Lepton decay

[p.1 omitted]

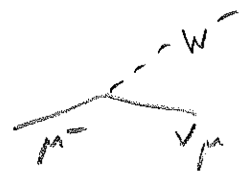
L-2

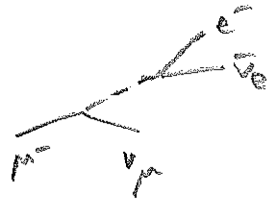
μ^- unstable w/ $\tau = 2.197 \times 10^{-6}\,s$

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

Conservation of lepton # for each generation

analogous to β^- decay of nuclei

weak vertex 

$$\Rightarrow A \sim$$  $$\sim \frac{\frac{1}{8}g^2}{p_W^2 - m_W^2} M \sim \frac{\frac{1}{8}g^2}{-m_W^2} M$$

since $m_\mu \ll m_W$

Earlier we calculated for nuclear β-decay

$$R = \frac{1}{60\pi^3\hbar}\frac{G_F^2}{(\hbar c)^6}|M|^2 Q^5$$

This is not valid here because $\bar{\nu}_\mu$ is not massive (& so it gets a lot of energy)

In the massless electron approximation, an accurate calculation gives

$$R = \frac{1}{192\pi^3\hbar}\frac{G_F^2}{(\hbar c)^6}Q^5, \quad \text{where} \quad Q = m_\mu c^2$$

need to do it a different way than we did (see my β decay notes)

$$= \frac{(1.1664\,E{-}11\ \text{MeV}^{-2})^2(105.66\ \text{MeV})^5}{192\pi^3(6.582\,E{-}22\ \text{MeV}\cdot s)}$$

$$= 4.572 \times 10^5\,s^{-1}$$

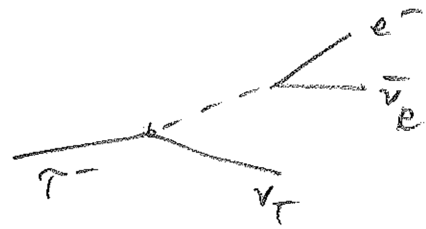
$$\tau = \frac{1}{R} = 2.187 \times 10^{-6}\,s. \quad \text{(very close!)}$$

Much larger phase space $\Rightarrow$ shorter τ (than, say, neutron)

[Incl. m_e does not improve]

Expect the τ to decay even faster

$$\tau^- \rightarrow e^- + \bar{\nu}_e + \nu_\tau$$



Same calculation as for μ^-;
only Q is different

$$\frac{R(\tau \rightarrow e \bar{\nu}_e \nu_\tau)}{R(\mu \rightarrow e \bar{\nu}_e \nu_\mu)} = \left(\frac{m_\tau}{m_\mu}\right)^5 = \left(\frac{1777}{105.66}\right)^5 = 1.345 \times 10^6$$

million times faster!

$$\text{Expect } \tau_\tau \sim \frac{(2.2 \times 10^{-6}\,s)}{(1.34 \times 10^{+6})} = 1.625 \times 10^{-12} \qquad (1.6\ ps)$$

Expt: $\tau_\tau \sim 2.90 \times 10^{-13}\,s$ which is 0.18 times expected

i.e. it decays 5 times faster than calculated

Look at PPB Branching ratio

$\tau \to e + \bar{\nu}_e + \nu_\tau$ 17.8%
$\to \mu + \bar{\nu}_\mu + \nu_\tau$ 17.4%
$\to \pi^- + \nu_\tau$ 10.8%
$\to \pi^- + \pi^0 + \nu_\tau$ 25.5%
$\to \pi^- + \pi^0 + \pi^0 + \nu_\tau$ 9%

(first five channels: 80%)

more decay channels → shorter mean life

The rates for mutually exclusive processes add.

$$R_{tot} = \sum_i R_i \qquad R_i = \text{partial rates}$$

$$\text{Branching ratio} \equiv \frac{R_i}{R_{tot}} = (BR)_i \qquad \sum_i (BR)_i = 1$$

$\therefore$ given R_i and $(BR)_i$ one can find $R_{tot} = \dfrac{R_i}{(BR)_i}$

$$\tau = \frac{1}{R_{tot}} = \frac{(BR_i)}{R_i}$$

$$\left[\cancel{\tau = \frac{1}{R_{tot}} = \frac{BR_i}{R_i} = (BR)_i \, \tau_i} \right]$$

"partial mean life"

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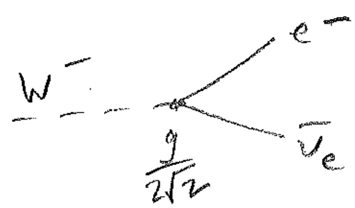
conservation of L_e, L_μ, L_τ guaranteed by vertices

Because of large masses of W and Z bosons,
not detected (produced) until 1983.

