalization)

To be able to normalize these, imagine the universe = box w/ volume $V = L^3$

w/ periodic boundary conditions (at end, take $V \to \infty$)

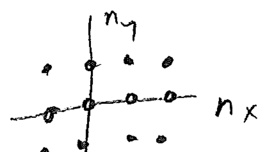


$$e^{\frac{ip(x+L)}{\hbar}} = e^{\frac{ipx}{\hbar}}$$

$$e^{\frac{ipL}{\hbar}} = 1 \Rightarrow p = \frac{2\pi\hbar}{L} n$$

3 dimensions $\vec{p} = \frac{2\pi\hbar}{L}\vec{n} = \frac{2\pi\hbar}{L}(n_x, n_y, n_z)$

Set of possible states characterized by lattice of integers



$$\sum_{n_x, n_y, n_z} \approx \int dn_x\, dn_y\, dn_z$$

$$= \left(\frac{L}{2\pi\hbar}\right)^3 \int dp_x\, dp_y\, dp_z$$

$$= \cancel{\frac{V}{(2\pi\hbar)^3}\int d^3p} = \frac{V}{\hbar^3}\int \frac{d^3p}{(2\pi)^3}$$

integral over phase space

Sum over final states

$$\boxed{\sum_f = \prod_{j=1}^{n_f} \frac{V}{\hbar^3}\int \frac{d^3p_j}{(2\pi)^3}}$$

"Integral over phase space"

$\longrightarrow v_A \quad \vec{v_B}$

$\boxed{E_A E_B (v_A - v_B)}$ is Lorentz invariant under boosts in x-direction

Proof: $(m_A \cosh \gamma_A)(m_B \cosh \gamma_B)(\tanh \gamma_A - \tanh \gamma_B)$

$= m_A m_B (\cosh \gamma_A \sinh \gamma_B - \cosh \gamma_B \sinh \gamma_A)$

$= m_A m_B \sinh(\gamma_A - \gamma_B)$

$= m_B \, m_A \cosh \gamma_{AB} \tanh \gamma_{AB}$

$= m_B (E_A \underbrace{v_{AB}}_{\text{relative}})$ ← velocity in fixed target frame (i.e. $\tanh \gamma_{AB} = \tanh(\gamma_A - \gamma_B)$)

$\xrightarrow{v_{AB}} \; \overset{\circ}{m_B}$

In FT frame $= m_B p_A$ (lab frame)

In cm frame: $p_A = -p_B$

$m_A \sinh \gamma_A = -m_B \sinh \gamma_B$

so $\underline{E_A E_B (v_A - v_B)} = m_A \sinh \gamma_A \, m_B \cosh \gamma_B - m_B \sinh \gamma_B \, m_A \cosh \gamma_A$

$= m_A \sinh \gamma_A (m_B \cosh \gamma_B + m_A \cosh \gamma_A)$

$= p (E_B + E_A)$

$= \underline{p \, E_{cm}}$

where V_1 is the normalization volume for particle 1, and v_1 its velocity. In fact by (103)

$$V_1 = \frac{mc^2}{E_1} \qquad V_2 = \frac{mc^2}{E_2} \qquad (v_1 - v_2) = \frac{c^2 p_1}{E_1} - \frac{c^2 p_2}{E_2} \tag{151}$$

Hence the cross-section becomes

$$\sigma = \frac{w\,(mc^2)^2}{c^2\,|p_1 E_2 - p_2 E_1|} = |K|^2 \frac{(mc^2)^4}{c^2\,|E_2 p_{13} - E_1 p_{23}|\,|E_2' p_{13}' - E_1' p_{23}'|} \frac{1}{(2\pi\hbar)^2} dp_{11}'\, dp_{12}' \tag{152}$$

It is worth noting that the factor $p_1 E_2 - p_2 E_1$ is invariant under Lorentz transformations leaving the x_1 and x_2 components unchanged (e.g.boosts parallel to the x_3 axis.)[15] To prove this, we have to show that $p_{13}E_2 - p_{23}E_1 = \tilde{p}_{13}\tilde{E}_2 - \tilde{p}_{23}\tilde{E}_1$ (where $\tilde{}$ denotes the quantities after the Lorentz transformation) because we have chosen a Lorentz system in which the direction of the momentum vector is the x_3 axis. Then

$$\tilde{E} = E\cosh\theta - cp\sinh\theta$$
$$\tilde{p} = p\cosh\theta - \frac{E}{c}\sinh\theta$$

Since $E^2 = p^2c^2 + m^2c^4$, we can write

$$E = mc^2\cosh\phi \qquad pc = mc^2\sinh\phi, \qquad \text{which makes}$$
$$\tilde{E} = mc^2\cosh(\phi-\theta) \qquad \tilde{p}c = mc^2\sinh(\phi-\theta) \qquad \text{and thus}$$

$$\begin{aligned}\tilde{E}_2\tilde{p}_{13} - \tilde{E}_1\tilde{p}_{23} &= m^2c^3\left\{\cosh(\phi_2-\theta)\sinh(\phi_1-\theta) - \cosh(\phi_1-\theta)\sinh(\phi_2-\theta)\right\}\\ &= m^2c^3\sinh(\phi_1-\phi_2)\end{aligned}$$

independently of θ. Hence we see that σ is invariant under Lorentz transformations parallel to the x_3 axis.

Results for Møller Scattering

One electron initially at rest, the other initially with energy $E = \gamma mc^2$;

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

scattering angle $= \theta$ in the lab system

$= \theta^*$ in the center-of-mass system

Then the differential cross-section is (Mott and Massey, *Theory of Atomic Collisions*, 2nd ed., p. 368)

$$2\pi\sigma(\theta)\,d\theta = 4\pi\left(\frac{e^2}{mv^2}\right)^2\left(\frac{\gamma+1}{\gamma^2}\right)dx\left\{\frac{4}{(1-x^2)^2} - \frac{3}{1-x^2} + \left(\frac{\gamma-1}{2\gamma}\right)^2\left(1+\frac{4}{1-x^2}\right)\right\} \tag{153}$$

with

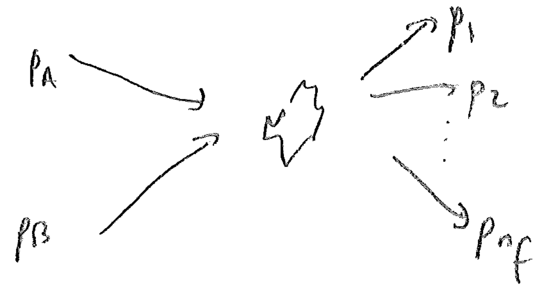
$$x = \cos\theta^* = \frac{2-(\gamma+3)\sin^2\theta}{2+(\gamma-1)\sin^2\theta}$$

Without spin you get simply

$$4\pi\left(\frac{e^2}{mv^2}\right)^2\left(\frac{\gamma+1}{\gamma^2}\right)dx\left\{\frac{4}{(1-x^2)^2} - \frac{3}{1-x^2}\right\}$$

Effect of spin is a measurable *increase* of scattering over the Mott formula. Effect of exchange is roughly the $\frac{3}{1-x^2}$ term. Positron-electron scattering is very similar. Only the exchange effect is different because of annihilation possibility.

$\boxed{2 \to n_f \text{ scattering process}}$



4-mom conserved

$$p_A^\mu + p_B^\mu = \sum_{j=1}^{n_f} p_j^\mu$$

~~From QFT, one can derive the formula for the cross section~~

The cross section for the process is given in QFT by

$$\sigma = \frac{\text{Scattering rate}}{\text{Incident flux}} = \frac{\hbar^2}{(2E_A)(2E_B)|v_A - v_B|} \int (LIPS)_{n_f} \, |A|^2$$

Depends on

~~AAA~~ Lorentz invariant amplitude A

~~(LIPS)~~ The Lorentz invariant phase space measure

$$(LIPS)_{n_f} = \int \left[\prod_{j=1}^{n_f} \frac{d^3 p_j}{(2\pi)^3 (2E_j)} \right] (2\pi)^4 \underbrace{\delta^{(4)}(\Delta p^\mu)}_{\text{energy-momentum conserving } \delta\text{-fcn}}$$

$\delta^{(4)}$ enforces

$$\Delta p^\mu = \sum_{j=1}^{n_f} p_j^\mu - (p_A^\mu + p_B^\mu) = 0$$

$E_A E_B |v_A - v_B|$ is invariant under boosts along incident direction

$\Rightarrow \sigma$ is Lorentz invariant

Initial state in the CM

Oct 6

CM (center of mass) frame = zero momentum

~~Initial state particles:~~

$p_A = p_i$ $\qquad$ $p_B = -p_i$

Consider

$$E_A E_B (v_A - v_B) = E_B (\underbrace{E_A v_A}_{p_A}) - E_A (\underbrace{E_B v_B}_{p_B})$$

$$= (E_B + E_A) p_i$$

$$= E_{cm}\, p_i$$

where E_{cm} = total energy in CM frame

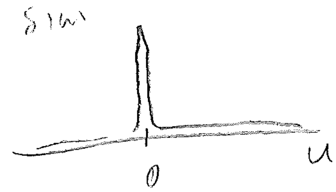
p_i = momentum of each pcl in CM frame

[NB $E_A E_B (v_A - v_B)$ is invariant under boosts in $\hat{z}$ direction, so is σ.]

