Nuclear transmutation

Rutherford & collaborators used α particles emitted by Po & Ra w/ typical $T = 5$ MeV to probe atomic structure.

For heavy targets (Z_2 large), Coulomb barrier prevents α from getting too close to nucleus, which therefore behaves as a point charge w/ $\frac{d\sigma}{d\Omega} \sim \frac{1}{\sin^4 \frac{\theta}{2}}$.

Scattering from lighter nuclei begins to depart from this suggesting a finite size for nucleus.

If α gets close enough (~ 1 fm), strong force can cause some of its particles to be absorbed

Nuclear transmutation (Rutherford, 1919)

$$\alpha + {}^{14}_{7}N \longrightarrow {}^{17}_{8}O + p$$

Cockcroft/Walton designed a linear accelerator to accelerate protons to ~ 0.5 MeV.
With this, they "split the atom" (1932)

$$p + {}^{7}Li \rightarrow \alpha + \alpha$$

Chadwick discovers the neutron (1932)

$$\alpha + {}^{9}_{4}Be \rightarrow {}^{12}_{6}C + n$$ [also Bothe & Becker 1930]

Artificially radioactive isotopes (Joliot & I. Curie, 1934)

$$\alpha + {}^{27}_{13}Al \rightarrow {}^{30}_{15}P + n$$

$$\downarrow 3 \text{ minutes}$$

$$ {}^{30}_{14}Si + e^+ + \nu_e$$

Neutrons as probes [no Coulomb barrier]

$$n + {}^{16}_{8}O \longrightarrow {}^{17}_{8}O \longrightarrow {}^{13}_{6}C + \alpha$$ (Meitner & Philipp)

$$n + {}^{19}_{9}F \rightarrow {}^{20}_{9}F \longrightarrow {}^{16}_{7}N + \alpha$$ (Fermi & Rasetti)

$$\downarrow$$

$$ {}^{16}_{8}O + e^- + \bar{\nu}_e$$

Many artificially radioactive isotopes produced [nothing below fluorine]

even beyond uranium

$$n + {}^{238}_{92}U \longrightarrow {}^{239}_{92}U \longrightarrow {}^{239}_{93}Np + e^- + \bar{\nu}_e$$

$$\downarrow$$

$$ {}^{239}_{94}Pu + e^- + \bar{\nu}_e$$

It was found that for medium energy neutrons ($T \approx 20\,\mathrm{MeV}$),

the absorption cross section

$$\sigma \sim \pi R_X^2$$

Suggests most neutrons are absorbed by an nucleus they strike

Recall $R_X \sim A^{1/3} r_0$ where $r_0 = 1.2$ fm

$$\sigma \sim A^{2/3} \pi r_0^2$$

Uranium ($A = 238$) $\Rightarrow$ $\sigma \sim (13\ \mathrm{fm})^2$

Nuclear physicists define

$$1 \text{ barn} = (10\ \mathrm{fm})^2 = 100\ \mathrm{fm}^2 = 10^{-28}\ \mathrm{m}^2$$

"broadside of a barn"

uranium $\sim$ 1.7 barns

For higher energies, $\sigma \downarrow$ $\Rightarrow$ not all neutrons are absorbed

"partial translucency"

[cf graph from Evans]