$m_W \sim 80.4$ GeV

Decay of W^-

$W^- \longrightarrow e^- + \bar{\nu}_e$ (10.7%)



$$\left[\begin{array}{l} \text{Rest frame of } W^- \Rightarrow E_{init} = m_W c^2 \\ \text{Momentum conservation} \Rightarrow |\vec{p}_e| = |\vec{p}_\nu| \\ \qquad\qquad \Rightarrow E_e \approx E_\nu \\ \qquad\qquad E_{final} = E_e + E_\nu = 2E_e \end{array}\right]$$

Decay rate

$$R = \left(\frac{L}{h}\right)^3 \int d^3\vec{p}_e \; \frac{2\pi}{\hbar} \, \delta(E_i - E_f) \, |A|^2$$

$$A \sim \frac{g}{2\sqrt{2}} \, M$$

As before, we'll assume M is simple, but get units correct
(ignoring the spin of W, e^-, $\bar{\nu}_e$)

$|A|^2 = \frac{g^2}{8}|M|^2$ must have units of (energy)^2

$|M|^2$ must contain a factor of $\frac{1}{L^3}$ to cancel L^3
so that decay rate is independent of fictional box

Write $|A|^2 = \frac{g^2}{8L^3}(?)$

What are units of ?

Recall $\alpha_W = \frac{g^2}{4\pi\hbar c}$ so g^2 has units of $\hbar c = E \cdot L$

Unit wise we have

$$E^2 = \frac{E \cdot L}{L^3} \cdot ?$$

So ? has units of $L^2 \cdot E$

skip

What combinations of $\hbar$ + c can we use?

$\hbar c = E \cdot L$ so we can write

$? = (\hbar c)^2$ but still off by factor of energy

The only energy scale in the problem is $M_W c^2$

so let's use that too

$$? = \frac{(\hbar c)^2}{(m_W c^2)} \quad \text{has units } L^2 \cdot E.$$

$$\Rightarrow |A|^2 = \frac{g^2 \hbar^2}{8 L^3 m_W}$$ has the right units

[Now compute R as a problem]

Decay rate as imaginary mass

Quantum mechanically, a particle of energy E has wavefunction

$$\psi(x,t) = u(x) e^{-\frac{iEt}{\hbar}}$$

For a particle at rest $E = mc^2$

$$\psi(x,t) = u(x)\, e^{-\frac{i mc^2 t}{\hbar}}$$

Probability of finding particle

$P = |\psi(x,t)|^2 = |u(x)|^2$ independent of time (stationary state)

But an unstable particle has decreasing probability of existing

$$\frac{dP}{dt} = -RP \quad \Rightarrow \quad P = e^{-Rt} \quad (\text{if } P(0) = 1)$$

Thus

$$\psi(x,t) = u(x)\, e^{-\frac{imc^2 t}{\hbar}}\, e^{-\frac{1}{2}Rt}$$

$$= u(x)\, e^{-\frac{i}{\hbar}\left(\underbrace{mc^2 - i\frac{\hbar R}{2}}_{\text{complex "rest energy" of a particle}}\right)t}$$

This suggests $-\frac{\hbar R}{2}$ = Imaginary part of rest energy

Define the $\underbrace{\text{width}}_{\text{units of energy}}$ Γ of an unstable particle as $\Gamma = \hbar R$

$\mathrm{Im}(\text{rest energy}) = -\frac{\Gamma}{2}$

Width is related to mean life $\tau = \frac{1}{R}$ by

$$\Gamma = \frac{\hbar}{\tau} = \frac{6.6 \times 10^{-22}\ \text{MeV} \cdot \text{s}}{\tau}$$

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neutron: $\tau = 880\ \text{s} \Rightarrow \Gamma = 7.5 \times 10^{-25}$ MeV

muon $\tau = 2.2 \times 10^{-6}\ \text{s} \Rightarrow \Gamma = 3 \times 10^{-16}$ MeV

[A tiny fraction of the real part of rest energy]

PPB often lists Γ rather than τ for short-lived particle

W^- $\Gamma = 2\ \text{GeV} \Rightarrow \tau = \frac{\hbar}{\Gamma} = 3.3 \times 10^{-25}$ sec

