

Intergenerational mixing

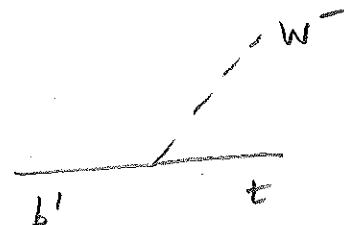
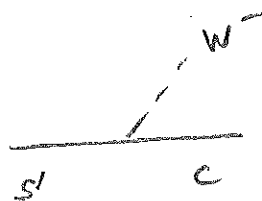
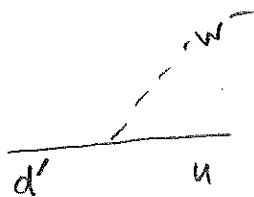
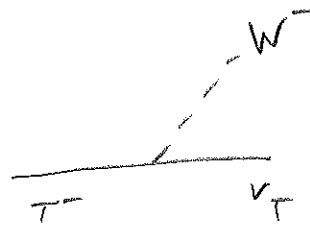
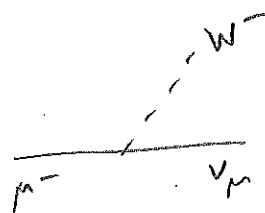
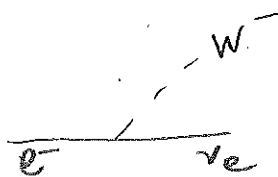
Force carriers: γ , $W^\pm$, Z^0 , gluons, gravitons

Matter constituents

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

Weak interaction vertices



[Before we wrote $\frac{d}{u}$ to discuss neutron decay]

d', s', b' are "almost" same as d, s, b

If they were exactly the same, then the strange + bottom quarks would be absolutely stable.

The "weak eigenstates" d', s', b' are related to the "mass eigenstates" d, s, b by a unitary transformation

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

"CKM matrix", determined experimentally

In particular $d' = V_{ud} d + V_{us} s + V_{ub} b$

so

$$\frac{d'}{g} \xrightarrow{W^-} u = \frac{d}{gV_{ud}} \xrightarrow{W^-} u + \frac{s}{gV_{us}} \xrightarrow{W^-} u + \frac{b}{gV_{ub}} \xrightarrow{W^-} u$$

[omit the $\frac{1}{2\sqrt{2}}$ as well as Dirac γ matrices]

[which explains the labels]

where $V_{ud} \sim 0.97$

$V_{us} \sim 0.23$

$V_{ub} \sim 0.004$

($\neq b$ quark)

Thus the s quark can decay to an up quark.

The CKM matrix can be parametrized by 3 angles $\theta_{12}, \theta_{13}, \theta_{23}$ and one complex phase $e^{i\delta}$ (which allows CP violation)

$$V_{CKM} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

$$c_{ij} = \cos \theta_{ij}$$

$$s_{ij} = \sin \theta_{ij}$$

"mixing matrix"

θ_{ij} is roughly the decay amplitude between i & j generations.

Experimentally $\theta_{13} \ll \theta_{23} \ll \theta_{12} \ll 1$. Let $\begin{cases} \theta_{12} \sim \epsilon \\ \theta_{23} \sim \epsilon^2 \\ \theta_{13} \sim \epsilon^3 \end{cases}$

Very roughly (ignoring CP violating phase) we can approximate

$$V_{CKM} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \quad \text{where } \epsilon \text{ is small.}$$

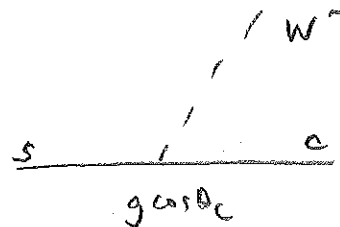
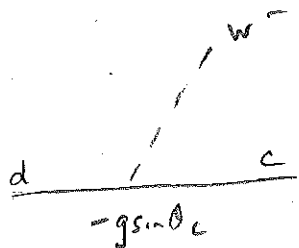
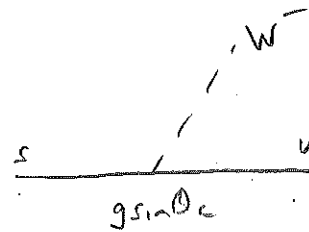
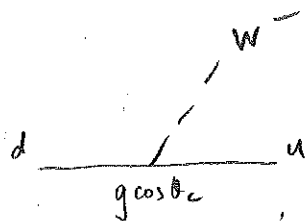
Hence top quark decays in stages:

$$t \xrightarrow{1} b \xrightarrow{\epsilon^2} c \xrightarrow{1} s \xrightarrow{\epsilon} u$$

