β-decay and weak interactions

Initially it was thought that

$${}^{3}_{1}H \longrightarrow {}^{3}_{2}He + e^{-}$$

kinetic energy released $Q = m_H - m_{He} - m_e = 18.6 \text{ keV}$
as we calculated earlier.

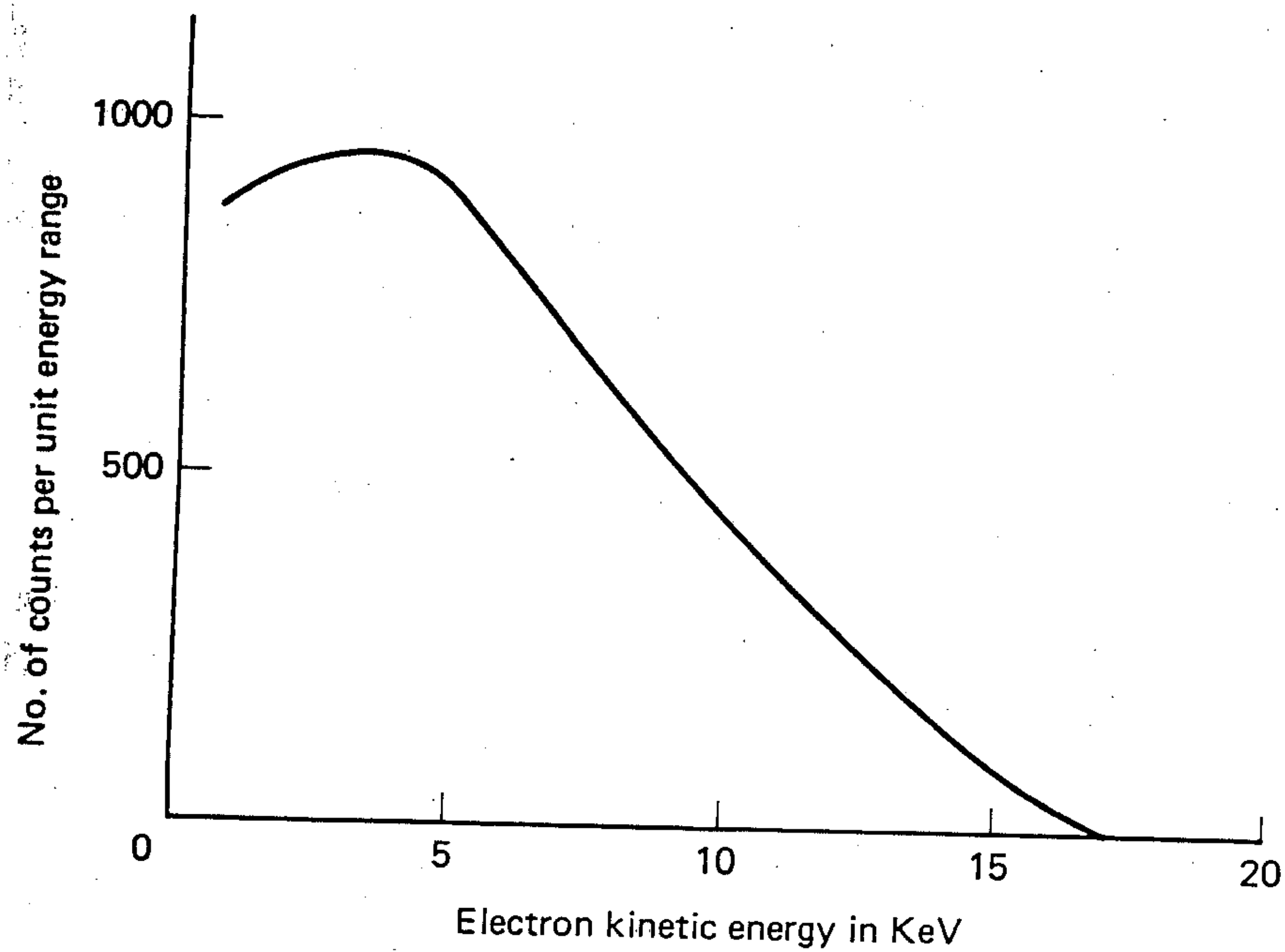
Initial state tritium at rest $\Rightarrow T_{He} + T_e = Q$

Since $m_e \ll m_H$, $T_{He} \ll T_e$ so expect $T_e \approx Q$

Experiments, however, revealed a
<u>continuous beta spectrum</u>

ie T_e can take on any value $0 < T_e < Q$

[Chadwick 1914] see graph →



The beta decay spectrum of tritium ($^{3}_{1}H \rightarrow\, ^{3}_{2}He$). (*Source:* G. M. Lewis,

Fig. 1.6 Griffiths

[Bohr suggested energy not conserved microscopically but only statistically on macro scales]

1930 Pauli proposed the existence of a neutral particle that carries off the missing energy (w/ $m < 10$ MeV)
Fermi dubbed it the neutrino (little neutral one)

[quote]

$${}^{3}_{1}H \longrightarrow {}^{3}_{2}He + e^- + \bar{\nu}_e$$

electron antineutrino

As before, T_{He} is negligible so

$$T_e + E_\nu = Q = 18.6 \text{ keV}$$

[NB not T_ν because didn't include m_ν in definition of Q]

$$(E_\nu)_{min} = m_{\bar{\nu}} c^2 \quad \text{so}$$

$$(T_e)_{max} = Q - m_\nu c^2$$

Since $(T_e)_{max} \gtrsim 18$ keV $\Rightarrow m_\nu \lesssim 500$ eV [Segrè, p. 334]

At any rate the earliest reference I know to the new particle is Heisenberg's mention of 'your neutrons' in a letter to Pauli[97] dated 1 December. More details are found in Pauli's letter (its main part follows) of 4 December to a gathering of experts on radioactivity in Tübingen.[60] 1930

Dear radioactive ladies and gentlemen,
I have come upon a desperate way out regarding the 'wrong' statistics of the N- and the Li 6-nuclei, as well as to the continuous β-spectrum, in order to save the 'alternation law' of statistics* and the energy law. To wit, the possibility that there could exist in the nucleus electrically neutral particles, which I shall call neutrons, which have spin 1/2 and satisfy the exclusion principle and which are further distinct from light-quanta in that they do not move with light velocity. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any case not larger than 0.01 times the proton mass.—The continuous β-spectrum would then become understandable from the assumption that in β-decay a neutron is emitted along with the electron, in such a way that the sum of the energies of the neutron and the electron is constant.

