Nuclear transmutation

Rutherford & collaborators used α particles emitted by Po & Ra
w/ typical $T = 5$ MeV to probe atomic structure.
For heavy targets (Z_2 large), Coulomb barrier prevents
α from getting too close to nucleus,
which therefore behaves as a point charge
w/ $\frac{d\sigma}{d\Omega} \sim \frac{1}{\sin^4 \frac{\theta}{2}}$.

Scattering from lighter nuclei begins to depart from this
suggesting a finite size for nucleus.

If α gets close enough (~ 1 fm), strong force
can cause some of its particles to be absorbed

Nuclear transmutation (Rutherford, 1919)

$$\alpha + {}^{14}_{7}N \rightarrow {}^{17}_{8}O + p$$

Cockcroft / Walton [designed a linear accelerator to]
accelerated protons to ~ 0.5 MeV, and
[with this, they] "split the atom" (1932)

$$p + {}^{7}Li \rightarrow \alpha + \alpha$$

Chadwick discovers the neutron (1932)

$$\alpha + {}^{9}_{4}\mathrm{Be} \rightarrow {}^{12}_{6}\mathrm{C} + n$$ [also Bothe & Becker 1930]

Artificially radioactive isotopes (Joliot & I. Curie, 1934)

$$\alpha + {}^{27}_{13}\mathrm{Al} \rightarrow {}^{30}_{15}\mathrm{P} + n$$

$$\downarrow 3 \text{ minutes}$$

$$ {}^{30}_{14}\mathrm{Si} + e^{+} + \nu_e$$

Neutrons as probes [no Coulomb barrier; but can't accelerate them either]

$$n + {}^{16}_{8}\mathrm{O} \longrightarrow {}^{17}_{8}\mathrm{O} \longrightarrow {}^{13}_{6}\mathrm{C} + \alpha$$ (Meitner & Philipp)

$$n + {}^{19}_{9}\mathrm{F} \rightarrow {}^{20}_{9}\mathrm{F} \longrightarrow {}^{16}_{7}\mathrm{N} + \alpha$$ (Fermi & Rasetti)

$$\downarrow$$

$$ {}^{16}_{8}\mathrm{O} + e^{-} + \bar{\nu}_e$$

Many artificially radioactive isotopes produced [nothing below fluorine]

even beyond uranium (transuranic)

$$n + {}^{238}_{92}\mathrm{U} \longrightarrow {}^{239}_{92}\mathrm{U} \longrightarrow {}^{239}_{93}\mathrm{Np} + e^{-} + \bar{\nu}_e$$

$$\downarrow$$

$$ {}^{239}_{94}\mathrm{Pu} + e^{-} + \bar{\nu}_e$$

[It was found that] for medium energy neutrons ($T \approx 20\,MeV$))

the absorption cross section for neutrons is

$\sigma \sim \pi R_X^2$

Suggests most neutrons are absorbed by an nucleus they strike

Recall $R_X \sim A^{1/3} r_0$ where $r_0 = 1.2\ fm$

$\sigma \sim A^{2/3} \pi r_0^2$ [$A \sim 180$; tungsten]

For heavy nuclei: $\pi A^{2/3} \sim 100$

$\sigma \sim (10\ fm)^2 \sim 1$ barn...

Nuclear physicists define

$1\ \text{barn} = (10\ fm)^2 = 100\ fm^2 = 10^{-28}\ m^2$

"broadside of a barn"

[uranium (A 238) $\Rightarrow$ $\sigma \sim (13\ fm)^2$ } (4 ...)
uranium ~ 1.7 barns

For higher energies, $\sigma \downarrow$ $\Rightarrow$ not all neutrons are absorbed

"partial translucency"

For slow neutrons, $\sigma \uparrow$

[cf graph from Evans]