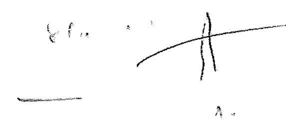
in FT frame $E_A E_B (v_A - v_B) = E_A v_A m_B = p_A m_B$

Dirac delta function

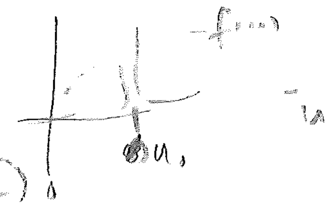
$$\delta(u) = \begin{cases} 0 & u \neq 0 \\ \infty & u = 0 \end{cases}$$



such that $$\int_{-\infty}^{\infty} du\ \delta(u) = 1$$



Also $$\int_{-\infty}^{\infty} du\ f(u)\,\delta(u-u_0) = f(u_0)$$ for arbitrary function f



$$\delta^{(4)}(\Delta p^\mu) = \delta(\Delta E)\,\delta(\Delta p_x)\,\delta(\Delta p_y)\,\delta(\Delta p_z)$$

Two-particle final state

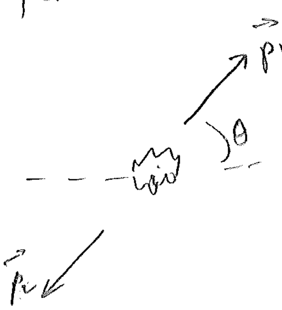
$$\int (LIPS)_2 = \int \frac{d^3p_1}{(2\pi)^3(2E_1)} \frac{d^3p_2}{(2\pi)^3(2E_2)} (2\pi)^4 \delta^{(4)}(\Delta p^\mu)$$

$$\Delta p^\mu = p_1^\mu + p_2^\mu - (p_A^\mu + p_B^\mu)$$

$$\Delta p^0 = E_1 + E_2 - \underbrace{(E_A + E_B)}_{E_{cm}}$$

$$\Delta \vec{p} = \vec{p}_1 + \vec{p}_2 - \vec{P}_{cm}$$

In cm frame, $\vec{P}_{cm} = 0$



$|\vec{p}_1| = |\vec{p}_2| = p_f$

$$\int (LIPS)_2 = \frac{1}{(2\pi)^2} \int \frac{d^3p_1}{(2E_1)(2E_2)} \underbrace{\int d^3p_2 \; \delta^{(3)}(\vec{p}_1 + \vec{p}_2)}_{1,\ \text{enforcing } \vec{p}_2 = -\vec{p}_1} \delta(E_1 + E_2 - E_{cm})$$

$$d^3p_1 = p_f^2 \, dp_f \underbrace{\sin\theta \, d\theta \, d\phi}_{d\Omega}$$

θ = angle $\vec{p}_1$ makes wrt. incident

$$\int (LIPS)_2 = \frac{1}{(2\pi)^2} \int d\Omega \frac{p_f^2 \, dp_f}{(2E_1)(2E_2)} \delta(E_1 + E_2 - E_{cm})$$

Observe that p_f is fixed by the energy δ-funct.

$$0 = E_1 + E_2 - E_{cm} = \sqrt{p_f^2 + m_1^2} + \sqrt{p_f^2 + m_2^2} - E_{cm}$$

[HW: solve for p_f]

Define $u = E_1 + E_2 - E_{cm}$

Consider $$\frac{du}{dp_f} = \frac{p_f}{\sqrt{p_f^2+m_1^2}} + \frac{p_f}{\sqrt{p_f^2+m_2^2}} = \frac{p_f}{E_1} + \frac{p_f}{E_2}$$

$$= \frac{(E_1+E_2)\, p_f}{E_1 E_2} = \frac{E_{cm}\, p_f}{E_1 E_2}$$

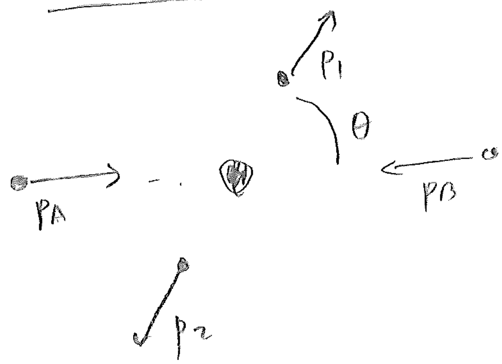
$$\Rightarrow dp_f = \frac{E_1 E_2}{E_{cm}\, p_f}\, du$$

$$\int (LIPS)_2 = \frac{1}{(2\pi)^2} \int d\Omega\, \frac{p_f}{4 E_{cm}} \underbrace{\int du\, \delta(u)}_{1}$$

$$\boxed{\int (LIPS)_2 = \frac{p_f}{(4\pi)^2 E_{cm}} \int d\Omega}$$

Sep 10

$2 \to 2$ scattering in CM frame



$|p_A| = |p_B| = p_i$

$|p_1| = |p_2| = p_f$

$$\sigma = \frac{\hbar^2}{(2E_A)(2E_B)|v_A - v_B|} \int (LIPS)_2 \, |A|^2$$

$$= \frac{\hbar^2}{4 E_{cm} p_i} \, \frac{p_f}{(4\pi)^2 E_{cm}} \int d\Omega \, |A|^2 = \int \left(\frac{d\sigma}{d\Omega}\right) d\Omega$$

Differential cross-section in CM frame.

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{\hbar^2}{(8\pi E_{cm})^2} \, \frac{p_f}{p_i} \, |A|^2}$$

→ we'll use this later for
① Rutherford scattering
② $e^+e^- \to \mu^+\mu^-$
③ neutrino absorption

connect w/ nonrelativistic formula

If $|A|^2 = (2E_1)(2E_2)(2E_A)(2E_B)|\tilde{V}|^2$

then $$\frac{2E_1 \, 2E_2}{4E_{cm}} \cdot \frac{2E_A \, 2E_B}{4E_{cm}} = \frac{p_f}{v_{12}} \, \frac{p_i}{v_{AB}}$$

$$\Rightarrow \left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{\hbar^2}{(2\pi)^2} \, \frac{p_f^2}{v_{12} v_{AB}} \, |\tilde{V}|^2$$

QEV-4

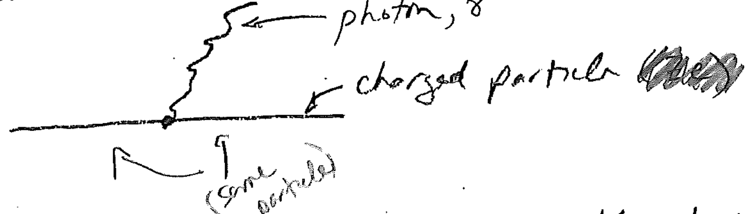
A = Amplitude = infinite sum of Feynman diagrams

- Not all contribute equally.
- If the strength of the interaction is weak, we can keep only diagrams w/ fewest # of vertices & neglect the rest.
- The more accuracy we need, the more diagrams we need to calculate (perturbation theory, Taylor series)
- Works well for EM & for the weak interaction (If the interaction is strong, the perturbative approach breaks down.)

Feynman diagrams constructed from <u>vertices</u> and <u>lines</u>

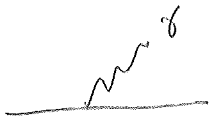
QFT for the EM field is called QED (quantum electrodynamics)
[cf popular book by Feynman]

The QED vertex is

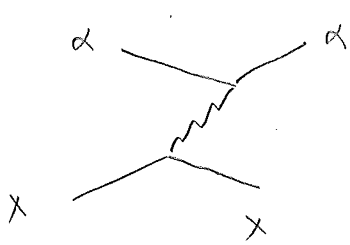


Vertex involves the <u>creation</u> of a new particle, the photon

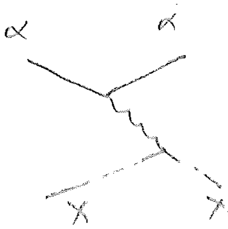
[classically, particles are not created or destroyed.
accel. a charge generates an EM wave
but Einstein's idea means photons are created



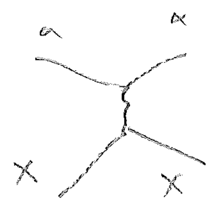
Rutherford scattering $\quad \alpha X \to \alpha X$



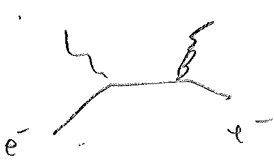
same as



just write



Compton scattering $\quad \gamma e^- \to \gamma e^-$



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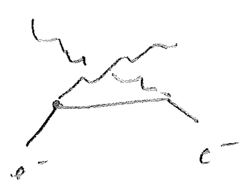
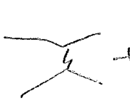
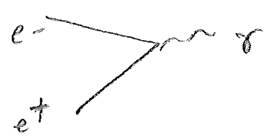
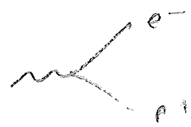


[not the same]

[Møller $e^- e^- \to e^- e^-$:  + ]

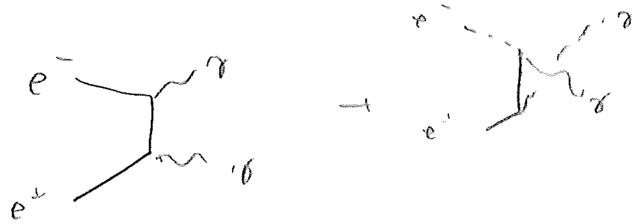
other vertex



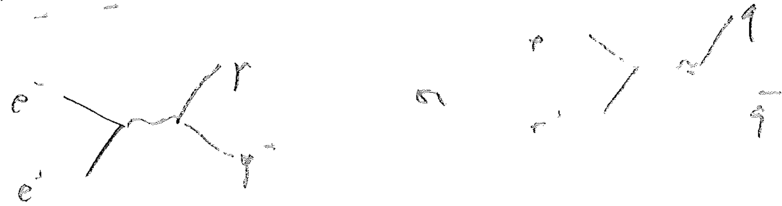


[not possible in isolation]

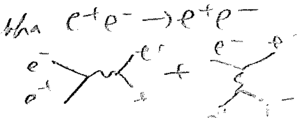
Pair annihilation into photons $e^+e^- \to \gamma\gamma$