There is the further question, which forces act on the neutron? On wave mechanical grounds . . . the most probable model for the neutron seems to me to be that the neutron at rest is a magnetic dipole with a certain moment μ. Experiments seem to demand that the ionizing action of such a neutron cannot be bigger than that of a γ-ray, and so μ may not be larger than $e \times 10^{-13}$ cm.

For the time being I dare not publish anything about this idea and address myself confidentially first to you, dear radioactive ones, with the question how it would be with the experimental proof of such a neutron, if it were to have a penetrating power equal to or about ten times larger than a γ-ray.

I admit that my way out may not seem very probable *a priori* since one would probably have seen the neutrons a long time ago if they exist. But only he who dares wins, and the seriousness of the situation concerning the continuous β-spectrum is illuminated by my honored predecessor, Mr. Debye, who recently said to me in Brussels: 'Oh, it is best not to think about this at all, as with new taxes'. One must therefore discuss seriously every road to salvation.—Thus, dear radioactive ones, examine and judge.—Unfortunately I cannot appear personally in Tübingen since a ball** which takes place in Zürich the night of the sixth to the seventh of December makes my presence here indispensable . . . Your most humble servant, W. Pauli'.

We now calculate the β-spectrum using the QFT describing the weak interaction

Electroweak theory (1968) Glashow, Weinberg, Salam

In addition to electromagnetic field, $\exists$ 3 additional fields

This gives rise to 3 new particles

W^+, W^-, Z_0

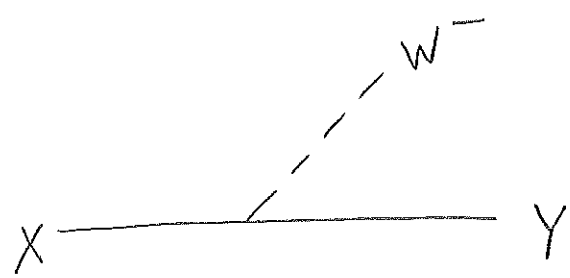
These are analogous to the photon γ, except massive

$m_{W^+} = m_{W^-} = 80.4$ GeV [PPB: 1st massive entry]

$m_Z = 91.2$ GeV

β-decay: $_{Z}X \longrightarrow {}_{Z+1}Y + e^- + \bar{\nu}_e$

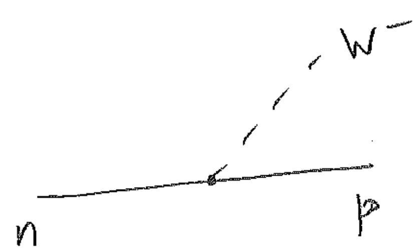
One of the vertices in this process is



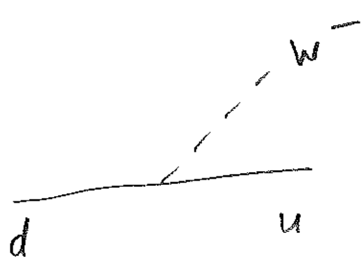
Electric charge is conserved at the vertex so charge of Y is one greater than charge of X

Electroweak vertex involves a change of identity for the particle emitting the W^- (unlike QED vertices)

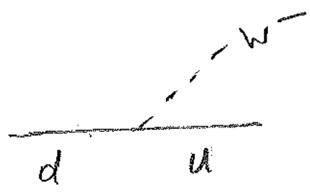
What's actually occuring is $n \to p + e^- + \bar{\nu}_e$



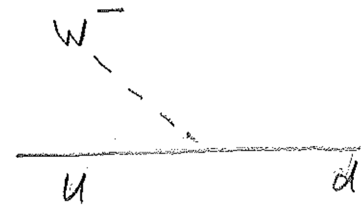
And on a deeper level $n = udd$, $p = uud$ so

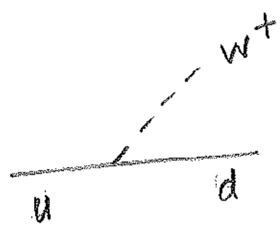


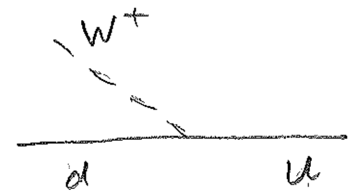
[NB let's not put on arrows]



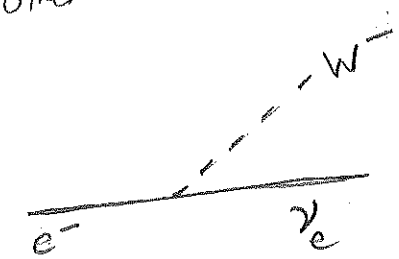
time reverse

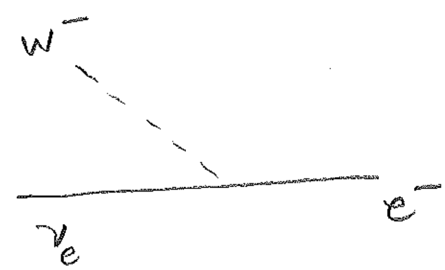


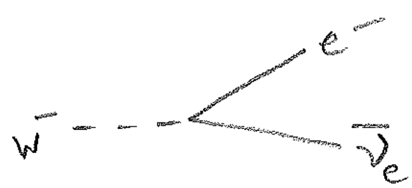




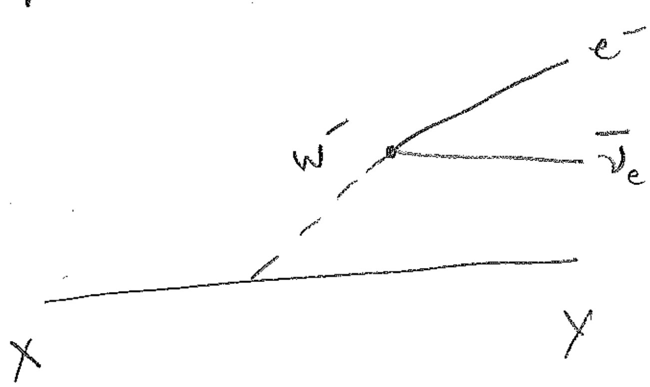
Other vertices



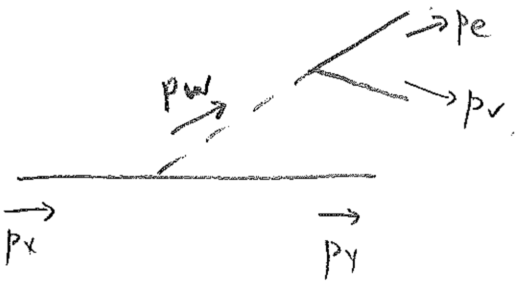




Therefore $X \to Y + e^- + \bar{\nu}_e$ obtained by assembling these



4-momentum is conserved at each vertex and overall



$$p_X = p_Y + p_e + p_\nu$$

$$p_W = p_X - p_Y$$

$$p_W = p_e + p_\nu \qquad \text{[be careful w/ arrows]}$$

For typical β-decay, momenta are of order of MeV or 10's of MeV

$\Rightarrow p_W \sim$ MeV

But $m_W \sim 80$ GeV

so $p_W^2 \neq M_W^2$

ie W^- is off-shell (virtual)