R. D. Evans, p. 458

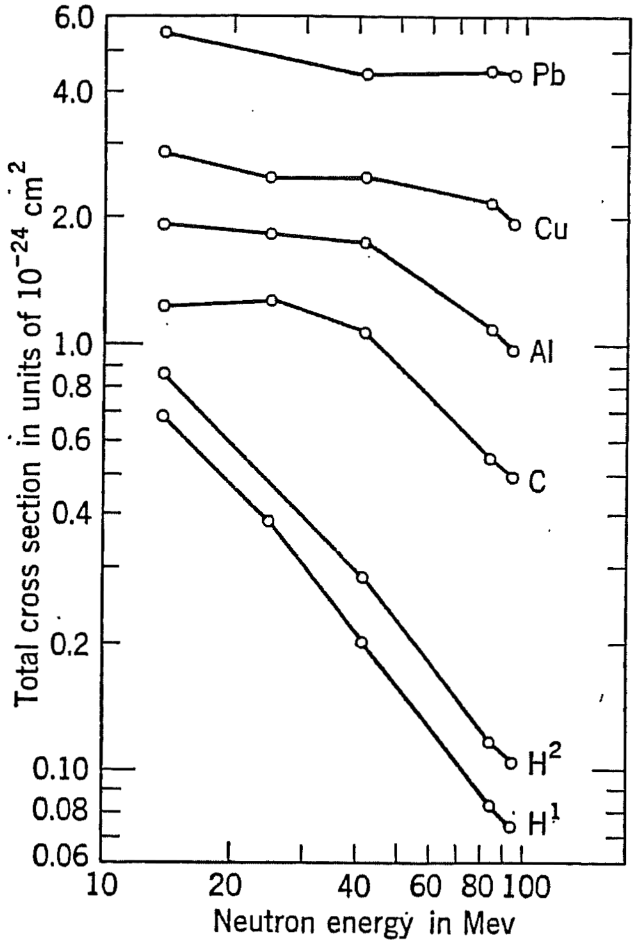
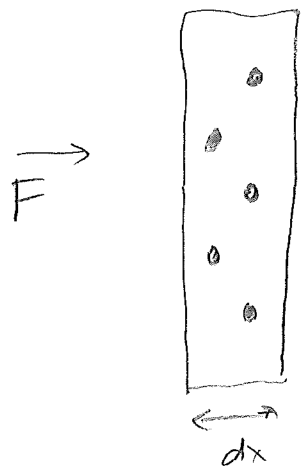


Fig. 2.3 Total neutron cross sections σ_t of five elements for neutrons of 14, 25, 42, 84, and 95 Mev. The decrease in σ_t at the higher neutron energies suggests that all neutrons which strike the target area $\pi(R + \lambda\!\!\!^-)^2$ are not captured and that nuclear matter is slightly transparent to swift neutrons. [*From Hildebrand and Leith* (H51).]

Let incident flux of neutrons be F



Probability of absorption by nuclei in film of thickness dx

$f = n\sigma(dx)$ $\quad$ n = # density of nuclei

$n(A\,dx)$ = # targets, area of targets: $n\sigma A\,dx$

$\Rightarrow$ Flux correspondingly diminishes

$$dF = -(n\sigma\, dx)F$$

$$\frac{dF}{F} = -n\sigma\, dx$$

$$\ln F = -n\sigma x + \ln F_0$$

$$F = F_0\, e^{-n\sigma x} = F_0\, e^{-\frac{x}{\ell}}$$

$$\boxed{\ell = \frac{1}{n\sigma} = \text{mean free path}} \text{ of neutrons}$$

← [HW]

$n = \frac{\rho}{A} N_A$ $\quad$ ρ = mass density

$U \rightarrow \rho = 19\ \frac{g}{cm^3} \Rightarrow n = 4.8E28\ m^{-3}$

$\sigma = 1.7E{-28}\ m^2$

$\ell \simeq 0.12\ m$

Neutrons, being neutral, cannot be accelerated or steered but they can be slowed down by collisions, or by absorption + re-emission

Fermi found that absorption $\sigma \uparrow$ for lower energy n

[see graph for nickel]

Cadmium ~ 2000 barns for slow neutrons

[empirically $\sigma \sim \frac{1}{v}$]