Pair annihilation into other pairs



[Bhabha $e^+e^- \to e^+e^-$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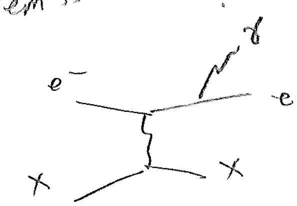
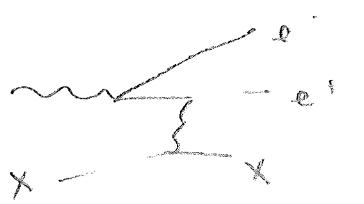
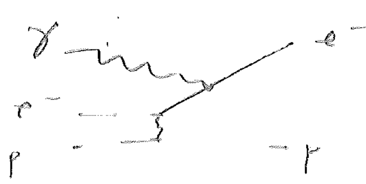
Bremsstrahlung $eX \to eX\gamma$



Pair production $\gamma X \to X e^+ e^-$



Photoelectric effect $\gamma\,(\text{atom}) \to (\text{ion})^+ e^-$



Recall dimensionless fine structure constant

$$\alpha = \frac{ke^2}{\hbar c} = \frac{e^2}{4\pi\epsilon_0 \hbar c} \simeq \frac{1}{137} \ll 1$$

↑ so perturbative expansion is useful

Particle physicists use "rationalized" units (Heaviside-Lorentz)

$c = 1, \quad \hbar = 1, \quad \epsilon_0 = 1$

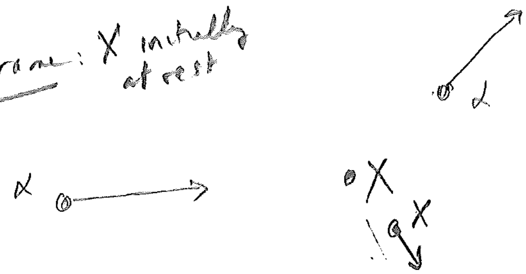
$$\Rightarrow \alpha = \frac{e^2}{4\pi} \quad \text{ie} \quad e = \sqrt{4\pi\alpha}$$

Each QED vertex  ↑ charge Ze

contributes a factor Ze to the amplitude

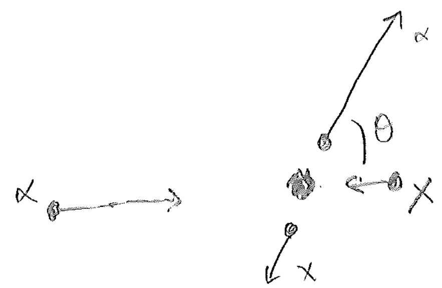
Rutherford scattering $\alpha X \to \alpha X$

Lab frame: X initially at rest



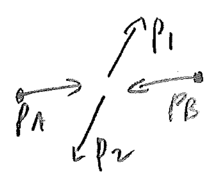
{classically we had to specify incident momentum as well as impact parameter. QM formula averages over all impact parameters so just specify momenta}

cm frame (nearly same as lab frame for $m_X \gg m_\alpha$)



If X is infinitely massive then lab $\cong$ cm frame

Kinematics in cm frame



$|p_A| = |p_B| = p_i$

$|p_1| = |p_2| = p_f$

energy conservation $E_A + E_B = E_1 + E_2$

$$\sqrt{m_\alpha^2 + p_i^2} + \sqrt{m_X^2 + p_i^2} = \sqrt{m_\alpha^2 + p_f^2} + \sqrt{m_X^2 + p_f^2}$$

Because masses don't change $\Rightarrow p_f = p_i \Rightarrow$ elastic collision (T is conserved)

All momenta are equal in magnitude.

$E_A = E_1 = \sqrt{m_\alpha^2 + p^2} = E_\alpha$

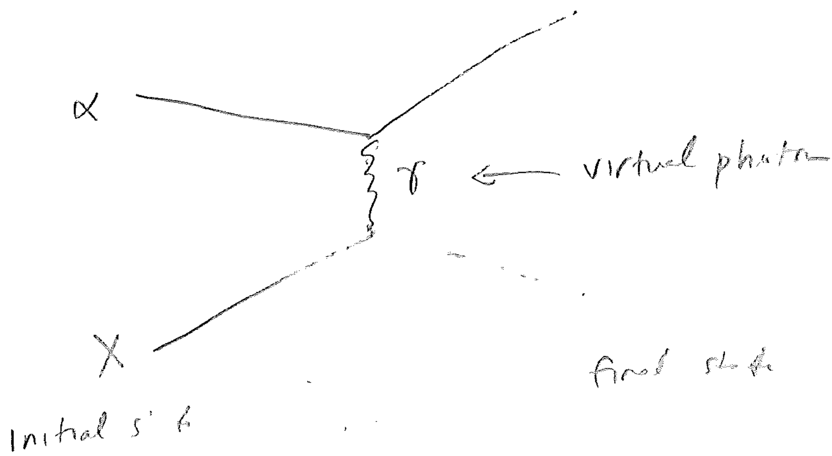
$E_B = E_2 = \sqrt{m_X^2 + p^2} = E_X$

$E_{cm} = E_X + E_\alpha$

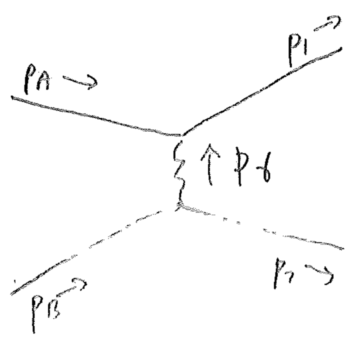
$p_A^\mu = (E_\alpha, 0, 0, p)$, $p_1^\mu = (E_\alpha, p\sin\theta, 0, p\cos\theta)$

$p_B^\mu = (E_X, 0, 0, -p)$, $p_2^\mu = (E_X, -p\sin\theta, 0, -p\cos\theta)$

The Feynman diagram that contributes most to Rutherford scattering is



Associate 4-momentum w/ each of the lines



Four-moment is conserved at each vertex

$$p_A^\mu + p_\gamma^\mu = p_1^\mu$$

$$\Rightarrow p_\gamma^\mu = p_1^\mu - p_A^\mu : \ (0,\ p\sin\theta,\ 0,\ p(\cos\theta - 1))$$

Virtual photon has 3-momentum but no energy (in cm frame) because collision is elastic (ie $E_A = E_1$)

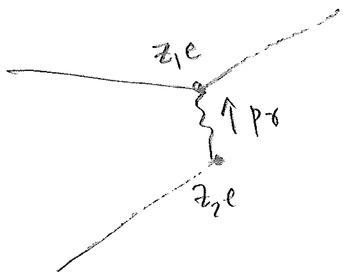
(space-like momentum)

For virtual photon, $E_\gamma \neq |\vec{p}_\gamma|$ $\Rightarrow$ we say it is "off shell" ie $p_\gamma^2 \neq 0$

(real photon has light-like momentum)

Each internal photon line contributes a factor $\frac{1}{p_\gamma^2}$ to the amplitude

(called the propagator)



Dirac gamma matrices γ^μ

$$A = \frac{(Z_1 e)(Z_2 e)}{p_\gamma^2} \cdot \begin{pmatrix}\text{spin}\\ \text{stuff}\end{pmatrix}$$

Note: if photon were physical (on-shell), $p_\gamma^2 = 0$

so $A \to \infty$, but it is virtual (off-shell)

Recall $p_\gamma^\mu = (0,\ p\sin\theta,\ 0,\ p(\cos\theta - 1))$

$$p_\gamma^2 = -(p\sin\theta)^2 - p^2(\cos\theta - 1)^2$$

$\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$

$$= 2p^2(\cos\theta - 1)$$

$$= -4p^2\sin^2\left(\frac{\theta}{2}\right)$$

($p_\gamma^2 < 0 \Rightarrow$ spacelike)

recall $e^2 = 4\pi\alpha$

$$A = \frac{Z_1 Z_2 (4\pi\alpha)}{-4p^2\sin^2\left(\frac{\theta}{2}\right)} \cdot \begin{pmatrix}\text{spin}\\ \text{stuff}\end{pmatrix}$$

Recall $\left(\frac{d\sigma}{d\Omega}\right)_{cm} \left(\frac{\hbar}{8\pi E_{cm}}\right)^2 \frac{p_f}{p_i} |A|^2$

Dimensional analysis: $\frac{d\sigma}{d\Omega} \sim (\text{length})^2$

$\hbar \sim (\text{length})(\text{energy})$

$\Rightarrow |A| \sim$ dimensionless for $2\to 2$ scattering

Rutherford scattering

$$A = \left(\frac{4\pi\alpha Z_1 Z_2}{-4p^2 \sin^2\frac{\theta}{2}}\right) (\text{spin stuff})$$

so (spin stuff) has units of $(\text{energy})^2$

We now make the apparently arbitrary but approximately correct assumption that

$$(\text{spin stuff}) \simeq \sqrt{(2E_A)(2E_B)(2E_1)(2E_2)} = \underline{4E_X E_\alpha}$$

Then $\left(\frac{d\sigma}{d\Omega}\right)_{cm} \left[\frac{\hbar\alpha Z_1 Z_2}{2p^2\sin^2\frac{\theta}{2}}\left(\frac{E_X E_\alpha}{E_X+E_\alpha}\right)\right]^2$ since $p_f = p_i = p$, $E_{cm} = E_X + E_\alpha$

Consider nonrelativistic limit: $E_\alpha \simeq m_\alpha$, $E_X \simeq m_X$, $p = m_\alpha v_\alpha$ $\Rightarrow \frac{E_X E_\alpha}{E_X+E_\alpha} = \frac{m_X m_\alpha}{m_X+m_\alpha}$ = reduced mass μ

Further assumption $m_\alpha \ll m_X \Rightarrow$ reduced mass $\mu \simeq m_\alpha$

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \left[\frac{\hbar\alpha Z_1 Z_2}{2m_\alpha v_\alpha^2 \sin^2\frac{\theta}{2}}\right]^2 \to$$ exact agreement w/ classical differential cross-section in nonrelativistic limit

[$\exists$ corrections at relativistic speeds]

Using our crude approximation, we obtained

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \left[\frac{Z_1 Z_2 \hbar\alpha}{2p^2 \sin^2\frac{\theta}{2}}\left(\frac{E_1 E_2}{E_1+E_2}\right)\right]^2$$

If $m_2 \gg m_1$, we can treat it as fixed.