The propagator for the internal line is $\dfrac{1}{p_W^2 - M_W^2}$

(it would $\rightarrow \infty$ if W were on-shell)

The strength of vertices involving $W^\pm$ is proportional to g, analogous to e for the QED vertex.

$g =$ "weak charge"

(set $\epsilon_0 = 1$)

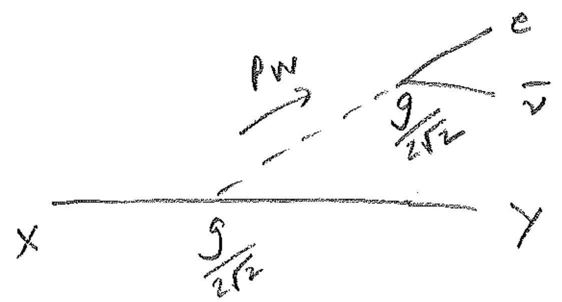
Fine structure constant for QED $\alpha = \dfrac{e^2}{4\pi\hbar c} \approx \dfrac{1}{137}$

" " " " weak $\alpha_W = \dfrac{g^2}{4\pi\hbar c} \approx \dfrac{1}{30}$ (as we'll see from expt.)

$\alpha_W > \alpha$! Why is weak force so much weaker than EM?

Because the $W^\pm$ is so massive!

$$\left[\begin{array}{l} g = \dfrac{e}{\sin\theta_W} \\ \sin^2\theta_W = \dfrac{e^2}{g^2} = \dfrac{\alpha}{\alpha_W} = 0.215 \end{array}\right]$$



[really $\frac{g}{2\sqrt{2}}(\gamma^\mu - \gamma^\mu\gamma^5)$]

Amplitude for decay $\quad A \sim \dfrac{\frac{1}{8}g^2}{p_W^2 - m_W^2} M$

For typical β-decay, $p_W^2 \ll m_W^2$ so $\qquad$ [HW]

$$A \sim \frac{g^2}{8 m_W^2} M$$

[minus sign irrelevant because $|A|^2$ to get rate.]

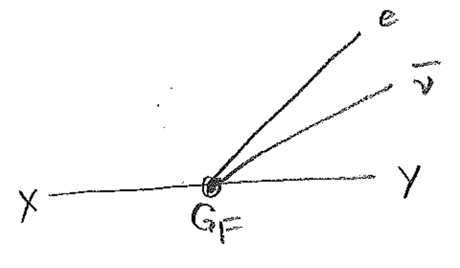
Large $m_W \longrightarrow$ small $A \longrightarrow$ small rate $\longrightarrow$ long mean life

Determine M using unit analysis (as before)

$$A \sim \frac{g^2}{8(m_W c)^2}\left(\frac{\hbar^2}{L^3}\right) = \frac{L}{L^3}\,\frac{1}{8}\left(\frac{g\hbar}{m_W c}\right)^2$$

1932 Fermi proposed the 1st theory of weak interactions

Knowing nothing of the W boson, he proposed the vertex



[N.B. 2 new particles created in the interaction]

He labelled the strength of the interaction G_F

determined from experiment to be [PPB]

$$\frac{G_F}{(\hbar c)^3} = 1.1664 \times 10^{-5}\ \text{GeV}^{-2}, \quad G_F = 8.9620\text{E-}5\ \text{MeV}\cdot\text{fm}^3$$

$$A \sim \frac{G_F}{L^3}$$ ⟶ compare of previous

The exact relationship between G_F and our previous constant turns out to be

$$G_F = \frac{1}{4\sqrt{2}}\left(\frac{g\hbar}{m_W c}\right)^2$$

[differs by $\sqrt{2}$ from previous $\frac{1}{8}\left(\frac{g\hbar}{m_W c}\right)^2$]

From this we can calculate the weak fine structure const

$$\alpha_W = \frac{g^2}{4\pi\hbar c} = \frac{4\sqrt{2}\,G_F\left(\frac{m_W c}{\hbar}\right)^2}{4\pi\hbar c} = \frac{\sqrt{2}}{\pi}\frac{G_F}{(\hbar c)^3}(m_W c^2)^2$$

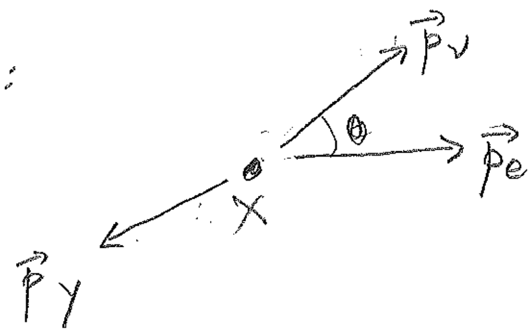
$$= \frac{\sqrt{2}}{\pi}(1.1664 \times 10^{-5})(80.38)^2 = 0.03392 = \frac{1}{29.48}$$

Let's calculate β-decay rate: $X \to Y + e^- + \bar{\nu}_e$

$Q = m_X - m_Y - m_e$

(does not incl. m_ν)

Rest frame of X:



3-momentum conservation determines $\vec{p}_Y = -\vec{p}_e - \vec{p}_\nu$

$$E_f^{tot} - E_i^{tot} = m_Y + T_Y + m_e + T_e + E_\nu - m_X$$

$$\simeq T_e + E_\nu - Q \qquad \text{(neglecting } T_Y\text{)}$$

Decay rate $R = \left(\frac{L}{h}\right)^6 \int d^3\vec{p}_e \, d^3\vec{p}_\nu \, \frac{2\pi}{\hbar} \delta(T_e + E_\nu - Q) \, |A|^2$

where $A = \frac{G_F}{L^3} M$

"matrix element"
depends on the structure
of the nuclei involved
(+ also e^- + $\bar{\nu}$ spin)

Could depend on T_e, T_ν, and θ

To proceed, we need to make some approximations.