R. D. Evans, p. 455

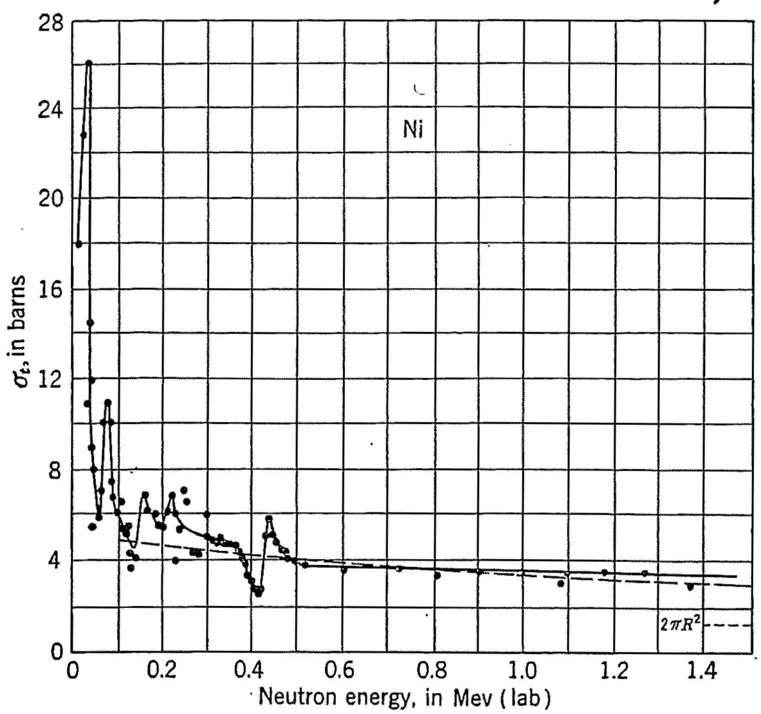


Fig. 2.1 The total cross section (absorption plus scattering) of nickel for neutrons from 0.01 to 1.5 Mev. Approximately monoenergetic neutrons (± 20 kev below 0.5 Mev; ± 150 kev above 0.5 Mev) were produced by bombarding a thin Li target with protons accelerated by an electrostatic generator. The dotted line is from the continuum theory of Feshbach and Weisskopf (F49), assuming a nuclear radius of 4.6×10^{-13} cm. [*The experimental data are from Barschall, Bockelman, and Seagondollar* (B15).]