Then $\frac{E_1 E_2}{E_1+E_2} \longrightarrow E_1$

and the cm frame is just the fixed target frame, w/ $p = p_1$

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \left[\frac{Z_1 Z_2 \hbar\alpha\, E}{2(\sin^2\frac{\theta}{2})\, p^2}\right]^2 = \left[\frac{Z_1 Z_2 \hbar\alpha}{2 \cdot m v^2 \sin^2\frac{\theta}{2}}\right]^2 (1-v^2)$$

which is correct for a relativistic spin-0 incident particle (provided $E_1 \ll m_2$)

For a spin-$\frac{1}{2}$ incident particle w/ have the Mott formula (Bjorken & Drell, RQM, p. 106)

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{Z_1 Z_2 \hbar\alpha}{2 m v^2 \sin^2\frac{\theta}{2}}\right)^2 (1-v^2)\left(1 - v^2 \sin^2\frac{\theta}{2}\right)$$

Coleman, 647n omits this.

(Same as above as $\theta \to 0$)

High energy limits

our crude result

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \left[\frac{z_1 z_2 \hbar \alpha}{2p^2 \sin^2\frac{\theta}{2}}\left(\frac{E_1 E_2}{E_1+E_2}\right)\right]^2$$

In high energy limit, $E_1 \approx E_2 \approx p$ so

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} \longrightarrow \left[\frac{\hbar\alpha}{2E_{cm}\sin^2\frac{\theta}{2}}\right]^2 = \frac{\alpha^2}{4E_{cm}^2}\left[\frac{1}{\sin^4\frac{\theta}{2}}\right]$$

For $e^-\mu^- \to e^-\mu^-$ scatt at high energies

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{\alpha^2}{8E_{cm}^2}\left[\frac{1+\cos^4\frac{\theta}{2}}{\sin^4\frac{\theta}{2}}\right]$$

← I think of B&D, (7.84) omitting the last 2 terms

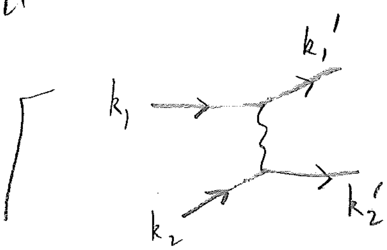
(so same as above as $\theta \to 0$)

For $e^-e^- \to e^-e^-$ (Moller) at high energy — B&D (7.84)

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\alpha^2}{8E_{cm}^2}\left[\frac{1+\cos^4\frac{\theta}{2}}{\sin^4\frac{\theta}{2}} + \frac{1+\sin^4\frac{\theta}{2}}{\cos^4\frac{\theta}{2}} + \frac{2}{\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}}\right]$$

(1-21-19) Rutherford scattering in cm frame

Relativistic vs nonrelativistic approach



$$Z_1 Z_2 e^2 \frac{(k_1+k_1')\cdot(k_2+k_2')}{(k_1-k_1')^2}$$

[e^2 here means $\frac{(4\pi K)e^2}{\hbar c} = 4\pi\alpha$]

Treat particles as charged scalars

$$k_1 + k_2 = k_1' + k_2'$$

$$\Rightarrow (k_1+k_1')\cdot(k_2+k_2') = (k_1+k_1')\cdot(k_1-k_1'+2k_2)$$

$$= \underbrace{k_1^2 - k_1'^2}_{0} + \underbrace{2\,k_1\cdot k_2}_{s-m_1^2-m_2^2} + \underbrace{2k_1'\cdot k_2}_{m_1^2+m_2^2-u} = s-u$$

$$s = (k_1+k_2)^2 = m_1^2 + m_2^2 + 2k_1\cdot k_2$$

$$u = (k_1'-k_2)^2 = m_1^2 + m_2^2 - 2k_1'\cdot k_2 = m_1^2 + m_2^2 - 2k_1\cdot k_2'$$

$$t = (k_1-k_1')^2 = 2m_1^2 - 2k_1\cdot k_1'$$

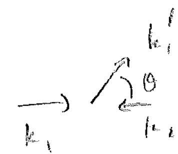
$$s+t+u = 4m_1^2 + 2m_2^2 + 2k_1\cdot(\underbrace{k_2 - k_2' - k_1'}_{-k_1}) = 2m_1^2 + 2m_2^2 \checkmark$$

$$\mathcal{M} = Z_1 Z_2 e^2 \frac{s-u}{t}$$

$k_1 \ (E_1, 0, 0, p_1)$

$k_2 = (E_2, 0, 0, -p_1)$

$k_1' = (E_1, p_1\sin\theta, 0, p_1\cos\theta)$



$$s = (E_1+E_2)^2$$

$$u = (E_1-E_2)^2 - p_1^2\sin^2\theta - p_1^2(\cos\theta+1)^2$$

$$= (E_1-E_2)^2 - p_1^2 - 2p_1^2\cos\theta - p_1^2$$

$$= (E_1-E_2)^2 - 2p_1^2(\cos\theta+1)$$

$$t = -p_1^2\sin^2\theta - p_1^2(\cos\theta-1)^2$$

$$= -2p_1^2 + 2p_1^2\cos\theta$$

$$= 2p_1^2(\cos\theta-1)$$

$$\frac{s-u}{t} = \frac{(E_1+E_2)^2-(E_1-E_2)^2+2p_1^2(\cos\theta+1)}{2p_1^2(\cos\theta-1)}$$

$$= \frac{4E_1E_2 + 2p_1^2(\cos\theta+1)}{2p^2(\cos\theta-1)}$$

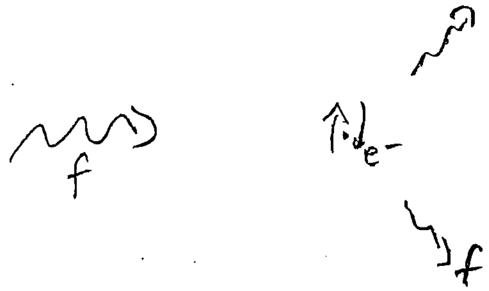
cancels the $\frac{1}{2E_1}\frac{1}{2E_2}$ normalization factors!

~~Physicists were skeptical of the photon concept until Arthur Compton did his expts on X-rays (1922)~~

~~X-rays were known to be EM waves~~

Scattering of X-ray by electrons (Thomson or Compton scattering)

Classical picture: electromagnetic waves of frequency f cause electrons to oscillate w/ frequency f.
Those accelerating electrons emit EM wave of frequency f.



Classical cross-section $\sigma = \frac{R}{F}$
R = rate at which energy is radiated by electron
F = flux of incident EM waves

From 1140, $F = \epsilon_0 c |\vec{E}|^2$
From 3120, Larmor formula $R = \frac{e^2 a^2}{6\pi \epsilon_0 c^3}$

$$\vec{a} = \frac{\vec{F}}{m_e} = \frac{e\vec{E}}{m_e} \Rightarrow R = \frac{e^4 |\vec{E}|^2}{6\pi\epsilon_0 m_e^2 c^3} \Rightarrow \sigma =$$

$$\Rightarrow \sigma = \frac{e^4}{6\pi\epsilon_0^2 (m_e c^2)^2} = \frac{8\pi}{3}\left(\frac{ke^2}{m_e c^2}\right)^2 = \frac{8\pi}{3} r_0^2$$

where $r_0 \equiv \frac{ke^2}{m_e c^2}$ = classical electron radius = $\frac{1.44 \text{ MeV fm}}{0.511 \text{ MeV}} = 2.8$ fm

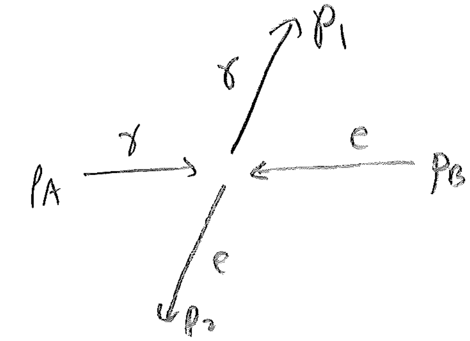
$\sigma = 66.5 \text{ fm}^2 = \frac{2}{3}$ barn = Thomson cross-section

[Thomson used it to measure # electrons in atoms]
Nucleus too massive to radiate much.

(1-26-23)

Compton scattering in cm frame

$e^-\gamma \to e^-\gamma$



Kinematics same as Rutherford $\Rightarrow$ all momenta equal

Also $E_\gamma = p$, $E_e = \sqrt{p^2+m_e^2} = E$

$p_A = (p, 0, 0, p)$ $\quad$ $p_1 = (p, p\sin\theta, 0, p\cos\theta)$

$p_B = (E, 0, 0, -p)$ $\quad$ $p_2 = (E_e, -p\sin\theta, 0, -p\cos\theta)$

$p_A + p_B = (E+p, 0, 0, 0)$

$s = (p_A+p_B)^2 = (E+p)^2 = m^2 + 2p(E+p)$

$p_B - p_1 = (E-p, -p\sin\theta, 0, -p(1+\cos\theta))$

$u = (p_B-p_1)^2 = (E-p)^2 - p^2 s^2 - p^2(1+2c+c^2) =$

$= E^2 - 2Ep + p^2 - p^2 - p^2 - 2p^2 c$

$= E^2 - 2Ep - p^2 - 2p^2\cos\theta = m^2 - 2p(E+p\cos\theta)$

$p_1 - p_A = (0, p\sin\theta, 0, p(\cos\theta - 1))$

$t = (p_1-p_A)^2 = -p^2 s^2 - p^2(1-2c+c^2) = -2p^2(1-\cos\theta)$

$s+t+u = E^2 + \cancel{2Ep} + \cancel{p^2} + E^2 - \cancel{2Ep} - \cancel{p^2} - 2\cancel{p^2c} - 2p^2 + \cancel{2p^2c} = 2(E^2-p^2) = 2m^2$ ✓

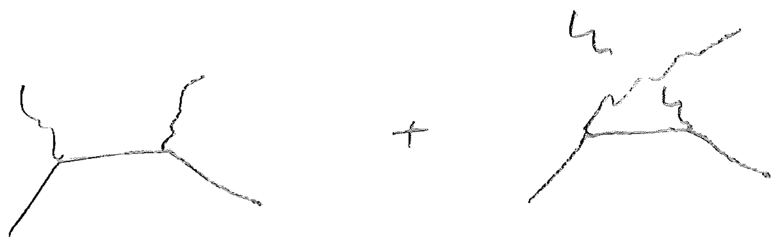
$s - m^2 = 2p(E+p)$

$u - m^2 = -2p(E+p\cos\theta)$

$t = -2p^2(1-\cos\theta)$

Compton scattering discussed by most authors.