R.D. Evans, p. 458

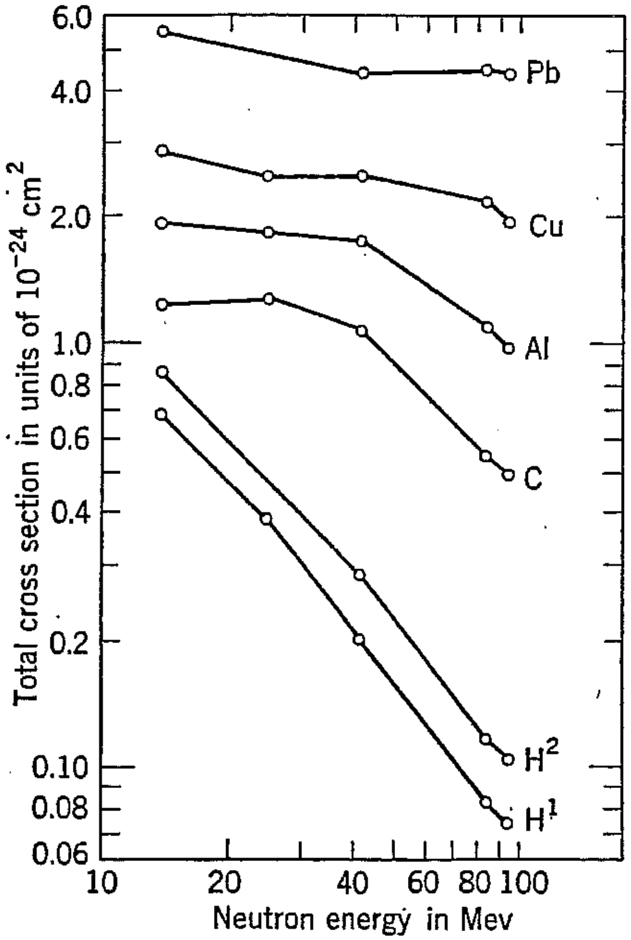
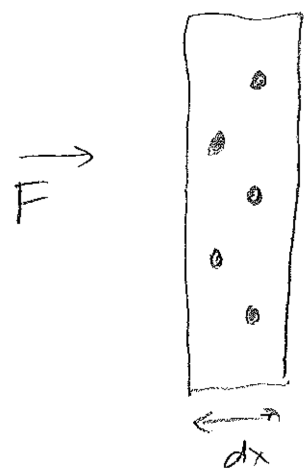


Fig. 2.3 Total neutron cross sections σ_t of five elements for neutrons of 14, 25, 42, 84, and 95 Mev. The decrease in σ_t at the higher neutron energies suggests that all neutrons which strike the target area $\pi(R + \lambda\!\!\!^{-})^2$ are not captured and that nuclear matter is slightly transparent to swift neutrons. [*From Hildebrand and Leith* (H51).]

Let incident flux of neutrons be F



Probability of absorption by nuclei in film of thickness dx

$f = n\sigma(dx)$ $\quad n = \#$ density of nuclei

$\Rightarrow$ Flux correspondingly diminishes

$$dF = -(n\sigma\, dx)\,F$$

$$\frac{dF}{F} = -n\sigma\, dx$$

$$\ln F = -n\sigma x + \ln F_0$$

$$F = F_0\, e^{-n\sigma x} = F_0\, e^{-\frac{x}{l}}$$

$l = \frac{1}{n\sigma}$ = mean free path of neutrons

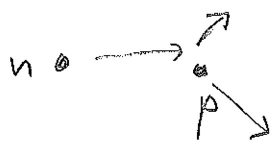
$n = \frac{\rho}{A} N_A$ $\quad \rho$ = mass density

$U \to \rho = 19\ \frac{g}{cm^3} \Rightarrow n = 4.8E28\ m^{-3}$

$\sigma = 1.7E-28\ m^2$

$l \simeq 0.12\ m$

Neutrons, being neutral, cannot be accelerated or steered but they can be slowed down by collisions, or by absorption + re-emission



Fermi found that absorption $\sigma \uparrow$ for lower energy n

[see graph for nickel]

Cadmium ~ 2000 barns for slow neutrons

[empirically $\sigma \sim \frac{1}{v}$]