R.D Evans, p. 443

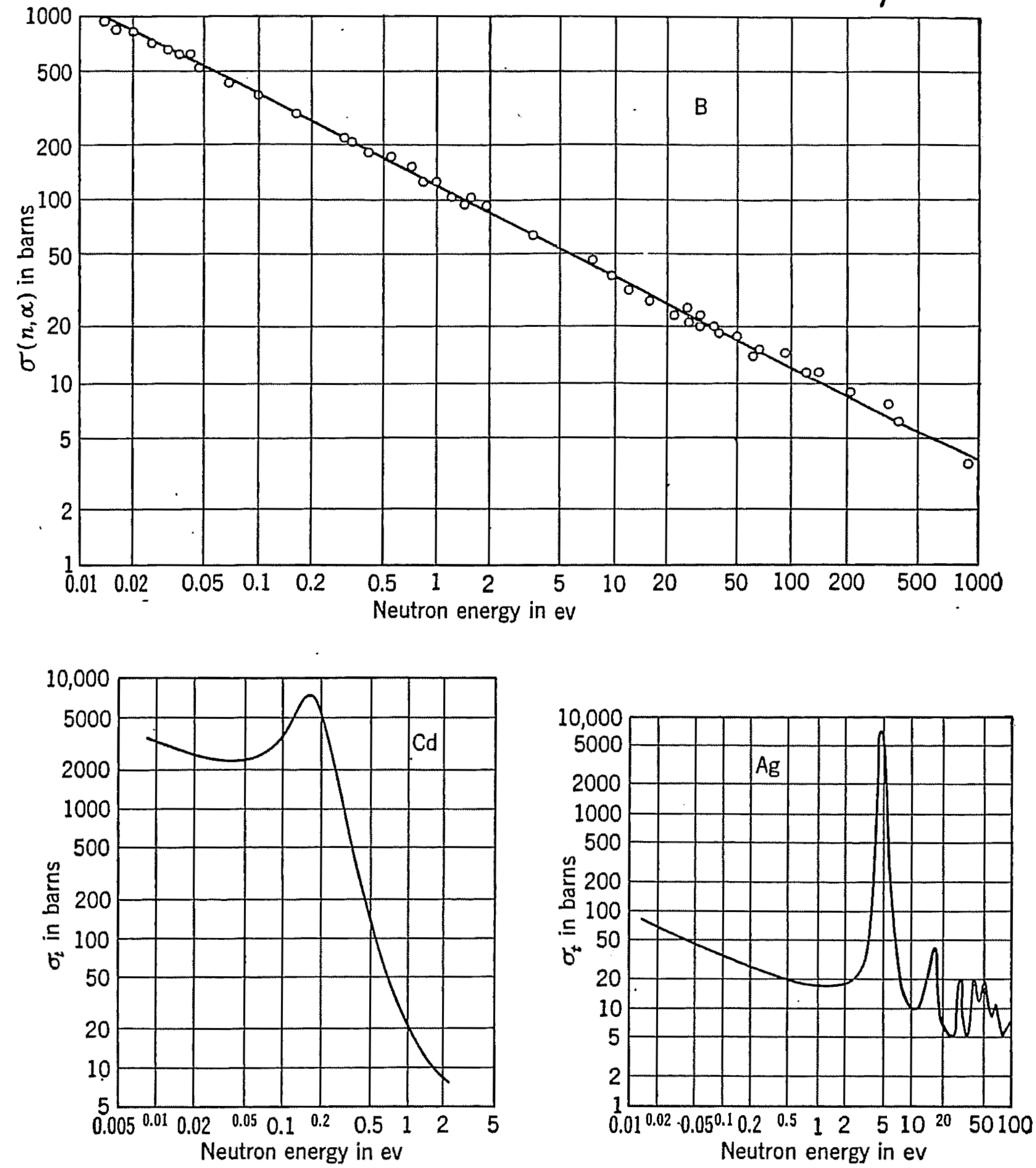


Fig. Introduction.1 Total neutron cross sections for B, Cd, and Ag, illustrating typical combinations of $1/V$ dependence (pure, in the case of boron) and slow-neutron resonances. The resonance in Cd at $E_0 = 0.176$ ev has a total width $\Gamma = 0.115$ ev and a maximum value of $\sigma_0 = 7{,}200$ barns. The curves are for the normal isotopic mixtures. In Cd, the 0.176-ev resonance is due entirely to Cd^{113} (relative abundance, 12.3 per cent); hence $\sigma_0 \simeq 57{,}000$ barns for Cd^{113} alone. The complex γ-ray spectrum leading to the ground level of Cd^{114} has been measured by Bartholomew and Kinsey (B16). [*From Goldsmith, Ibser, and Feld* (G29) *and Hughes* (H68).]

[skipped in 2019]

Why? wave properties of neutrons

de Broglie wavelength $\lambda = \frac{h}{p}$

Define reduced wavelength $\lambda\!\!\!^{-} = \frac{\lambda}{2\pi} = \frac{\hbar}{p}$

Consider non-relativistic neutrons ($T \ll m_n c^2 \sim 1000$ MeV)

$$T = \frac{p^2}{2m} = \frac{1}{2m}\left(\frac{\hbar}{\lambda\!\!\!^{-}}\right)^2 = \frac{1}{2mc^2}\left(\frac{\hbar c}{\lambda\!\!\!^{-}}\right)^2 = \frac{1}{2(1000\text{ MeV})}\left(\frac{200\text{ MeV fm}}{\lambda\!\!\!^{-}}\right)^2$$

$$= (20\text{ meV})\left(\frac{1\text{ fm}}{\lambda\!\!\!^{-}}\right)^2$$

$$\frac{\lambda\!\!\!^{-}}{1\text{ fm}} = \sqrt{\frac{20\text{ meV}}{T}}$$

For $T \gtrsim 20$ meV, $\lambda\!\!\!^{-} < 1$ fm, neutron acts as a particle ($\lambda\!\!\!^{-} \ll R$)

Small T, $\lambda\!\!\!^{-} \gg R$, acts like a wave, ~~σ~~ ϕ

Neutrons of $T \sim \frac{3}{2}kT \sim 0.05$ eV are called "thermal neutrons"

1938 Hahn & Strassmann (chemists)
thought they observed

$$n + {}_{92}U \xrightarrow{?} {}_{88}Ra$$

but no physical process could explain this
- why did they think it was radium?
- Because it had similar chemical properties to Ba and a radioactive product was precipitating out of soln w/ Ba
- But they then tried to separate Ra from Ba and failed
- What did they do? consulted a physicist.

Former colleague Lise Meitner
an Austrian Jew. when Hitler annexed Austria, she lost her citizenship
In 1938, she escaped to Sweden;
She discussed it w/ nephew, Otto Frisch
and decided it <u>was</u> Ba.