- I'll use results from Peskin + Schroeder (p. 161–2)
- Mandl + Shaw is also good (p. 144 – 5)
- Srednicki also has results for pair annihilation (p. 359–60) that can be converted to Compton using prob 59.1



$$A = e^2 \left[\frac{N_s}{s-m^2} + \frac{N_u}{u-m^2} \right]$$

where N_s, N_u depend on spins + polarizations

$$|A|^2 = e^4 \left[\frac{|N_s|^2}{(s-m^2)^2} + \frac{(N_s^* N_u + N_u^\dagger N_s)}{(s-m^2)(u-m^2)} + \frac{|N_u|^2}{(u-m^2)^2} \right]$$

Now imagine summing over the spins + polarizations

eq (5.81) $$= \frac{e^4}{4} \left[\frac{\mathrm{I}}{(s-m^2)^2} - \frac{(\mathrm{II} + \mathrm{III})}{(s-m^2)(u-m^2)} + \frac{(\mathrm{IV})}{(u-m^2)^2} \right]$$

eq.(5.84) see errata

$$\mathrm{I} = 16 \left[2m^4 + m^2(s-m^2) - \tfrac{1}{2}(s-m^2)(u-m^2) \right]$$

$$\mathrm{IV} = 16 \left[2m^4 + m^2(u-m^2) - \tfrac{1}{2}(s-m^2)(u-m^2) \right]$$

$$\mathrm{II} + \mathrm{III} = -16 \left[4m^4 + m^2(s-m^2) + m^2(u-m^2) \right]$$

Thus

$$\begin{array}{cccc} & \mathrm{I} & \mathrm{II}+\mathrm{III} & \mathrm{IV} \\ \langle |A|^2 \rangle = 4e^4 \Big[& \dfrac{2m^4}{(s-m^2)^2} + & \dfrac{4m^4}{(s-m^2)(u-m^2)} + & \dfrac{2m^4}{(u-m^2)^2} \Big] \end{array}$$

$$+ \frac{m^2}{(s-m^2)} + \frac{m^2}{(s-m^2)} + \frac{m^2}{(u-m^2)} + \frac{m^2}{(u-m^2)}$$

$$- \frac{\frac{1}{2}(u-m^2)}{(s-m^2)} \qquad - \frac{\frac{1}{2}(s-m^2)}{(u-m^2)} \Big]$$

$$= 4e^4 \Big[2m^4 \left(\frac{1}{s-m^2} + \frac{1}{u-m^2} \right)^2$$

$$+ 2m^2 \left(\frac{1}{s-m^2} + \frac{1}{u-m^2} \right)$$

$$- \frac{1}{2}\left(\frac{u-m^2}{s-m^2} \right) - \frac{1}{2}\left(\frac{s-m^2}{u-m^2} \right) \Big]$$

which agrees w/ eq (5.87), using eqn (5.83), of Peskin & Schroeder

Let's take the $p \ll m$ limit (Thomson) of $\langle |A|^2 \rangle$

$\Rightarrow E \to m$

since $s - m^2 = 2p(E+p)$
$u - m^2 = -2p(E + p\cos\theta)$

we have $\frac{u-m^2}{s-m^2} \to -1$

and
$$\frac{1}{s-m^2} + \frac{1}{u-m^2} = \frac{1}{2p(E+p)} - \frac{1}{2p(E+p\cos\theta)}$$
$$= \frac{(\cos\theta - 1)}{2(E+p)(E+p\cos\theta)}$$
$$\to \frac{\cos\theta - 1}{2m^2}$$

Then

$$\langle |A|^2 \rangle = e^4 \Big[\underbrace{8m^4 \left(\frac{\cos\theta - 1}{2m^2}\right)^2}_{2\cos^2\theta - 4\cos\theta + 2} + \underbrace{8m^2 \left(\frac{\cos\theta - 1}{2m^2}\right)}_{4\cos\theta - 4} + 4 \Big]$$

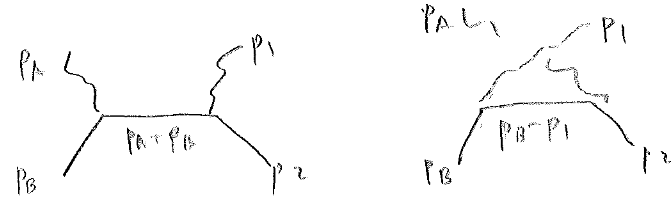
$$= 2e^4 [1 + \cos^2\theta]$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{\hbar^2}{(8\pi E_{cm})^2} \left(\frac{p_f}{p_i}\right) |A|^2 \xrightarrow[p \ll m]{} \frac{\hbar^2 (4\pi\alpha)^2}{(8\pi m)^2} 2(1+\cos^2\theta)$$

$$= \frac{\hbar^2 \alpha^2}{m^2} \frac{(1+\cos^2\theta)}{2} = \frac{\hbar^2\alpha^2}{m^2}\left(1 - \frac{1}{2}\sin^2\theta\right)$$

$$\sigma = \frac{\hbar^2\alpha^2}{m^2} \underbrace{\int_0^{2\pi} d\phi}_{2\pi} \underbrace{\int_{-1}^{1} d(\cos\theta) \left(\frac{1+\cos^2\theta}{2}\right)}_{\frac{4}{3}} = \frac{8\pi}{3} \cdot \frac{\hbar^2\alpha^2}{m^2}$$ Thomson cross section

Compton scattering is too complicated for our crude approach, whereby to make the amplitude dimensionless, we multiply by $\sqrt{(2E_A)(2E_B)(2E_1)(2E_2)} = (2E)(2p)$



$$A = e^2\left[\frac{(2E)(2p)}{s-m^2} + \frac{(2E)(2p)}{u-m^2}\right]$$

$$= e^2\left[\frac{(2E)(2p)}{(2p)(E+p)} - \frac{(2E)(2p)}{(2p)(E+p\cos\theta)}\right]$$

$$= e^2\left[\frac{2E}{E+p} - \frac{2E}{E+p\cos\theta}\right] \quad p\ll E \;\to\; O(p) \text{ not const!}$$

The actual computation shows that the numerators go (in the limit $p \ll E \sim m$) as m^2, not mp and that there is cancellation between the terms so that the leading contribution to the amplitude $\sim$ const.

In 2019, I used the crude approx above and also neglected the 2nd Feynman diag (although I was using a different convention) to get

$$A = e^2\,\frac{2E}{E+p} \to 2e^2 \quad \text{(isotropic!)}$$

+ then $\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi m)^2}|A|^2 = \frac{4(4\pi\alpha)^2}{(8\pi m)^2} = \frac{\alpha^2}{m^2}$

$\sigma = 4\pi\frac{\alpha^2}{m^2}$ but this is not really kosher

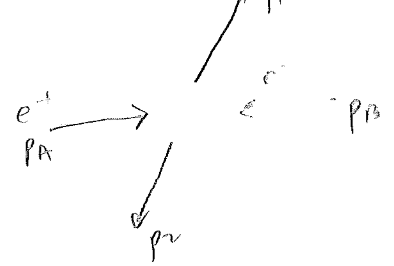
(1-27-23)

Pair production $e^+e^- \to \gamma\gamma$

In cm frame the kinematics are the same as $e^+e^- \to \mu^+\mu^-$ w/ all energies equal, and w/ $p_\gamma = E$, $p = p_e = \sqrt{E^2 - m^2}$

$p_A = (E, 0, 0, p)$ $\quad$ $p_1 = (E, E\sin\theta, 0, E\cos\theta)$

$p_B = (E, 0, 0, -p)$ $\quad$ $p_2 = (E, -E\sin\theta, 0, -E\cos\theta)$



$p_A + p_B = (2E, 0, 0, 0)$

$s = (p_A + p_B)^2 = 4E^2$

$p_1 - p_A = (0, E\sin\theta, 0, E\cos\theta - p)$

$t = (p_1 - p_A)^2 = -(E\sin\theta)^2 - (E\cos\theta - p)^2 = -E^2 + 2Ep\cos\theta - p^2$

$t - m^2 = -2E^2 + 2Ep\cos\theta = -2E(E - p\cos\theta)$

$p_2 - p_A = (0, -E\sin\theta, 0, -E\cos\theta - p)$

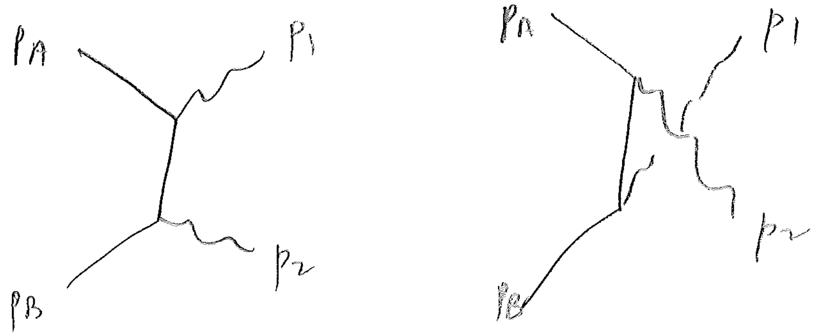
$u = (p_2 - p_A)^2 = -(E\sin\theta)^2 - (E\cos\theta + p)^2 = -E^2 - 2Ep\cos\theta - p^2$