R. D. Evans, p. 455

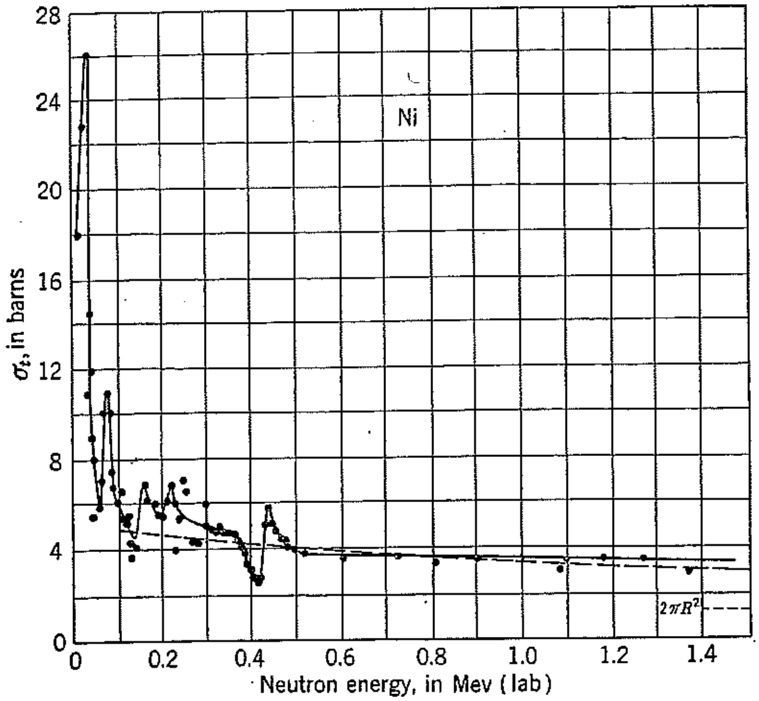


Fig. 2.1 The total cross section (absorption plus scattering) of nickel for neutrons from 0.01 to 1.5 Mev. Approximately monoenergetic neutrons (± 20 kev below 0.5 Mev; ± 150 kev above 0.5 Mev) were produced by bombarding a thin Li target with protons accelerated by an electrostatic generator. The dotted line is from the continuum theory of Feshbach and Weisskopf (F49), assuming a nuclear radius of 4.6×10^{-13} cm. [*The experimental data are from Barschall, Bockelman, and Seagondollar* (B15).]