1939 Meitner & Frisch propose

$$n + U \longrightarrow Ba + Kr$$

radioactive

"nuclear fission"

Why didn't anyone think of this before?

$\frac{B}{A}$ for U ~ 7.5 MeV

$\frac{B}{A}$ for Ba, Kr ~ 8.2 MeV

about 0.7 MeV released per nucleon

⇒ ~ 170 MeV per fission.

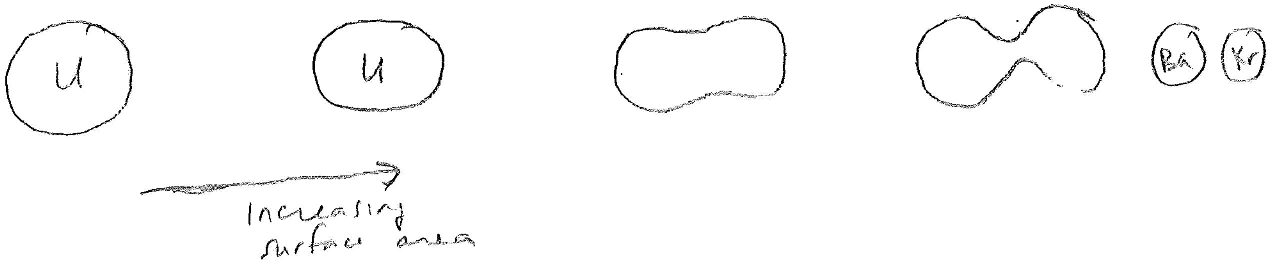
→ kinetic energy of decay products

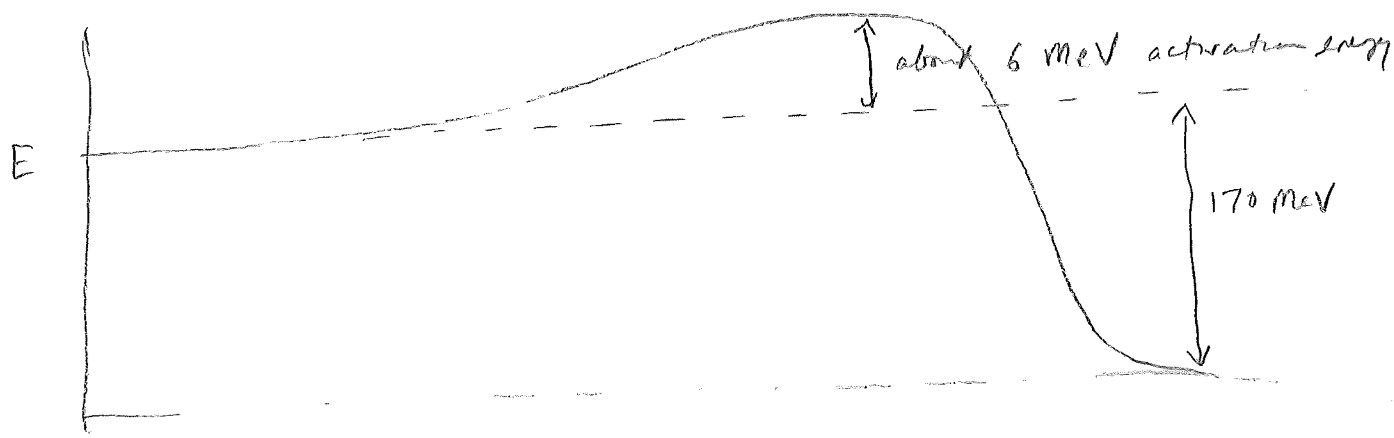
Frisch in his lab verified energy of fragments.

If U less stable than Ba, Kr why doesn't it just decay?

Think α-decay. Coulomb barrier holds them in.