$u - m^2 = -2E(E + p\cos\theta)$

$s + t + u = 2E^2 - 2p^2 = 2m^2$ ✓

In limit of e^+e^- initially at rest. $s = 4m^2$, $t - m^2 = -2m^2$, $u - m^2 = -2m^2$

$t = -m^2$, $u = -m^2$



$$A = e^2 \left[\frac{N_t}{t-m^2} + \frac{N_u}{u-m^2} \right]$$

Our <u>crude</u> approximation says $N_t = N_u = (2E)^2 = s$

$$A = e^2 \left[\frac{(2E)^2}{-2E(E-p\cos\theta)} + \frac{(2E)^2}{-2E(E+p\cos\theta)} \right]$$

$$= e^2 \left[\frac{-(2E)^2}{(E-p\cos\theta)(E+p\cos\theta)} \right]$$

That is:

$$e^2 \left[\frac{s}{t-m^2} + \frac{s}{u-m^2} \right] = e^2 \frac{-s^2}{(t-m^2)(u-m^2)}$$

<u>This is far from the mark see Srednicki p. 359-60 (next page)</u>

However assume $p \ll m$ i.e $\boxed{e^+e^- \text{ initially at rest}}$

then our <u>crude assumption</u> gives $A = e^2 \left[\frac{-(2m)^2}{m \cdot m} \right] = -4e^2$

which lo & behold! agrees w/ Griffiths (2e) ch 7, eq. 7.163.

(But this is just a <u>lucky fluke</u>. E.g. in Srednicki the cross term vanishes in this limit)

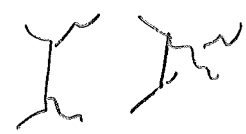
Then $\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi E_{cm})^2} \frac{p_f}{p_i} |A|^2 = \frac{1}{(8\pi(2m))^2} \frac{m}{(mv)} [4(4\pi\alpha)]^2 = \frac{\alpha^2}{m^2 v}$

$$\sigma = \frac{4\pi\alpha^2}{m^2 v} \qquad \text{(Griffiths 7.168)}$$

Srednicki
p. 359-60

$$\boxed{e^+e^- \to \gamma\gamma}$$

invt under $t \leftrightarrow u$
due to



$$\langle |A|^2 \rangle = e^4 \Big[\frac{2(tu - m^2[3t+u] - m^4)}{(t-m^2)^2}$$

eq (59.18)
eqs (59.22-25)

$$+ \frac{2(tu - m^2[3u+t] - m^4)}{(u-m^2)^2}$$

cross term

$$+ \frac{4m^2(s-4m^2)}{(t-m^2)(u-m^2)} \Big]$$

e^+e^- initially at rest

$\Downarrow$

$s = 4m^2$
$t = -m^2$
$u = -m^2$

note cross term vanishes!

$$\Rightarrow e^4 \Big[\frac{2(1+4-1)}{4} + \frac{2(1+4-1)}{4} + 0 \Big] = 4e^4$$

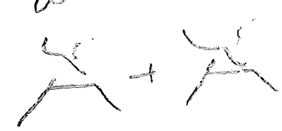
seems to be $\frac{1}{4}$ of Griffiths' result

$$e^-\gamma \to e^-\gamma \quad \text{(Compton)}$$

Srednicki
prob. 59.1

Same as above, but w/ $s \leftrightarrow t$ and overall minus

invt under $s \leftrightarrow u$
due to



$$|A|^2 = -e^4 \Big[\frac{2(su - m^2[3s+u] - m^4)}{(s-m^2)^2}$$

$$+ \frac{2(su - m^2[3u+s] - m^4)}{(u-m^2)^2}$$

$$+ \frac{4m^2(t-4m^2)}{(s-m^2)(u-m^2)} \Big]$$

$s \sim m^2$
$u \sim m^2$
$t \sim 0$

$$\Big\} \Rightarrow = + e^4 \Big[\frac{8m^4}{(s-m^2)^2} + \frac{8m^4}{(u-m^2)^2} + \frac{16m^4}{(s-m^2)(u-m^2)} + \ldots \Big]$$

mm / misc / compton.nb

Compton scattering

```
In[33]:= s = m^2 + 2 p (e + p);
In[34]:= u = m^2 - 2 p (e + p cos);
In[35]:= t = -2 p^2 (1 - cos);
In[36]:= e = Sqrt[p^2 + m^2];
```

Srednicki, p. 359

```
In[37]:= num1 = - 2 (s u - m^2 (3 s + u) - m^4);
In[38]:= num2 = -2 (s u - m^2 (3 u + s) - m^4);
In[39]:= num3 = - 4 m^2 (t - 4 m^2);
In[40]:= T1 = Series[num1 / (s - m^2) ^2, {p, 0, 2}];
In[41]:= T2 = Series[num2 / (u - m^2) ^2, {p, 0, 2}];
In[42]:= T3 = Series[num3 / (s - m^2) / (u - m^2), {p, 0, 2}];
In[43]:= sred = Simplify[T1 + T2 + T3]
```

Out[43]= $$2\left(1+\cos^2\right)-\frac{4\left(\cos\left(-1+\cos^2\right)\right)p}{\sqrt{m^2}}+\frac{\left(4-4\cos-6\cos^2+6\cos^4\right)p^2}{m^2}+O[p]^3$$

Peskin and Schroeder

```
In[44]:= X1 = Series[ 8 m^4 ( 1 / (s - m^2) + 1 / (u - m^2) ) ^2, {p, 0, 2}];
In[45]:= X2 = Series[ 8 m^2 ( 1 / (s - m^2) + 1 / (u - m^2) ), {p, 0, 2}];
In[46]:= X3 = Series[-2 ( (u - m^2) / (s - m^2) + (s - m^2) / (u - m^2)) , {p, 0, 2}];
In[47]:= ps = Simplify[X1 + X2 + X3]
```

Out[47]= $$2\left(1+\cos^2\right)-\frac{4\left(\cos\left(-1+\cos^2\right)\right)p}{\sqrt{m^2}}+\frac{\left(4-4\cos-6\cos^2+6\cos^4\right)p^2}{m^2}+O[p]^3$$

```
In[48]:= sred - ps
```

Out[48]= $O[p]^3$

old calc
based on Sakurai
notes ...

See 315 notes

QM Compton scattering : non rel limit ($\hbar\omega \ll mc^2$)

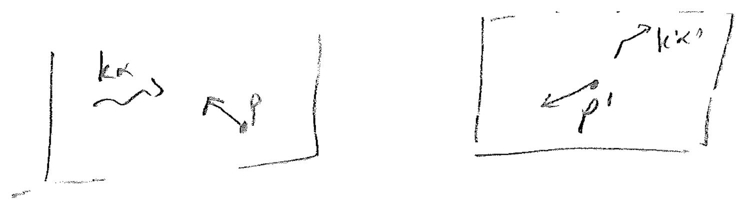
Thomson

Time dep. pert theory $\Rightarrow$ Fermi golden rule

(set $\hbar = c = 1$)

$\omega' = \omega$

$$dR = \text{transition rate} = 2\pi\, |\mathcal{M}|^2\, \delta(E_f - E_i)$$



$$\mathcal{M} \quad \langle p', k'\alpha' |\, H_{int}\, | p, k\alpha \rangle$$

$$H_{int} = -\frac{e}{m}\vec{A}\cdot\vec{p} + \frac{e^2}{2m}\vec{A}^2$$

$$\vec{A} = \frac{1}{\sqrt{2V}} \sum_{k\alpha} \frac{1}{\sqrt{\omega}}\left[a_{k\alpha}\vec{\epsilon}^{\,\alpha} e^{i\vec{k}\cdot\vec{x}} + a^\dagger_{k\alpha}\vec{\epsilon}^{\,\alpha} e^{-i\vec{k}\cdot\vec{x}}\right]$$

suppressed in $\hbar\omega \ll mc^2$ limit

$$A^2 \longrightarrow \langle k' |\, (a + a^\dagger)(a + a^\dagger)\, | k \rangle$$

$$= \langle k' |\, a_k a^\dagger_{k'} + a^\dagger_{k'} a_k\, | k \rangle$$

$$= 2$$

$$\mathcal{M} = \langle p' |\, \frac{e^2}{2m}\, \frac{\vec{\epsilon}\cdot\vec{\epsilon}'}{2V\omega}\, e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}\, | p \rangle$$

Now we'll set $\hbar = 1$

$$|M|^2 = \left(\frac{e^2}{2mV^2}\right)^2 \frac{(\vec{\epsilon}\cdot\vec{\epsilon}\,')^2}{\omega\omega'} \int d^3x \; e^{i(k+p-k'-p')\cdot x} \int d^3x \; e^{i(k+p-k'-p')\cdot x}$$

In a box, the moments are discrete, so the integral gives $V\,\delta_{k+p-k'-p'}$

but we can use the continuum approximation in which case

$\int d^3x \; e^{i(k+p-k'-p')\cdot x} = (2\pi)^3\,\delta(k+p-k'-p')$.