R.D Evans, p. 443

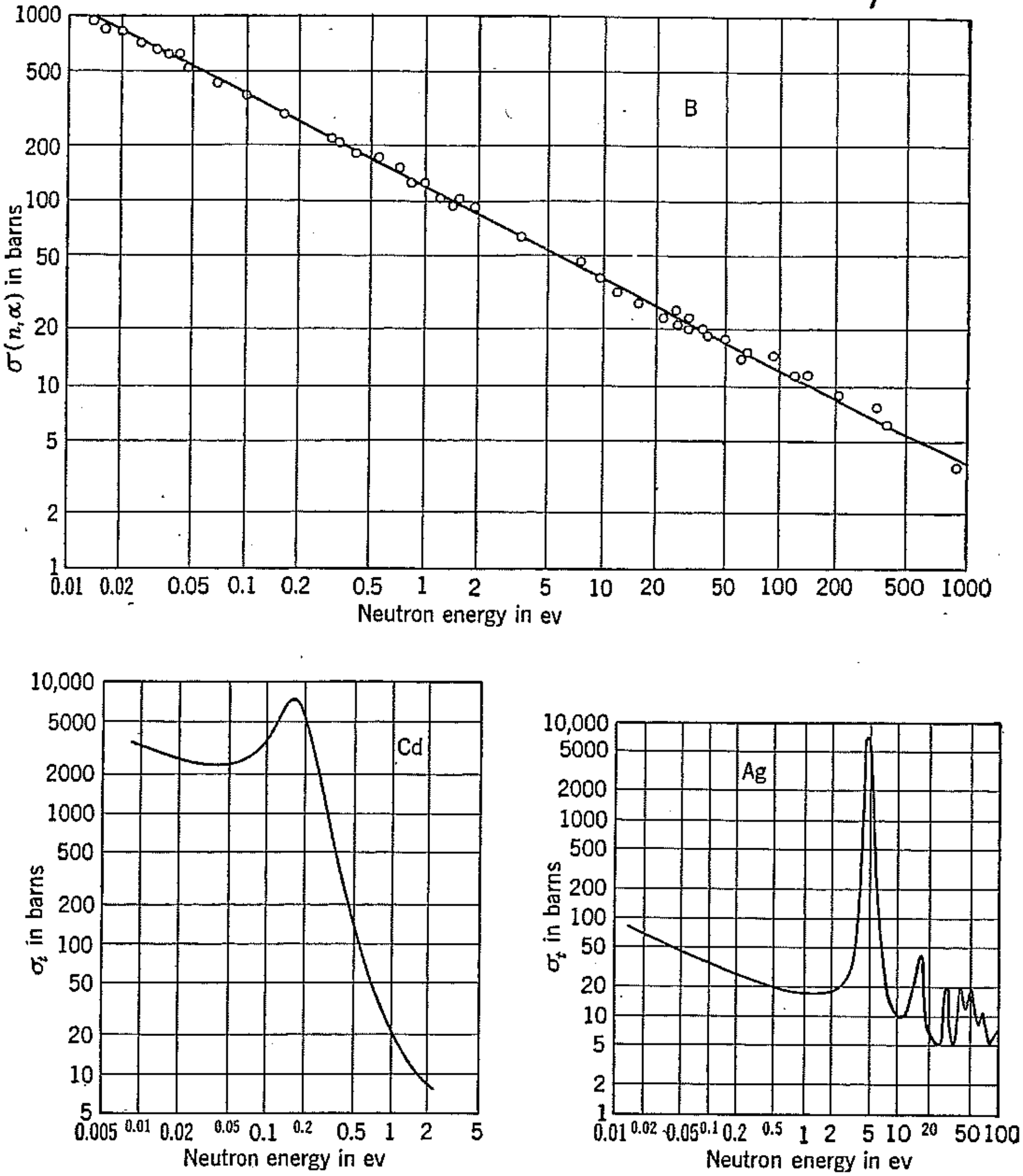


Fig. Introduction.1 Total neutron cross sections for B, Cd, and Ag, illustrating typical combinations of $1/V$ dependence (pure, in the case of boron) and slow-neutron resonances. The resonance in Cd at $E_0 = 0.176$ ev has a total width $\Gamma = 0.115$ ev and a maximum value of $\sigma_0 = 7{,}200$ barns. The curves are for the normal isotopic mixtures. In Cd, the 0.176-ev resonance is due entirely to Cd^{113} (relative abundance, 12.3 per cent); hence $\sigma_0 \simeq 57{,}000$ barns for Cd^{113} alone. The complex γ-ray spectrum leading to the ground level of Cd^{114} has been measured by Bartholomew and Kinsey (B16). [*From Goldsmith, Ibser, and Feld* (G29) *and Hughes* (H68).]

[skipped in 2019]

fin 6

Why? wave properties of neutrons

de Broglie wavelength $\lambda = \frac{h}{p}$

Define reduced wavelength $\lambda\!\!\!^{-} = \frac{\lambda}{2\pi} = \frac{\hbar}{p}$

Consider non-relativistic neutrons ($T \ll m_n c^2 \sim 1000$ MeV)

$$T = \frac{p^2}{2m} = \frac{1}{2m}\left(\frac{\hbar}{\lambda\!\!\!^{-}}\right)^2 = \frac{1}{2mc^2}\left(\frac{\hbar c}{\lambda\!\!\!^{-}}\right)^2 = \frac{1}{2(1000\,\text{MeV})}\left(\frac{200\,\text{MeV fm}}{\lambda\!\!\!^{-}}\right)^2$$

$$= (20\,\text{MeV})\left(\frac{1\,\text{fm}}{\lambda\!\!\!^{-}}\right)^2$$

$$\frac{\lambda\!\!\!^{-}}{1\,\text{fm}} = \sqrt{\frac{20\,\text{MeV}}{T}}$$

For $T \gtrsim 20$ MeV, $\lambda\!\!\!^{-} < 1$ fm, neutron acts as a particle
($\lambda\!\!\!^{-} \ll R$)

Small T, $\lambda\!\!\!^{-} \gg R$, acts like a wave, ~~or~~ ϕ

Neutrons of $T \sim \frac{3}{2}kT \sim 0.05$ eV are called "thermal neutrons"

1938 Hahn & Strassmann (chemists)
thought they observed

$$n + {}_{92}U \xrightarrow{?} {}_{88}Ra$$

but no physical process could explain this

- why did they think it was radium?
- Because it had similar chemical properties to Ba
  and a radioactive product was precipitating out of soln w/ Ba
- But they then tried to separate Ra from Ba and failed
- What did they do? consulted a physicist.
  Former colleague Lise Meitner
  an Austrian Jew.
  When Hitler annexed Austria, she lost her citizenship
  In 1938, she escaped to Sweden.
  She discussed it w/ nephew Otto Frisch
  and decided it <u>was</u> Ba.

1939 Meitner & Frisch propose

$$n + U \longrightarrow \underbrace{Ba}_{\text{radioactive}} + \underset{?}{Kr}$$

"nuclear fission"

Why didn't anyone think of this before?

$\frac{B}{A}$ for U ~ 7.5 MeV

$\frac{B}{A}$ for Ba, Kr ~ 8.2 MeV

about 0.7 MeV released per nucleon

⇒ ~170 MeV per fission.