Alternative picture. Liquid drop model (surface tension)





Tunnelling through barrier can occur but is rare

"spontaneous fission"

$T_{\frac{1}{2}} \sim 3.5 \times 10^{17}$ years [HRK, p. 1169]

(α-decay is more likely)

Incident neutron supplies energy

$$n + {}^{238}U \longrightarrow {}^{239}U$$

$n \longrightarrow$ (U) (U) wobble

$$Q = \Delta({}^{238}U) + \Delta(n) - \Delta({}^{239}U) = 4.8\ \text{MeV}$$

$$\begin{bmatrix} 47.306 \\ 8.071 \\ -51.571 \\ \hline 4.806 \end{bmatrix}$$

energy released causes drop to wobble

but activation energy of ^{239}U is 6.6 MeV

so neutrons must have $T > 1.8$ MeV to initiate fission

[?5.7 MeV in HRK, 1156]

"fast neutron fission of ^{238}U", $\frac{v}{c} \sim \sqrt{\frac{2T}{mc^2}} \sim 0.06$

But Hahn & Strassmann were using slow (thermal) neutrons ($T < 1$ eV)

1939 Bohr realized that they were observing "slow neutron fission of ^{235}U"

^{238}U 99.3%

^{235}U 0.7%

$$n + {}^{235}U \longrightarrow {}^{236}U$$

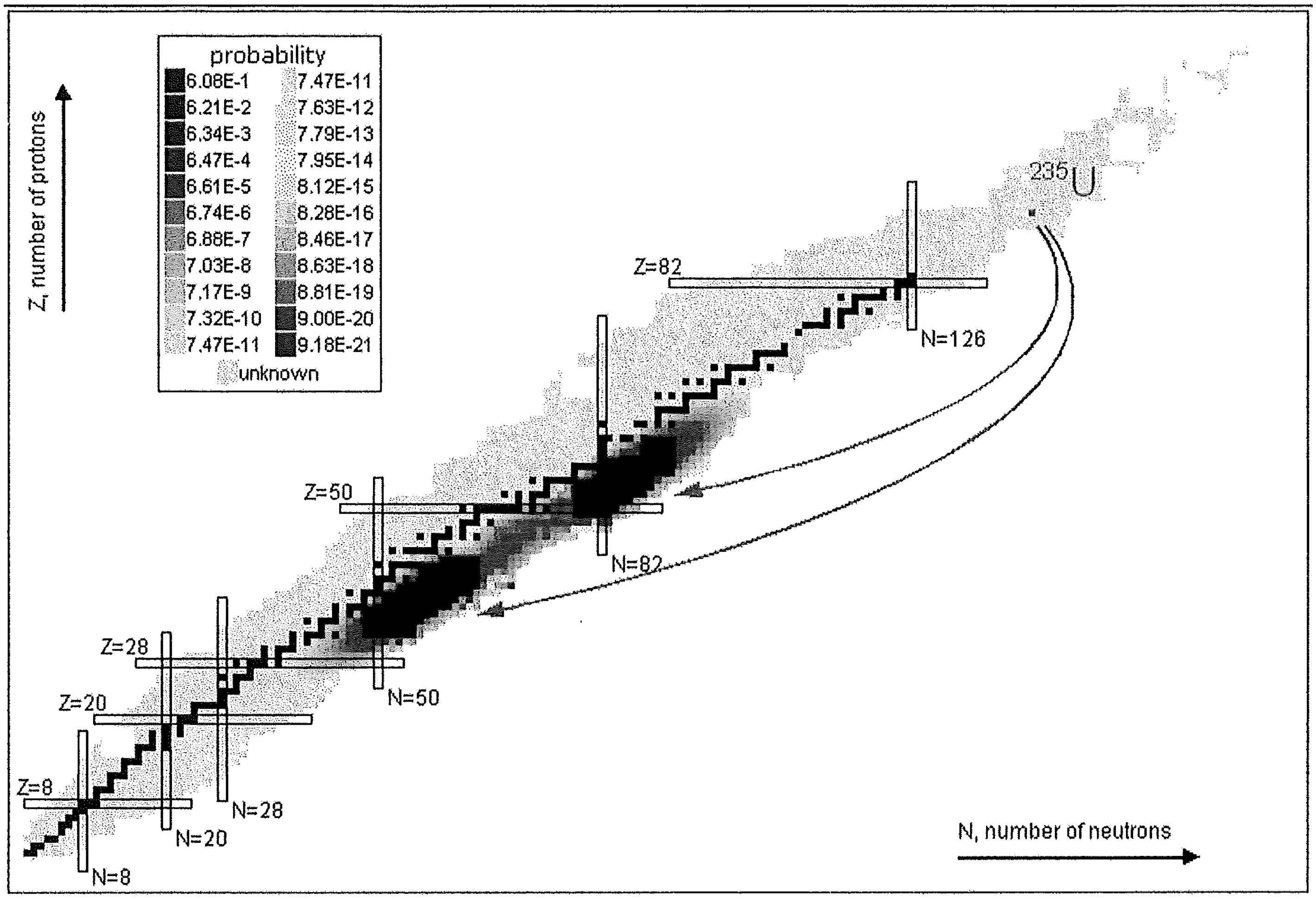
$Q = 6.5$ MeV $\Longrightarrow$ why different? even $\to$ odd vs odd $\to$ even