Assume that M is relatively insensitive to θ, the angle between e^- and $\bar{\nu}_e$

Then we can integrate independently over the directions of e^- & $\bar{\nu}$

$$\int d^3\vec{p}_e = \int \vec{p}_e^{\,2}\, dp_e\, d\Omega = 4\pi \int \vec{p}_e^{\,2}\, dp_e$$

$$R = \left(\frac{L}{2\pi\hbar}\right)^6 \int 4\pi \vec{p}_e^{\,2}\, dp_e \; 4\pi \vec{p}_\nu^{\,2}\, dp_\nu \; \frac{2\pi}{\hbar}\, \delta(T_e + E_\nu - Q)\, \frac{G_F^2 |M|^2}{L^6}$$

$$= \frac{G_F^2}{2\pi^3 \hbar^7} \int \vec{p}_e^{\,2}\, dp_e \; \vec{p}_\nu^{\,2}\, dp_\nu\, \delta(T_e + E_\nu - Q)\, |M|^2$$

(Segre 9-5.4)
p.350

Next, assume ν is massless $\Rightarrow E_\nu = c|\vec{p}_\nu|$

$$R = \frac{G_F^2}{2\pi^3 \hbar^7 c^3} \int \vec{p}_e^{\,2}\, dp_e \underbrace{E_\nu^2\, dE_\nu\, \delta(T_e + E_\nu - Q)\, |M|^2}_{(Q-T_e)^2 |M|^2}$$

evaluated at $E_\nu = Q - T_e$

$$= \int dp_e \left(\frac{dR}{dp_e} \right)$$

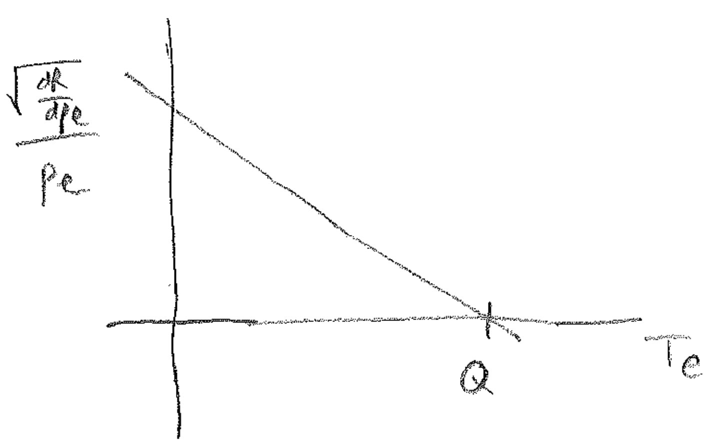
where $\frac{dR}{dp_e} = \frac{G_F^2}{2\pi^3 \hbar^7 c^3} \vec{p}_e^{\,2} (Q - T_e)^2 |M|^2 =$ rate of decay to an electron w/ momentum between p_e and $p_e + dp_e$

$$\frac{\sqrt{\frac{dR}{dp_e}}}{p_e} = \frac{G_F}{(2\pi^3 \hbar^7 c^3)^{1/2}} (Q - T_e) |M|$$

A plot of $\frac{\sqrt{\frac{dR_e}{dp_e}}}{p_e}$ vs T_e is called a Kurie plot.

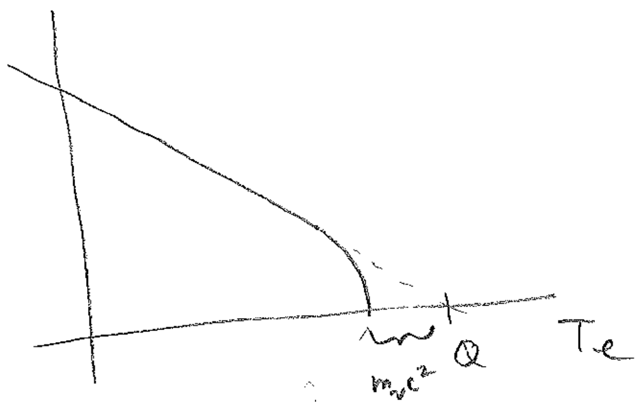
If $|M|$ is insensitive to T_e, the plot will be linear

In fact, Kurie plots are often linear, so the assumption about $|M|$ is good



If the neutrino has mass, then expect [Hw]

$$\frac{\sqrt{\frac{dR}{dp_e}}}{p_e} \sim (Q - T_e)\underbrace{\left[1 - \left(\frac{m_\nu c^2}{Q - T_e}\right)^2\right]^{\frac{1}{4}}}_{\text{vanishes if } T_e = Q - m_\nu c^2} |M|$$



(see plots)

