(3-21-19)

W-decay

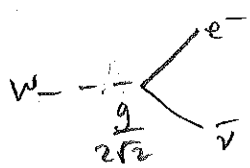
$$\frac{G_F}{(\hbar c)^3} = \frac{\pi}{\sqrt{2}} \frac{\alpha_W}{(m_W c^2)^2}$$

Decay products are massless, so $f = 1$ in prior formulas

$$R = \frac{E_0^2}{8\pi\hbar(\hbar c)^3} |A|^2 L^9 \quad \text{where } \left[|A|^2 L^9 \right] = \frac{(\hbar c)^3}{E}$$

Using relativistic amplitude $A = \dfrac{\mathcal{M}}{\left[(2E_0)(2E_1)(2E_2)L^9\right]^{1/2}} = \dfrac{\mathcal{M}}{(2E_0^3 L^9)^{1/2}}$

$$R = \frac{1}{16\pi\hbar(\hbar c)^3} \frac{|\mathcal{M}|^2}{E_0} \qquad \text{where } \left[|\mathcal{M}|^2\right] = (\hbar c)^3 E^2$$



$$A \sim \frac{g}{2\sqrt{2}}$$

$$L^9 |A|^2 \sim \frac{g^2}{8} \cdot ? = \frac{4\pi\alpha_W \hbar c}{8} \cdot ?$$

$$\left[|A|^2 L^9\right] = \frac{(\hbar c)^3}{E} \Rightarrow [\,?\,] = \frac{(\hbar c)^2}{E}$$

Since only mass involved is m_W, we assume <u>for simplicity</u>: $? = \dfrac{(\hbar c)^2}{m_W c^2}$

$$L^9 |A|^2 = \frac{4\pi\alpha_W \hbar c}{8} \cdot \frac{(\hbar c)^2}{m_W c^2} = \frac{\pi}{2}\alpha_W \frac{(\hbar c)^3}{(m_W c^2)} = \frac{1}{\sqrt{2}} G_F m_W c^2$$

$$\Rightarrow R = \frac{(m_W c^2)^2}{8\pi\hbar} \frac{\pi\alpha_W}{2 m_W c^2} = \frac{\alpha_W m_W c^2}{16\hbar} = \frac{1}{8\sqrt{2}\pi\hbar} \frac{G_F}{(\hbar c)^3} (m_W c^2)^3$$

[Actual: $\dfrac{1}{6\sqrt{2}\pi\hbar} \dfrac{G_F}{(\hbar c)^3} (m_W c^2)^3$, Perkins p. 367]

$$R = \frac{1}{16}\left(\frac{1}{29.5}\right) \frac{80{,}400 \text{ MeV}}{(6.6\text{ E-22 MeV}\cdot s)}$$

$= 2.6$ E23

Really, a bit lucky to have gotten so close

$\tau = 4$E-24 s

$\Gamma = \hbar R = 170$ MeV

[actual partial width $= (0.107)(2.086\text{ GeV}) = 220$ MeV

$\nu X \longrightarrow Y e^-$

$$\frac{d\sigma}{d\Omega} = \frac{1}{(2\pi)^2}\frac{1}{\hbar^4}\frac{p_{1f}^2}{v_f v_i} L^{12}|A|^2$$

$$v_i = c, \quad v_f = \frac{c^2 p_{1f}}{E_{1f}} \quad \Rightarrow \quad \frac{d\sigma}{d\Omega} = \frac{1}{(2\pi)^2}\frac{(cp_{1f})E_{1f}}{(\hbar c)^4} L^{12}|A|^2$$

$$\underbrace{L^6 A}\ = G_F M \quad \text{(as in class)}$$

(N.B. A here & in my class notes differ by factor of L^3)

$$\frac{d\sigma}{d\Omega} = \frac{1}{(2\pi)^2}\frac{(cp_{1f})E_{1f}}{(\hbar c)^4} G_F^2 |M|^2$$

$$\sigma = \frac{1}{\pi}\left[\frac{G_F}{(\hbar c)^3}\right]^2 (\hbar c)^2 (cp_e)E_e |M|^2 \quad ✓$$

[Perkins, p.227: $\frac{4G^2E^2}{\pi}$ but Perkins, PA 1.276: $\frac{G^2 s}{\pi}$]