$$\begin{bmatrix} 40.916 \\ 8.071 \\ -42.442 \\ \hline 6.545 \end{bmatrix}$$

and activation energy of ^{236}U is 6.2 MeV

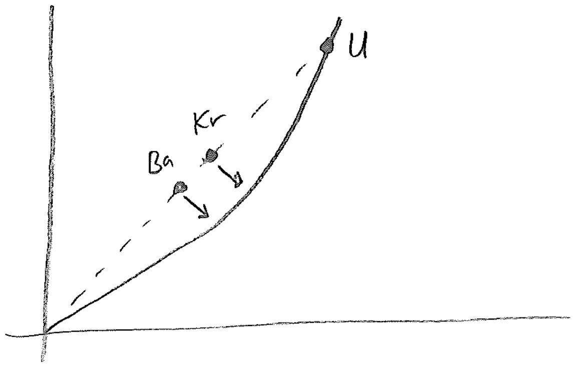
no additional kinetic energy required!

[?5.2 in HRK]



RobDemmingLounge.blogspot.com

[Why are Ba + Kr radioactive? Because they are neutron rich]



Fission often releases 2 or 3 neutrons in addition to fission fragments

For example

$$n + {}^{235}_{92}U \xrightarrow{e.g.} {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3n$$

$$\xrightarrow{e.g.} {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2n$$ [note: asymmetric]

Released neutrons can initiate further fissions

$\Rightarrow$ chain reaction

controlled $\Rightarrow$ nuclear reactor
uncontrolled $\Rightarrow$ weapons

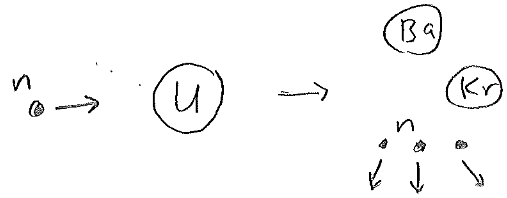
about 110 MeV released per fission
vs 1-10 eV released in chemical reactions [$\Rightarrow$ factor of 10^7 to 10^8]

$\rightarrow$ Leo Szilard convinced Einstein to write to Roosevelt...

First controlled nuclear reaction (Dec 2, 1942)

U of Chicago squash courts (Fermi)

$$n + {}^{235}U \longrightarrow {}^{236}U \xrightarrow{e.g.} {}^{141}Ba + {}^{92}Kr + 3n$$



Secondary neutrons
avg # ~ 2.47/fission
avg T ~ 2 MeV
(some are delayed)

- escape through surface w/o causing fission
  - solution: Increase volume to surface ratio (critical mass)
  - depends on mean free path
- fast n fission of ^{238}U
  - [possible since $\bar{E} > 1.8$ MeV]
  - occurs < 20% of time [because of low cross-sect for fast n absorption] → not sustainable [need at least 40%]
- slowed by inelastic collisions w/ ^{238}U (> 80% of time)
  - captured, re-emitted w/ E < 1 MeV
  - slow neutrons absorbed by $^{238}U \to {}^{239}U$ w/o subsequent fission (between 1-100 eV) capture resonance
- slowed by a moderator (water, heavy water, ultrapure graphite) below 1 eV
  - slow n fission of ^{235}U w/ thermal neutrons (0.04 eV)

For sustained rxn, need
① critical mass,
② pure moderator
or ③ enrichment above natural abundance

Def k = multiplication factor : # secondary neutrons that go on to produce subsequent fission

k < 1 subcritical
k = 1 critical (self-sustaining)
k > 1 supercritical

$$P(t) = P_0\, k^{\frac{t}{t_{gen}}}$$

(use neutron absorbers to keep reaction from going supercritical
e.g. Cadmium control rods)

[Richard Rhodes, Making of the Atomic Bomb]

Just talk about this :

Fri-13

[What if want a supercritical rxn, ie a bomb?]