$m_\nu < 0.3$ eV

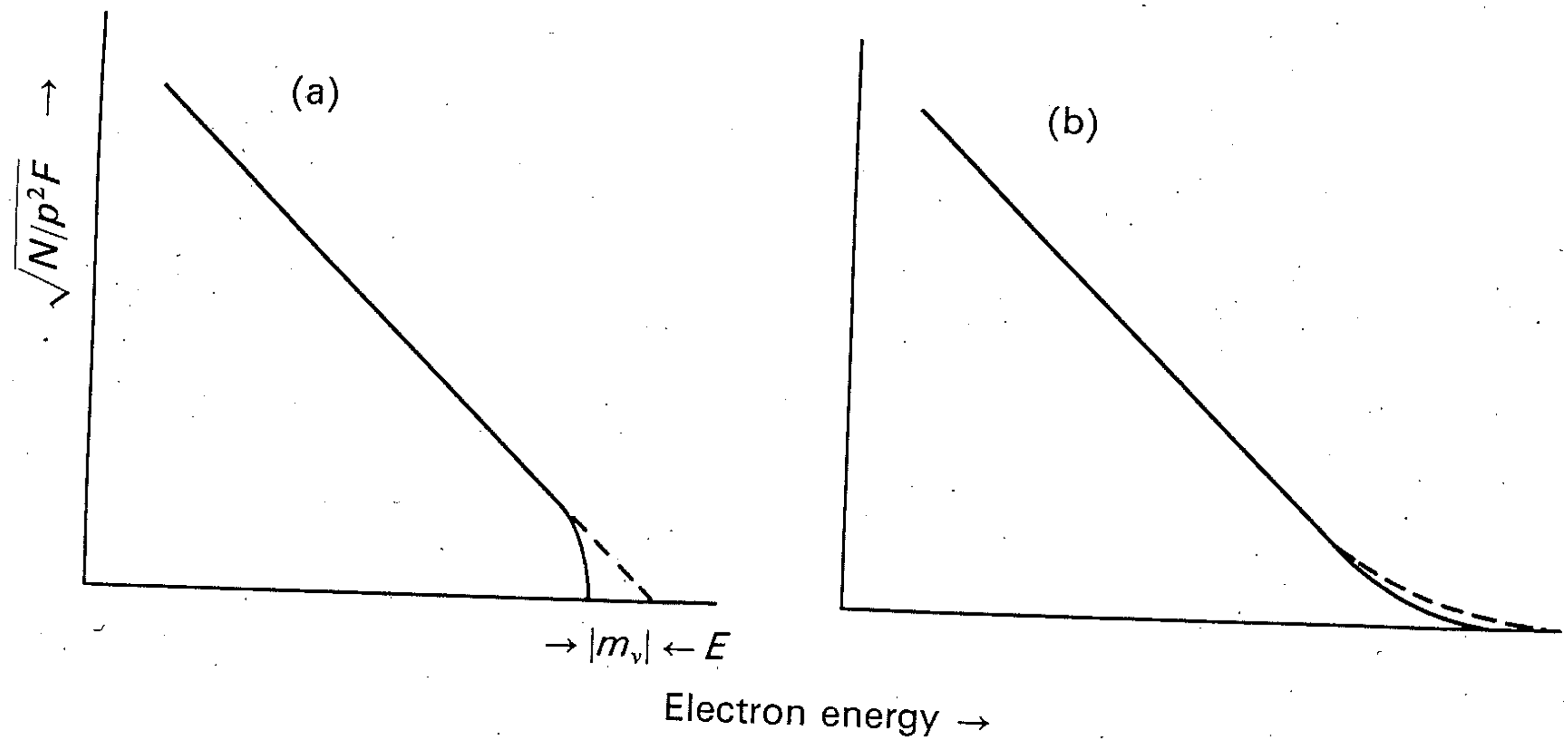


Fig. 6.2 Kurie plot of allowed transition for zero and finite neutrino mass: (a) pei resolution, (b) finite resolution.

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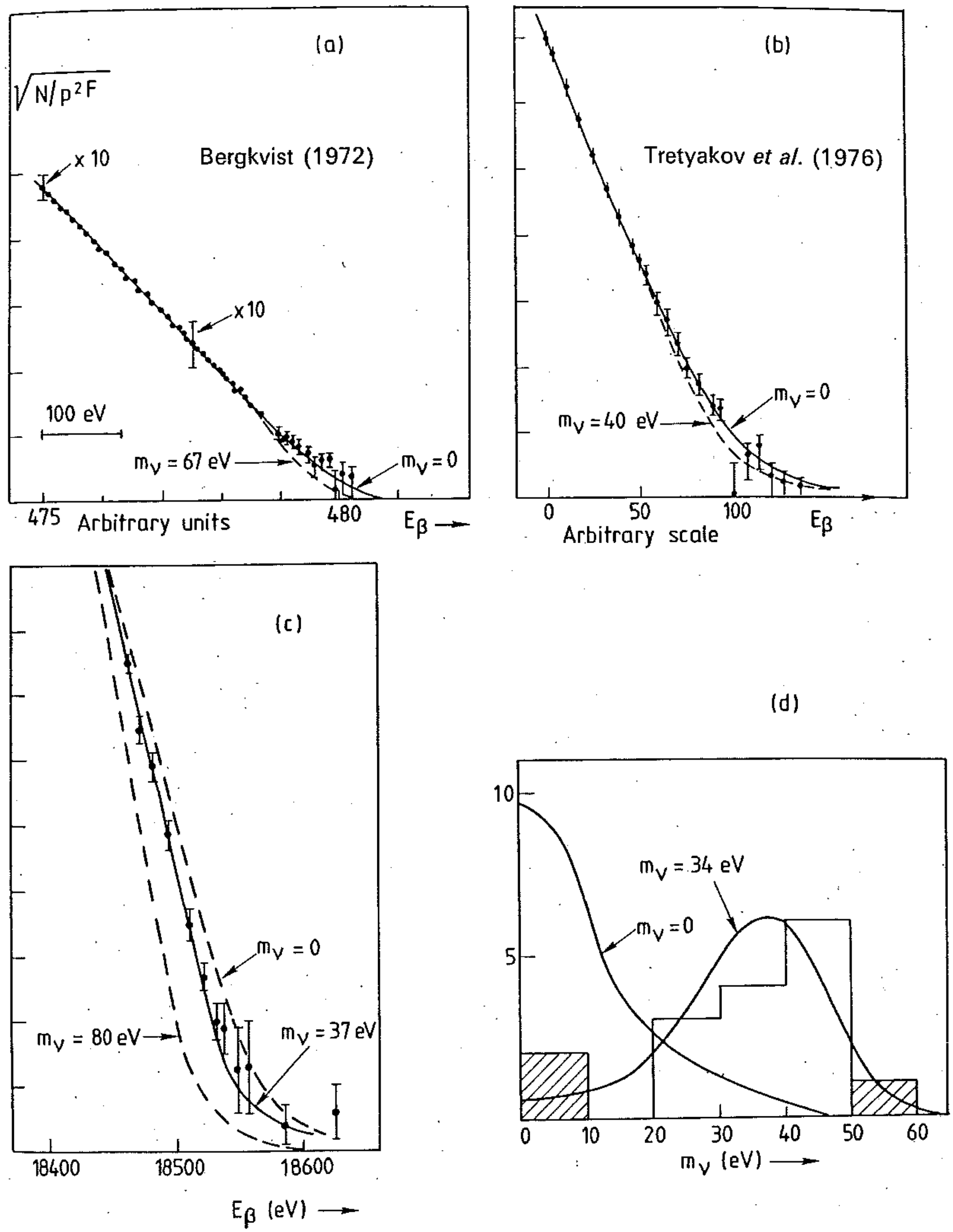

Fig. 6.3 Kurie plots from recent measurements of tritium β-decay. (a) Bergkvist (1972); (b) Tretyakov *et al.* (1976); (c) and (d) Lyubimov *et al.* (1980). (c) shows the average of several runs, and (d) the "best" mass estimate for m_ν from each of 16 runs. The expected distributions for $m_\nu = 0$ and $m_\nu = 35$ eV/c^2 are indicated.