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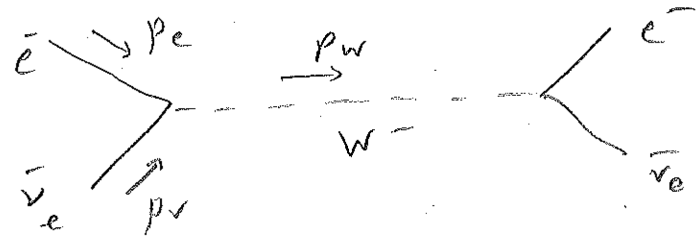
Partial width $\Gamma_i = \hbar R_i$

Full width $\Gamma = \sum \Gamma_i$

$$(BR)_i = \frac{\Gamma_i}{\Gamma}$$

How does one detect the existence of and measure the properties (m, Γ) of a very short-lived particle such as the W?

Consider a process $\bar{\nu}_e + e^- \to W^- \to \bar{\nu}_e + e^-$



$$p_W = p_e + p_\nu$$

In cm frame $\vec{p}_e + \vec{p}_\nu = 0$

$$E_e + E_\nu = E_{cm}$$

$$\Rightarrow p_W^2 = E_{cm}^2 - \vec{p}_{cm}^{\,2} = E_{cm}^2$$

Amplitude for the process above [neglect $\frac{g^2}{8}$]

$$A \sim \frac{1}{p_W^2 - m_W^2} \sim \frac{1}{E_{cm}^2 - m_W^2}$$

Scattering cross section $\sigma \sim |A|^2$

If $E_{cm} \sim m_W$, then $A \to \infty \Rightarrow \sigma \to \infty$

But if W is unstable, m_W has an imag. component

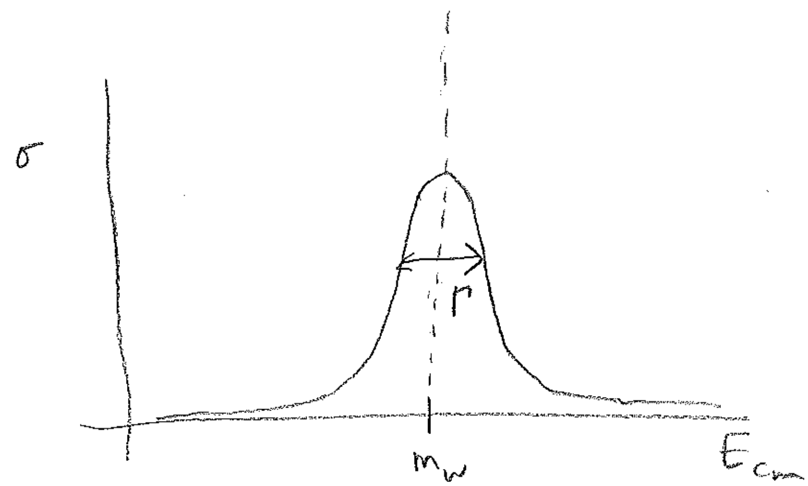
$$A \sim \frac{1}{E_{cm}^2 - (m_W - \frac{i\Gamma}{2})^2}$$

$$\sim \frac{1}{(E_{cm} - m_W + \frac{i\Gamma}{2})(E_{cm} + m_W - i\frac{\Gamma}{2})}$$

For $E_{cm} \sim m_W$, 2nd term $\sim 2m_W - \frac{i\Gamma}{2} \approx 2m_W$

$$A \sim \frac{1}{2m_W(E_{cm} - m_W + \frac{i\Gamma}{2})}$$

$$\sigma \sim |A|^2 \sim \frac{1}{(2m_W)^2\left[(E_{cm} - m_W)^2 + \frac{\Gamma^2}{4}\right]}$$ (Breit-Wigner curve)



Analogous to resonance of a damped driven harmonic osc.

→ resonance curve

Γ is the width of the resonance

(σ = half maximum for $E_{cm} - m_W = \frac{\Gamma}{2}$
(Γ = full width half-maximum

unstable particles are detected through bumps in the scattering cross-section as a fn of E_{cm}

Heisenberg uncertainty principle for energy + time

$$\Delta E = \Gamma$$

but $\Gamma = \frac{\hbar}{\tau}$

$$\Rightarrow \quad \Delta E \cdot \tau \sim \hbar$$

Therefore if we interpret Δt to mean τ

[N.B. there is no strict QM meaning for Δt]

then

$$\Delta E \, \Delta t \gtrsim \hbar$$