- slow n fission not suitable, energy release causes U to expand & ∴ n's will all escape
- fast n fission of ^{238}U impossible because $k<1$.
- fast n fission of ^{235}U is possible

- enrichment: electromagnetic separation, gaseous diffusion } Oak Ridge, TN ($2 Billion)

Little Boy ~ 80% ^{235}U (about 50 kg)

(critical mass ~10 cm)

$$[\; n + {}^{238}U \rightarrow {}^{239}U' \rightarrow {}^{239}Np \rightarrow {}^{239}Pu \;]$$

^{239}Pu is fissionable

[Fat Man]

Hanford, WA

Nuclear physics

Neutrino biography (Sime)
(also Philip Ball: serving the Reich)

1919 Rutherford induced nuclear rxns w/ α

$$\alpha + {}^{14}N \rightarrow {}^{17}O + p$$

but not until 1924 was it realized that α was absorbed, not scattered. (Blackett). These rxns $\Rightarrow$ existence of a strong attractive force that could overcome Coulomb repulsion.

Nuclei heavier than K never absorbed α's, however, nor did Li, Be, C, ?.

→ ~~But~~ (1930) Bothe & Becker discovered a new penetrating type of radiation from α (from Po) bombardment of Be, Li, B. Joliot & Irene Curie (1932) found that the radiation ejected p from paraffin, & interpreted it as γ, but Chadwick proved it was really neutrons (1932)

$$\alpha + {}^{9}Be \rightarrow {}^{12}C + n$$

→ Then $n + {}^{14}_{7}N \rightarrow {}^{11}_{5}B + \alpha$ was observed by Norman Feather
$n + {}^{16}_{8}O \rightarrow {}^{13}_{6}C + \alpha$ " " " Meitner & Philipp

1934 Fermi theory of β decay, which used Pauli's ν idea

1934 Joliot & Curie produced artificial radioactivity

$$\alpha + {}^{27}Al \rightarrow {}^{30}P + n$$
$$\hookrightarrow {}^{30}Si + e^{+} + \nu$$
3 minutes

1934 Fermi ~~Segrè~~ ~~[illegible]~~ & Rasetti bombarded elts w/ n producing artificial radioactivity of F, Al, Si, P, Cu, ... ~~all~~ up thru U

$$n + {}^{238}U \rightarrow {}^{239}U \rightarrow {}^{239}Np \rightarrow {}^{239}Pu$$

this was hypothesized but not proved

Fermi observed (n, α) rxns & (n, p) rxns any lighter elts

$$n + {}^{27}Al \rightarrow {}^{24}Na + \alpha$$

$$n + {}^{28}Si \rightarrow {}^{28}Al + p$$

and (n, γ) rxns w/ heavier

$$n + {}^{127}I \rightarrow \underbrace{{}^{128}I + \gamma}_{} \rightarrow {}^{128}Xe + e^- + \bar{\nu}$$

and for the radioactive product always emitted e^-

~~Meitner & Hahn decided~~ to start studying ~~transuranics~~

Szilard + Chalmers observed

$\gamma + {}^{9}Be \ldots + n$ followed by $n + {}^{127}I \rightarrow {}^{128}I$

~~[illegible]~~ (lower energy)

Meitner found that the n could not be absorbed by Al, Na, Si which required higher energies to split off an α or ps but only in capture processes by heavier nuclei Ag, I, Au

She suggested that slower neutrons are more easily [illegible] the fast neutrons

1934 Fermi showed that paraffin, etc slowed the n, after which they were more easily absorbed by Ag, Cu, I, ~~[illegible]~~

Uranium enrichment

Manh. project
- → electromagnetic separation
- → gaseous diffusion

→ ~~large~~ gas centrifuges (most common + economical)

3–5% ^{235}U ⇒ power reactors

70–90% ^{235}U ⇒ weapons

Hiroshima ⇒ 64 kg of ^{235}U ⇒ 15 kilotons (size of a melon)
could do it w/ 25 kg

Trinity ⇒ 6 kg of ^{239}Pu ⇒ 20 kilotons (size of a plum)
could do it w/ 5 kg or less