(1-19)

Crude estimates
↳ based on dim'l analysis

19

$\pi^0 \to \gamma\gamma$

$$R = \frac{E_0^2 \, |A|^2 L^9}{8\pi\hbar \, (\hbar c)^3} \qquad \left[|A|^2 L^9\right] = \frac{(\hbar c)^3}{E}$$

Let's pretend there is a vertex

This vertex comes w/ 2 factors of e

ie $\sim 4\pi K e^2 = 4\pi\alpha\hbar c$

$$\Rightarrow L^9|A|^2 \sim (4\pi\alpha\hbar c)^2 \cdot ?$$

In order for $|M|^2$ to have correct units, we must choose ? to have units $\frac{\hbar c}{E}$

The only energy scale is $m_\pi c^2$ so $? = \frac{\hbar}{m_\pi c}$

$$L^9|A|^2 = (4\pi\alpha)^2 \frac{(\hbar c)^3}{(m_\pi c^2)}$$

$$R = \frac{(m_\pi c^2)^2}{8\pi\hbar} \frac{(4\pi\alpha)^2}{(m_\pi c^2)}$$

$$= 2\pi\alpha^2 \frac{(m_\pi c^2)}{\hbar} = \frac{2\pi}{(137)^2} \frac{(140 \text{ MeV})}{(6.6E-22 \text{ MeV sec})} = 7E19 \text{ s}^{-1}$$

$$\Gamma = \hbar R = (2\pi\alpha^2)\, m_\pi c^2 \qquad \tau = 1.4E-20 \text{ sec}$$

(actual 10^{-16} sec)

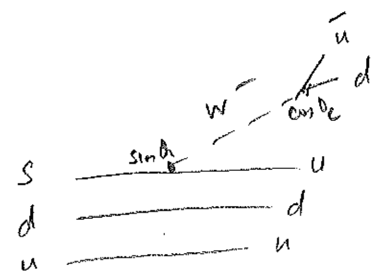
$$\frac{\Gamma}{m_\pi c^2} = 2\pi\alpha^2$$

$\Lambda^0 \to p\pi^-$

$\tau_\Lambda = 2.6\,E{-}10$ sec

BR = 0.64

$\Rightarrow$ partial $\tau = 4.1\,E{-}10$ sec



$$\mathcal{M} \sim \frac{g^2}{8} \cdot \frac{\sin\theta_c \cos\theta_c}{(m_W c^2)^2} \sim \frac{4\pi\alpha_W \hbar c}{8} \frac{\frac{1}{2}\sin 2\theta_c}{(m_W c^2)^2}$$

$$L^9|\mathcal{M}|^2 = \left(\frac{\pi\alpha_W}{2}\right)^2 \frac{(\hbar c)^2}{(m_W c^2)^4} \cdot \left(\frac{\sin 2\theta_c}{2}\right)^2 \cdot ?$$

But units of $L^9|\mathcal{M}|^2$ should be $\frac{(\hbar c)^3}{E}$ so $? = \underline{(\hbar c)(E_0)^3}$

$$L^9|\mathcal{M}|^2 = \left(\frac{\pi\alpha_W}{2}\right)^2 \frac{(\hbar c)^3 (E_0)^3}{(m_W c^2)^4} \cdot \left(\frac{\sin 2\theta_c}{2}\right)^2$$

$$R = \frac{f\,(m_\Lambda c^2)^2}{8\pi\hbar} \left(\frac{\pi\alpha_W}{2}\right)^2 \frac{(E_0)^3}{(m_W c^2)^4} \left(\frac{\sin 2\theta_c}{2}\right)^2$$