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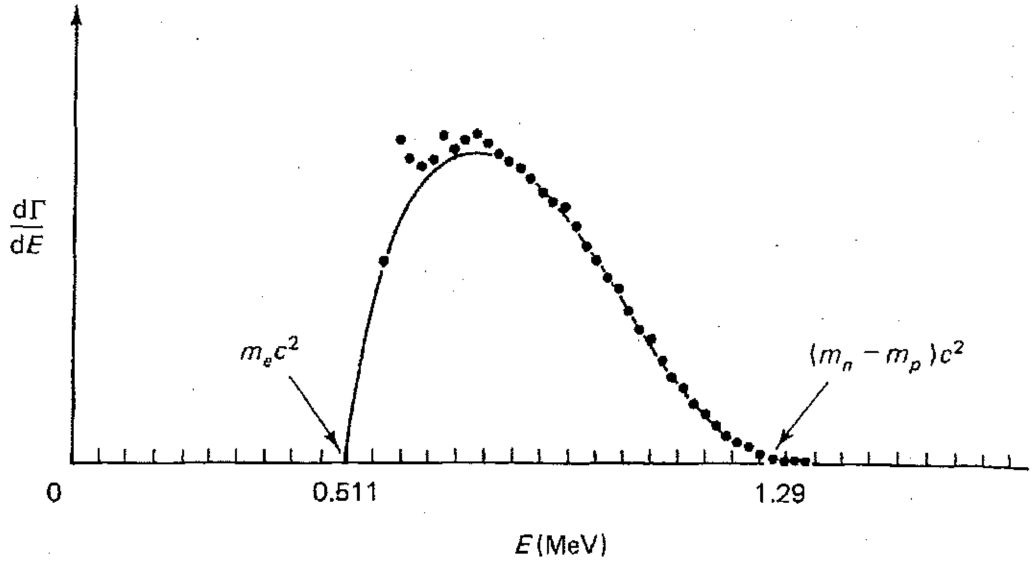
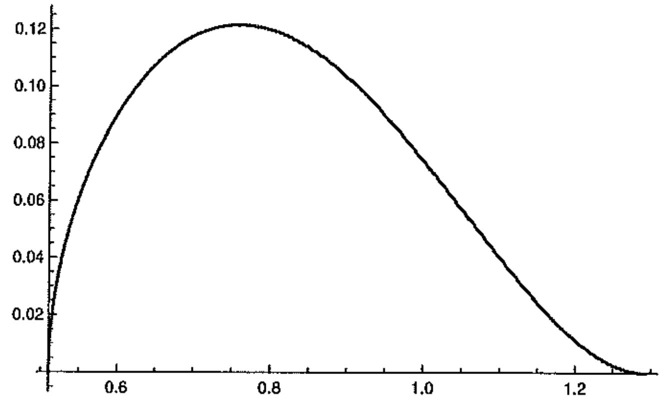


Fig. 9.2 Electron energy distribution from neutron beta decay. (Solid line is the theoretical curve; dots are experimental data.) (*Source*: Christensen, C. J. *et al.* (1972) *Physical Review*, **D5**, 1628. Figure (9.4).)

FermiIntegral.nb

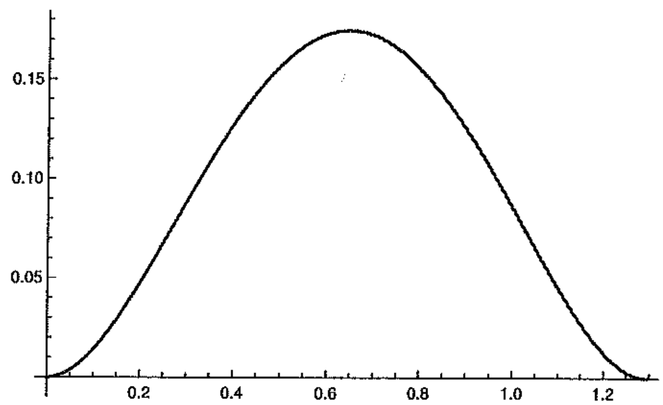
```
In[30]:= P[me_, del_] := Plot[e Sqrt[e^2 - me^2] (del - e)^2, {e, me, del}]
In[31]:= F[me_, del_] := Integrate[ e Sqrt[e^2 - me^2] (del - e)^2, {e, me, del}]
In[32]:= me = 0.511; del = 939.565 - 938.272;
In[33]:= P[me, del]
```

Out[33]=




```
In[34]:= F[.511, del]
Out[34]= 0.0569086
In[35]:= P[0, del]
```

Out[35]=




```
In[36]:= F[0, del]
Out[36]= 0.120468
```

Estimate decay rate, assuming $|M|$ is insensitive to E_e

$$R = \frac{G_F^2 \, |M|^2}{2\pi^3 \hbar^7 c^3} \int dp_e \, |\vec{p}_e|^2 \, (Q - T_e)^2$$

Recall $E_e^2 = (c\vec{p}_e)^2 - (m_e c^2)^2$

$E_e \, dE_e = c^2 |\vec{p}_e| \, dp_e$

$Q - T_e = m_x - m_y - m_e - T_e = m_x - m_y - E_e$

$$R = \frac{1}{2\pi^3 \hbar} \left[\frac{G_F}{(\hbar c)^3} \right]^2 |M|^2 \int_{m_e}^{m_x - m_y} dE_e \, E_e \sqrt{E_e^2 - (m_e c^2)^2} \, (m_x - m_y - E_e)^2$$

mean life $\tau = \frac{1}{R}$

Neutron decay

$n \rightarrow p + e^- + \bar{\nu}_e$

Numerically integrate

$$\int_{m_e}^{m_n - m_p} dE_e \, E_e \sqrt{E_e^2 - m_e^2} \, (m_n - m_p - E_e)^2 = 0.057 \ \mathrm{MeV}^5$$

If n were a fundamental spin-$\frac{1}{2}$ particle, then $|M| = 2$.

$$R = \frac{(1.166\mathrm{E}{-11}\ \mathrm{MeV}^{-2})^2}{2\pi^3 \, (6.6\mathrm{E}{-22}\ \mathrm{MeV \cdot s})} (2)^2 (0.057\ \mathrm{MeV}^5)$$

$= 7.6 \times 10^{-4}\ \mathrm{s}^{-1}$

$\tau = 1300\ \mathrm{s} \approx 22\ \mathrm{min}$

$\tau_{expt} \approx 15\ \mathrm{min}$ [close!]