Then the exponent in 2nd integral vanishes & $\int d^3x = V$

$$|M|^2 = \left(\frac{e^2\hbar\;\vec{\epsilon}\cdot\vec{\epsilon}\,'}{2m}\right)^2 \frac{1}{\omega\omega' V^4} \cdot (2\pi)^3\,\delta(k+p-k'-p')\;V$$

$$\text{Then } R = \sum_{\substack{\text{final}\\\text{state}}} dR = \underbrace{\frac{V}{(2\pi)^3}\int d^3p'}_{\text{this kills } (2\pi)^3\,\delta(\;)\,V} \;\frac{V}{(2\pi)^3}\int d^3k'$$

$$= \left(\frac{e^2\hbar^2\,\vec{\epsilon}\cdot\vec{\epsilon}\,'}{2m}\right)^2 \frac{1}{V^4\omega\omega'}\,\frac{V}{(2\pi)^3}\int d^3k'$$

We could have gotten to here by ignoring the phase space factor of the final electron &

At this pt we may proceed by ignoring the spatial dependence of the electrons (ie treat them as infinitely massive) + summing over the phase space of the final photon only (ie using momentum conservation to eliminate the final elect phase space)

Then

$$R = \int dR \cdot \frac{V}{(2\pi\hbar)^3}\int d^3k' \ \frac{2\pi}{\hbar}\left(\frac{e^2\hbar}{m\,2V}\right)^2 \frac{(\vec{\epsilon}\cdot\vec{\epsilon}\,')^2}{\omega\omega'}$$

My 3/5 note

Fermi golden rule

$$d\Gamma = \frac{2\pi}{\hbar} |\mathcal{M}|^2 \, \delta(E_f - E_i + \hbar\omega' - \hbar\omega)$$

$$\text{Rate} = \underset{\substack{\uparrow \\ \text{sum over final photon states}}}{\sum} d\Gamma = \frac{2\pi}{\hbar} \int \frac{V\, d^3k'}{(2\pi)^3} |\mathcal{M}|^2 \, \delta(E_f - E_i + \hbar\omega' - \hbar\omega)$$

$$= \frac{V}{4\pi^2} \frac{(\omega')^2}{\hbar^2 c^3} \int d\Omega' \sum_{\alpha'} |\mathcal{M}|^2$$

$$\text{Flux} = \frac{c}{V}$$

$$\frac{d\sigma}{d\Omega} = \frac{\text{Rate}}{\text{Flux}} : \frac{V^2 (\omega')^2}{4\pi^2 \hbar^2 c^4} \sum_{\alpha'} |\mathcal{M}|^2$$

$$= \frac{(\omega')^2}{4\pi^2 \cancel{\hbar^2} c^4} \frac{e^4 \cancel{\hbar^2}}{4m^2 \omega\omega'} \sum_{\alpha} \big| [\quad] \big|^2$$

$$= \left(\frac{\omega'}{\omega}\right) \underbrace{\left(\frac{e^2}{4\pi mc^2}\right)^2}_{r_0^2} \sum_{\alpha} \big| [\quad] \big|^2 \qquad \text{Kramers-Heisenberg}$$

$$= \left(\frac{m r_0^2}{\hbar}\right)^2 \omega\omega'^3 \sum_{\alpha} \left| \sum_{I} \frac{(\epsilon \cdot x)_{BI} (\epsilon' \cdot x)_{IA}}{\omega_{AI} + \omega} + \frac{(\epsilon \cdot x)_{IB} (\epsilon' \cdot x)_{IA}}{\omega_{AI} - \omega'} \right|^2$$

If $\omega \ll \omega_{AI}$ (Rayleigh) then $\sum |\quad|^2 \sim$ indep of ω

$$\Rightarrow \left(\frac{m r_0^2}{\hbar}\right)^2 \omega^4 \quad \#$$

(Rayleigh)

$$|M|^2 = \left(\frac{e^2\hbar}{2mV^2}\right)^2 \frac{(\vec{\epsilon}\cdot\vec{\epsilon}')^2}{\omega\omega'} \int d^3x\, e^{i(k+p-k'-p')\cdot x} \int d^3x\, e^{i(k+p-k'-p')x}$$

in a box, all momenta are discrete

$= 0$

$$= \delta_{k+p-k'-p'} \cdot V \quad \cdot \quad V$$

$$R = \sum_{\text{final states}} dR = \left(\frac{V}{(2\pi\hbar)^3}\int d^3p'\right)\left(\frac{V}{(2\pi\hbar)^3}\int d^3k'\right)$$

$\langle B| \quad |A\rangle$

$(\sum k'$

Or

$$\left[(2\pi)^3\,\delta(\quad)\right]\cdot V$$

$$\delta \qquad \int \delta(k-k)\, dk \quad 2\pi$$

$$\sum \delta$$

$$p \cdot \frac{n}{L} \qquad\qquad \int_0^L dx\, e^{i(p-p')\cdot x}$$

$=$

$$\sigma = \frac{R}{F}$$

T. Flux of photons $= \frac{c}{V} \sim \frac{1}{V}$

$$R = \sum dR = \frac{V}{(2\pi)^3} \int d^3k' \, (dR)$$

$$\sigma = \frac{V^2}{(2\pi)^3} \int d^3k' \, (2\pi) \left(\frac{e^2}{2m} \frac{\vec{\epsilon} \cdot \vec{\epsilon}\,'}{2\omega} \right)^2 \frac{1}{V^2} \left| \langle p' | e^{i(\vec{k} - \vec{k}') \cdot \vec{x}} | p \rangle \right|^2$$

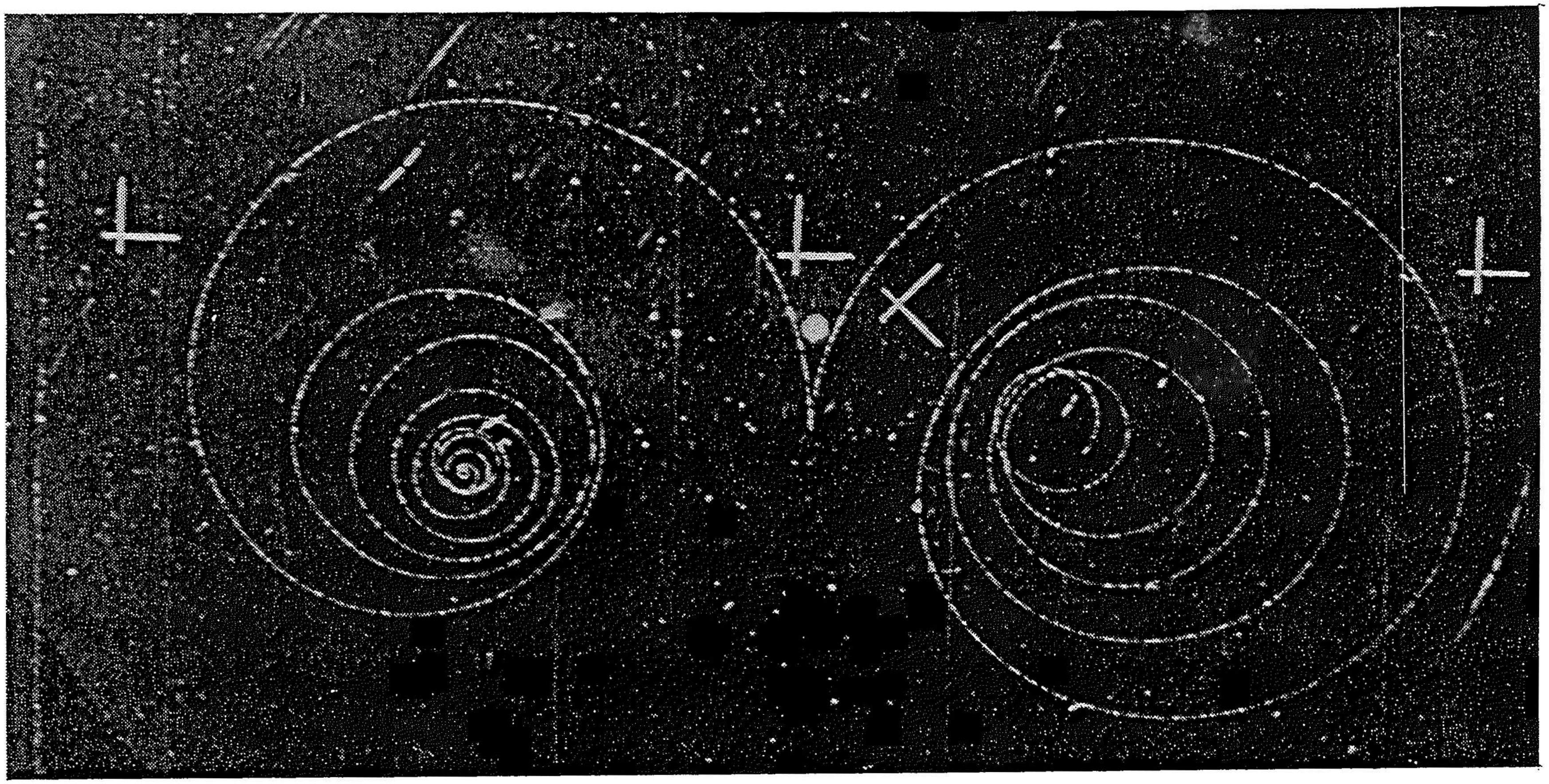
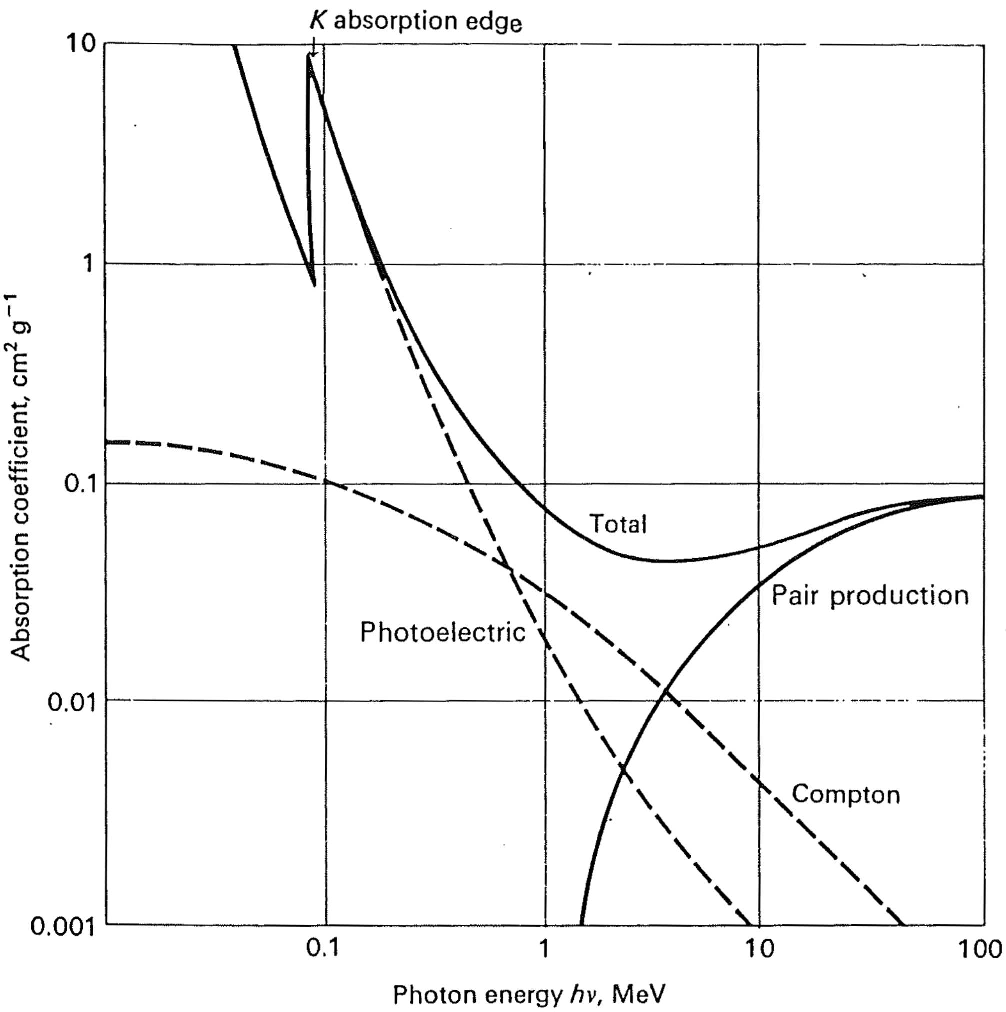