If we set $\theta_{13} = \theta_{23} = 0$ and $\theta_{12} = \theta_c = \text{Cabibbo angle}$

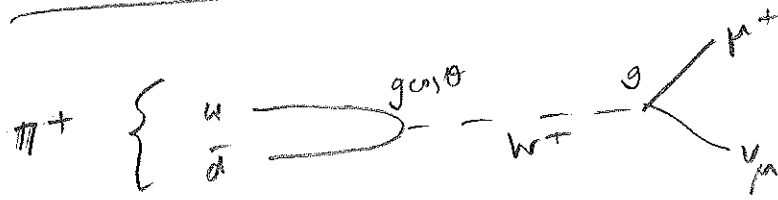
$$V_{ckm} = \begin{bmatrix} \cos \theta_c & \sin \theta_c & 0 \\ -\sin \theta_c & \cos \theta_c & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{aligned} \theta_c &\approx 12^\circ \\ \cos \theta_c &\approx 0.98 \\ \sin \theta_c &\approx 0.23 \end{aligned}$$

we only have mixing between first two generations.



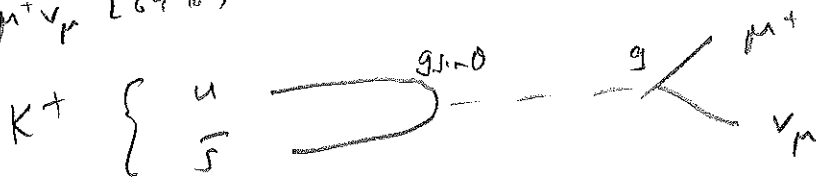
$$\cos \theta_c \downarrow \begin{pmatrix} u \\ d \end{pmatrix} \xleftarrow{\sin \theta_c} \begin{pmatrix} c \\ s \end{pmatrix} \xrightarrow{\cos \theta_c}$$

Meson decays

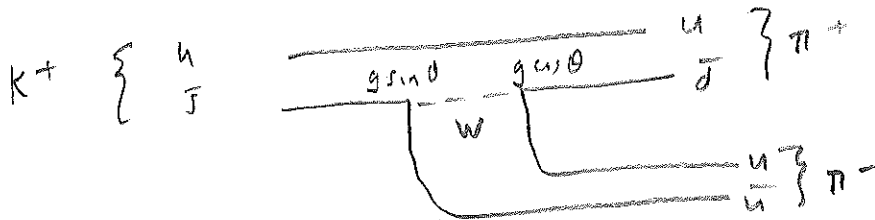


Cabibbo suppression
strange particles are
more "stable"
than expected

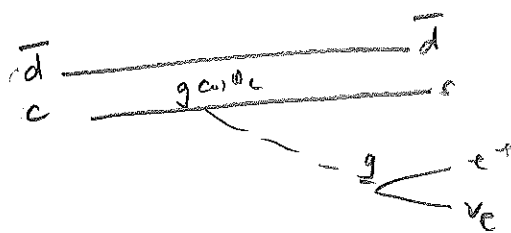
$K^+ \rightarrow \mu^+ \nu_\mu$ [64%]



$K^+ \rightarrow \pi^+ \pi^0$ [21%]

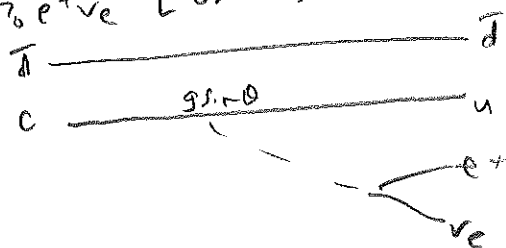


$D^+ \rightarrow \bar{K}_0 e^+ \nu_e$ [0.08%]



Ratio of amplitudes = $\frac{\sin \theta_c}{\cos \theta_c} = \tan \theta_c$

$D^+ \rightarrow \pi_0 e^+ \nu_e$ [0.004%]



Ratio of BR = $\tan^2 \theta_c = 0.045$

Prob. 2.5: $D^0 = c\bar{u} \rightarrow$
 $\rightarrow K^- \pi^+ [0.034]$
 $\rightarrow \pi^- \pi^+ [0.00140]$
 $\rightarrow K^+ \pi^- [0.00015]$