(n is <u>not</u> a fund. spin $\frac{1}{2}$ particle)

cf p 320 Griffiths for refinements

Suppose we have a β-decay in which $Q \gg m_e c^2$.
Then generally $T_e \gg m_e c^2$ (except near endpoint)
and so electron is effectively massless.

The integral (*) then becomes

$$R = \frac{1}{2\pi^3 \hbar} \frac{G_F^2}{(\hbar c)^6} |M|^2 \underbrace{\int_0^Q dE_e \, E_e^2 (Q - E_e)^2}_{\frac{1}{30} Q^5}$$

$$= \frac{1}{60\pi^3 \hbar} \frac{G_F^2}{(\hbar c)^6} |M|^2 Q^5$$

This Q^5 dependence is called Sargent's rule
and shows that decay rate grows rapidly w/ Q
(due to increasing phase space!)

So whereas neutron has $\tau \approx 15$ min ($Q = 0.8$ MeV)
expect that other nuclei can decay much faster

=

Can use exptl data on half-~~lif~~ lives to measure $|M|^2$
& ∴ to explore nuclear structure

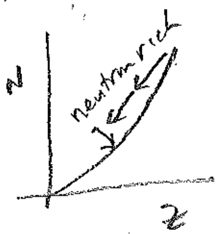
[β-decay gives indirect evidence for neutrinos
+ an upper bound (~ few MeV) on their mass.
Can one detect them directly?

Absorption cross-section for neutrinos $\sigma \sim 10^{-19}$ barns
[HW prob]

[which is ridiculously small (a very small barn)]

mean free path ~ light years. [HW prob: $l = \frac{1}{n\sigma} \sim 10^{18}\,m \sim 100\,ly$]

[To have any hope of detecting ν, need a lot of them.
Nuclear reactors produce a lot of them.]



$n \to p + e^- + \bar{\nu}$

Fission products copiously produce $\bar{\nu}$

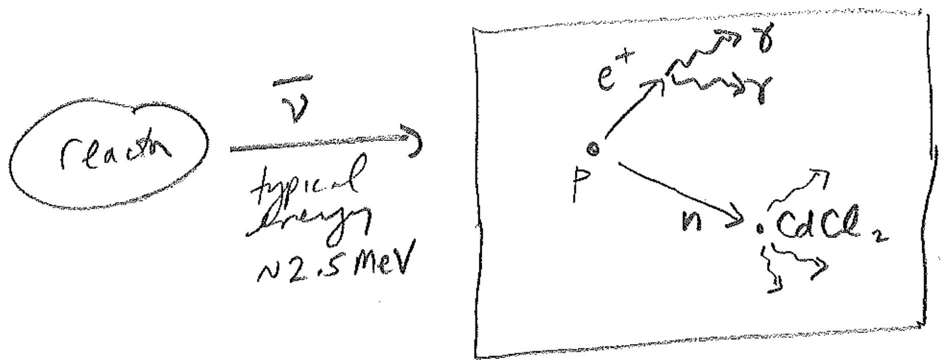
Absorption of antineutrinos

$$\bar{\nu} + p \to n + e^+$$

provided E_ν is sufficiently large

1956 Cowan-Reines antineutrino detection experiment
at Savannah River commercial reactor (S. Carolina)

[show picture]

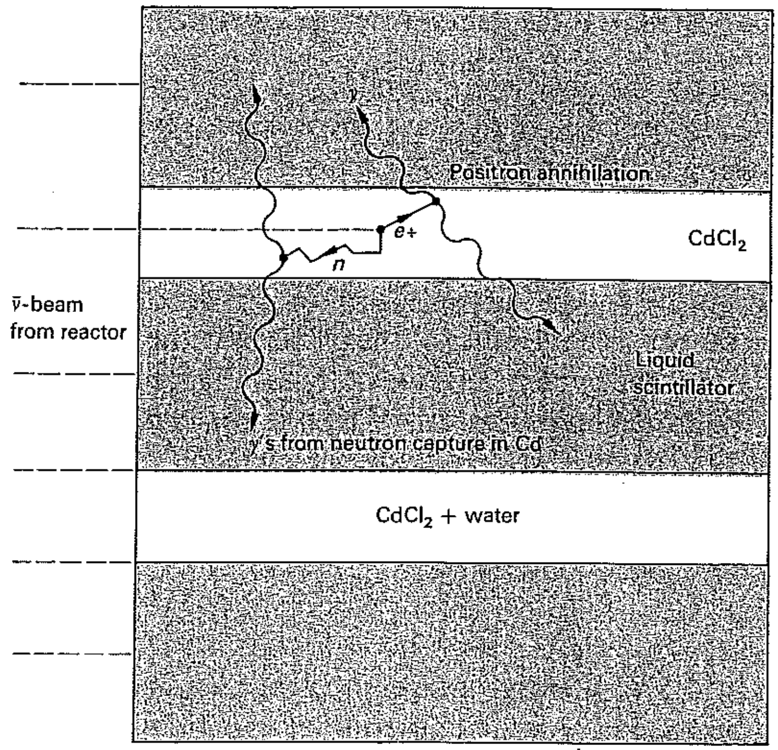


[recall cadmium good at absorbing neutrons]

experimental signature: 2 0.511 MeV γ
followed by 3 to 4 γ w/ $E_{tot} \sim 9$ MeV
after $\sim 10^{-6}$ s.

given in HW problem (no need to write) → [reactor flux $F \sim 1.2 \times 10^{17} \frac{\bar{\nu}}{m^2 \cdot s}$
detector efficiency $\epsilon \sim 0.025$

[HW: calculate # events per day]



Positron annihilation
CdCl$_2$
$e+$
n
$\bar{\nu}$-beam
from reactor
Liquid
scintillator
γ's from neutron capture in Cd
CdCl$_2$ + water