$\theta_c = 13^\circ$

$$= \frac{\pi}{32\hbar} f \cdot \alpha_W^2 \frac{(m_\Lambda c^2)^2 E_0^3}{(m_W c^2)^4} \left(\frac{\sin 2\theta_c}{2}\right)^2$$

$$\frac{\Gamma}{m_\Lambda c^2} = \frac{\hbar R}{m_\Lambda c^2} = \frac{\pi}{32} \cdot (0.34)\left(\frac{1}{29.5}\right)^2 \frac{1116\, E_0^3}{(80400)^4} (0.05) = (5.1E{-}23)\, E_0^3\ [\text{MeV}]$$

$$\Gamma = 5.7E{-}20\, E_0^3 \Rightarrow \tau = \frac{0.0115}{E_0^3} = 4.1E{-}10 \Rightarrow E_0 = 30.5\ \text{MeV}$$

$$f = \underbrace{\left(\frac{m_0+m_1+m_2}{m_0}\right)^{5/2} \left(\frac{m_0+m_1-m_2}{m_0}\right)^{3/2} \left(\frac{m_0-m_1+m_2}{m_0}\right)^{3/2} \left(\frac{m_0-m_1-m_2}{m_0}\right)^{1/2}}$$

Since the mass dependence of A is unknown, it really isn't good to rely too much on f!

$\Lambda^0 \to p\pi^-$ 64%

$\tau = 2.6E{-}10$ sec

$\Rightarrow \tau_{\Lambda\to p\pi^-} = 4.1E{-}10$ s

$\Xi^- \to \Lambda\pi^-$ 100% $\Rightarrow \tau_{\Xi\to\Lambda\pi^-} = 2.9E{-}10$ sec

$\tau = 2.9E{-}10$ sec

$$\frac{R_\Xi}{R_\Lambda} = 1.4 \quad \text{(expt)}$$

My calculation $\Rightarrow$

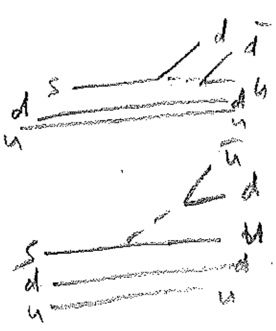
$$\frac{R_\Xi}{R_\Lambda} = \frac{f_\Xi\, m_\Xi^2}{f_\Lambda\, m_\Lambda^2} = \frac{0.365}{0.340} \cdot \underbrace{(1.185)^2}_{} = 1.51$$

alone gives 1.4!

So do we need f?

what's dependence of A on mass

$\Lambda^0 \to n\pi^0$ 36% $\Rightarrow \tau_{\Lambda\to n\pi^0} = 7.2E{-}10$ s



$$\frac{\tau_{\Lambda\to n\pi^0}}{\tau_{\Lambda\to p\pi^-}} = 1.8$$

overlap of $d\bar{d}$ w/ $\pi_0 = \frac{1}{\sqrt{2}}$ so expect this to be 2 times slower?

Seems good

$\pi^- \rightarrow \mu^- \bar{\nu}_\mu$ (100%) $\quad \tau = 2.6\text{E-}8 \text{ sec}$

As for Λ^0 decay

$$R = \frac{\pi}{32\hbar} f \cdot \alpha_W^2 \frac{(m_\pi c^2)^2 E_0^3}{(m_W c^2)^4} (\cos\theta_c)^2$$

$$\frac{\Gamma}{m_\pi c^2} = \frac{\pi}{32}(0.562)\left(\frac{1}{29.5}\right)^2 \frac{139.6\, E_0^3}{(80400)^4}(0.95) = (2.0\text{E-}22)E_0^3$$

$$\Gamma = 2.8\text{E-}20\ E_0^3$$

$$\tau = \frac{0.0235}{E_0^3} = 2.6\text{E-}8 \text{ sec} \Rightarrow E_0 = 97 \text{ MeV}$$

really don't expect to be able to do this because of chirality

$\Rightarrow R \rightarrow 0$ if $m_\mu \rightarrow 0$ which is not captured in our approach

$\pi \to \mu \nu_\mu$ 100% $\quad \tau = 2.6E-8$

$K \to \mu \nu$ 63.6% $\quad \tau_{K\to\mu\nu} = 1.95 E-8$

$\tau = 1.24E-8 s$

$$\frac{R_{K\to\mu\nu}}{R_{\pi\to\mu\nu}} = 1.33 \quad (\text{expt})$$

My calc gives $\frac{R_K}{R_\pi} = \frac{f_K}{f_\pi}\left(\frac{m_K}{m_\pi}\right)^2 (\tan\theta_c)^2$

$$= \frac{1.34}{0.56}\left(\frac{493.7}{139.6}\right)^2 (0.05) = 1.5$$

Looks good

probably just lucky!