Fig. 6–7 Production of an electron-positron pair in a liquid hydrogen bubble chamber in a magnetic field.



The absorption coefficient per g cm^{-2} of lead for γ-rays as a function of energy.

Perkins

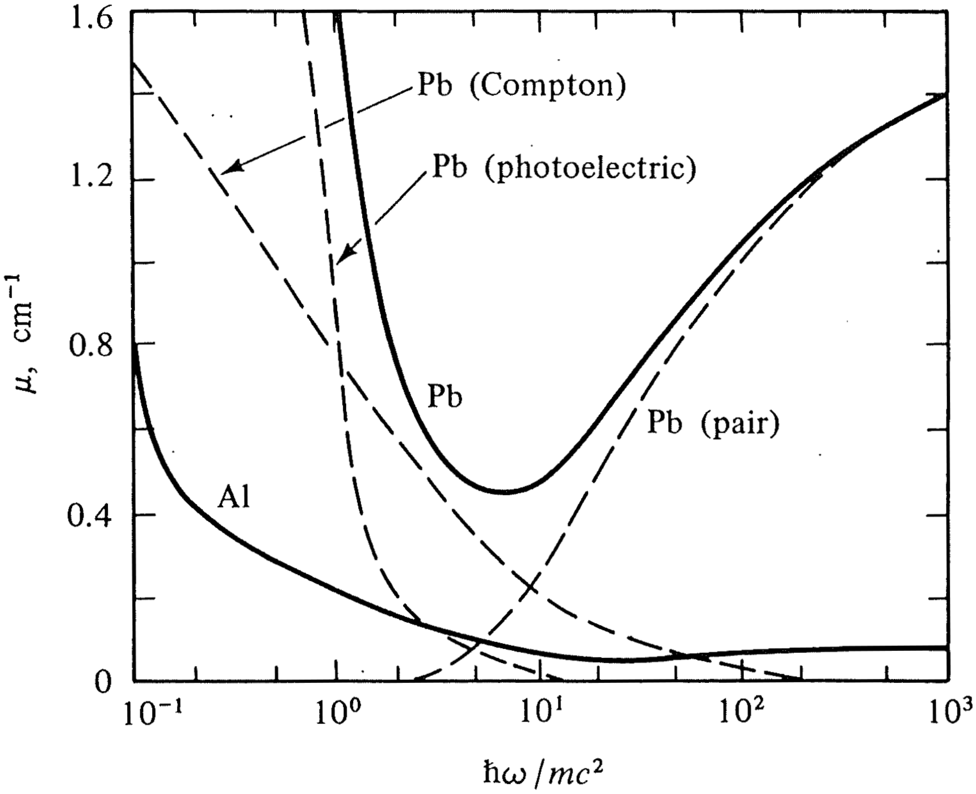
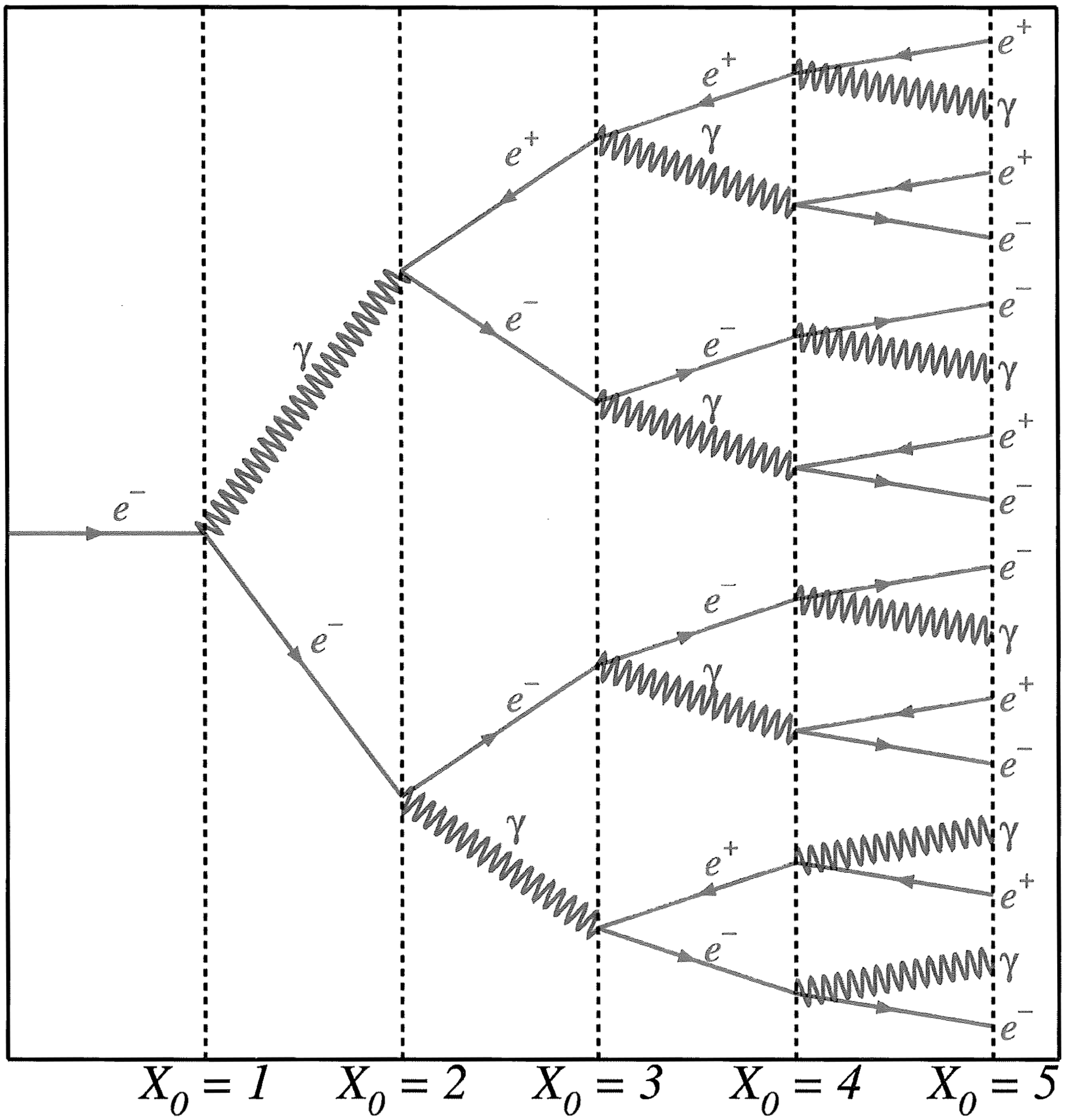


Fig. 3.7. Total absorption coefficients of γ rays by lead and aluminum as a function of energy (solid lines). Photoelectric absorption of aluminum is negligible at the energies considered here. Dashed lines show separately the contributions of photoelectric effect, Compton scattering, and pair production for Pb. Abscissa, logarithmic energy scale; $\hbar\omega/mc^2 = 1$ corresponds to 511 keV. (From W. Heitler, *The Quantum Theory of Radiation*, The Clarendon Press, Oxford, 1936, p. 216.)

Frauenfelder & Henley



e^-
γ
e^-
e^+
e^-
e^-
γ
e^+
γ
e^-
γ
e^-
γ
e^+
e^-
e^+
γ
e^+
e^-
e^-
γ
e^+
e^-
e^-
γ
e^+
e^-
γ
e^+
γ
e^-
$X_0 = 1$
$X_0 = 2$
$X_0 = 3$
$X_0 = 4$
$X_0 = 5$