183 cm x 132 cm

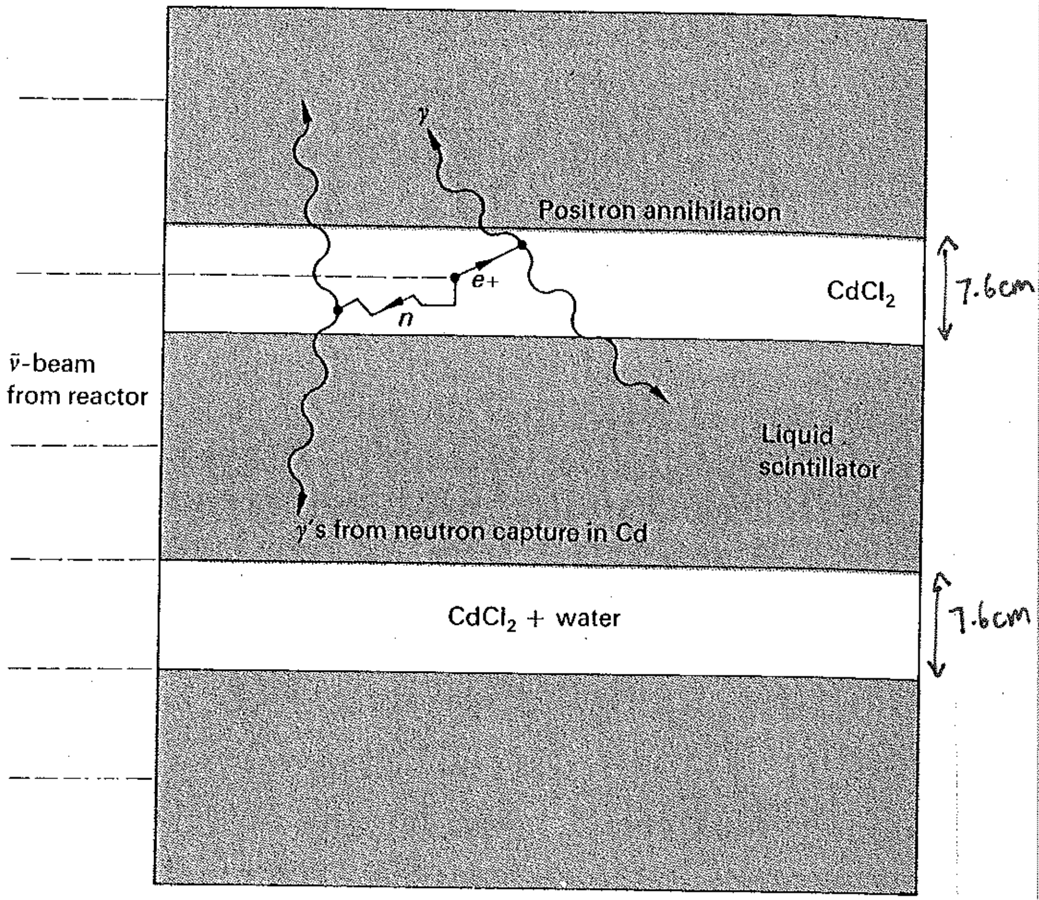


Fig. 6.4 Schematic diagram of the experiment by Reines and Cowan (1959), interactions of free antineutrinos from a reactor.

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water volume ≈ 400,000 cm^3

6' $\quad$ $4\frac{1}{3}$'

183cm x 132cm x 2(7.6cm) = 367,000 cm^3

Asimov: $6\frac{1}{4}$' x $4\frac{1}{2}$' x 6''

What about ν detection?

$$\nu + n \longrightarrow p + e^-$$

but no free neutrons. Neutrons must be in nuclei

$$\nu + {}^{A}_{Z}Y \longrightarrow {}^{A}_{Z+1}X + e^-$$

e.g. $\quad \nu + {}^{37}_{17}Cl \longrightarrow {}^{37}_{18}Ar + e^- \qquad Q = -0.814\,\text{MeV}$

Ray Davis experiment (1955) [F + H, p. 187]

[Davis + Harmon, 1959]

$\nu \longrightarrow$

Drycleaning fluid
CCl_4 carbon tetrachloride
C_2Cl_4 perchloroethylene

^{37}Ar [flush out + concentrate]
(unstable w/ $T_{\frac{1}{2}} = 35$ days)

$^{36,38,40}Ar$ stable

EC: $\quad e^- + {}^{37}_{18}Ar \longrightarrow {}^{37}_{17}Cl + \nu \qquad (Q = 0.814\,\text{MeV})$

(Auger electrons)

Absence of such reactions demonstrated ~~that~~ ν and $\bar{\nu}$ are distinct

Lepton # conservation [Konopinski, Mahmoud 1953]

$L(e^-) = L(\nu) = 1$

$L(e^+) = L(\bar{\nu}) = -1$

$\rightarrow$ Solar neutrino problem!

Using relativistic field theory approach

Srednicki

$$\langle f|i\rangle = (2\pi)^4\, \delta^4(\Sigma k)\; iT$$

$$\langle f|i\rangle^2 = VT\; (2\pi)^4\, \delta^4(\Sigma k)\, |T|^2$$

$$\langle i|i\rangle = 2mV$$

$$\langle f|f\rangle = 2E_i V$$

$$\text{Prob} = \frac{|\langle f|i\rangle|^2}{\langle f|f\rangle\langle i|i\rangle} = \frac{T(2\pi)^4\, \delta^4(\Sigma k)\, |T|^2}{2m \quad \prod 2E_i V}$$

$$d\Gamma = \frac{\text{prob density}}{\text{time}} = \frac{(2\pi)^4\, \delta^4(\Sigma k)\, |T|^2}{2m} \prod \frac{\frac{V}{(2\pi)^3}\int d^3k_i}{2E_i V}$$

$$= \frac{(2\pi)^4\, \delta^4(\Sigma k)\, |T|^2}{2m} \left(\prod_i \int d\tilde{k}_i\right) \qquad d\tilde{k}_i = \frac{d^3k_i}{(2\pi)^3(2E_i)}$$

$$X \to Y e^- \bar{\nu}_e$$

Y gets momentum but no energy $\Rightarrow E_y = m_y$

Griffiths
Golden Rule for decays

$$d\Gamma = \frac{(2\pi)\; |T|^2}{(2m_X)(2m_Y)} \underbrace{\int \frac{d^3k_e}{(2\pi)^3 2E_e} \int \frac{d^3k_\nu}{(2\pi)^3 2E_\nu}} \;\delta(K_e + E_\nu - Q)$$

$$\frac{k_e^2\, dk_e}{2\pi^2(2E_e)} \qquad \frac{k_\nu^2\, dk_\nu}{4\pi^2(2E_\nu)}$$

This essentially same as non-relativistic calculation (Fermi Golden rule) except for factors of $(2m_X)(2m_Y)(2E_e)(2E_\nu)$. But the relativistic $|T|^2 \approx (\underbrace{2m_X\, 2m_Y}_{\text{eq (12.19)}}\; \underbrace{2E_e 2E_\nu}_{\text{eq (12.21)}})$(angular dependence) so these cancel out.

Hadron Motion