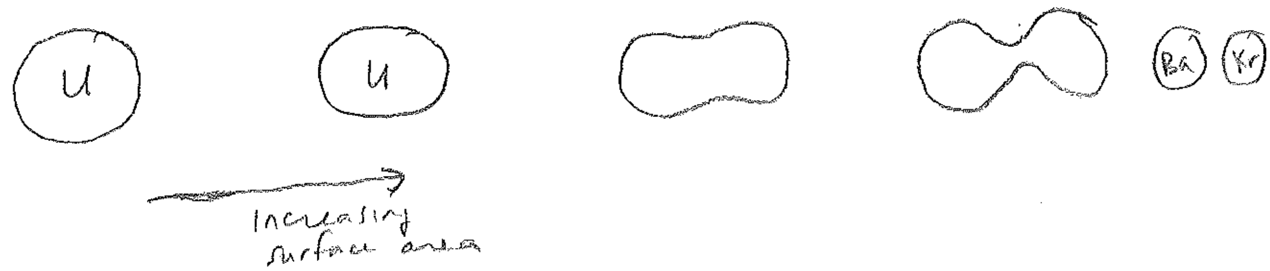
⇒ kinetic energy of decay products

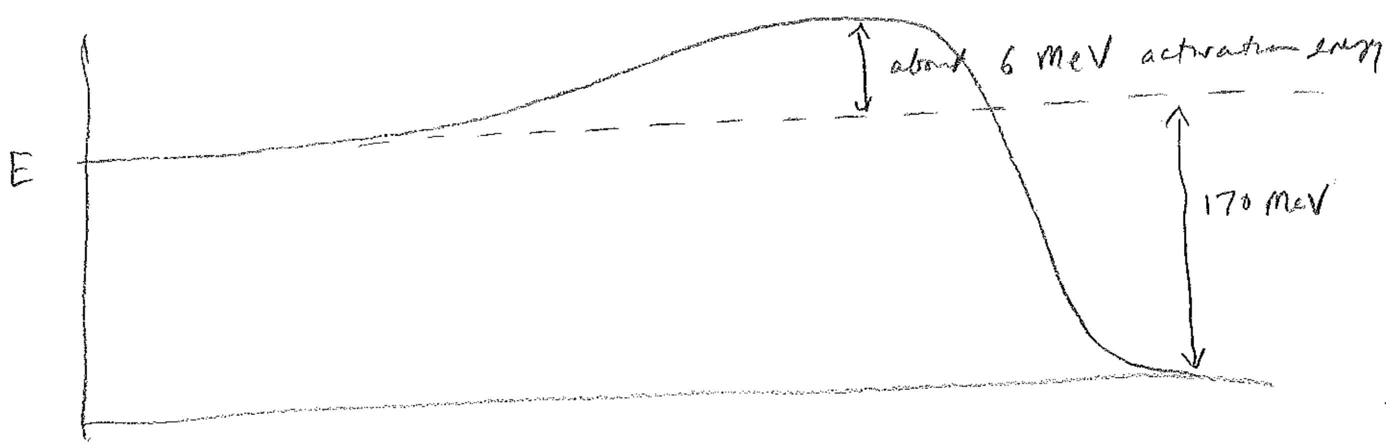
Frisch in his lab verified energy of fragments.

If U less stable than Ba, Kr why doesn't it just decay?

Think α-decay. Coulomb barrier holds them in.

Alternative picture. Liquid drop model (surface tension)





Tunnelling through barrier can occur but is rare

"spontaneous fission"

$T_{\frac{1}{2}} \sim 3.5 \times 10^{17}$ years [HKR, p. 1169]

(α-decay is more likely)

Incident neutron supplies energy

$$n + {}^{238}U \longrightarrow {}^{239}U$$

$n \longrightarrow$ (U) (U) wobble

$$Q = \Delta({}^{238}U) + \Delta(n) - \Delta({}^{239}U) = 4.8\ \text{MeV}$$

$$\begin{bmatrix} 47.306 \\ 8.071 \\ -50.571 \\ \hline 4.806 \end{bmatrix}$$

energy released causes drop to wobble
but activation energy of ^{239}U is 6.6 MeV

[?5.7 MeV in HRK, 1156]

so neutrons must have $T > 1.8$ MeV
to initiate fission

"fast neutron fission of ^{238}U", $\frac{v}{c} \sim \sqrt{\frac{2T}{mc^2}} \sim 0.06$

But Hahn & Strassmann were using slow (thermal) neutrons
($T < 1\,\text{eV}$)

1939 Bohr realized that they were observing
"slow neutron fission of ^{235}U"

^{238}U 99.3%
^{235}U 0.7%

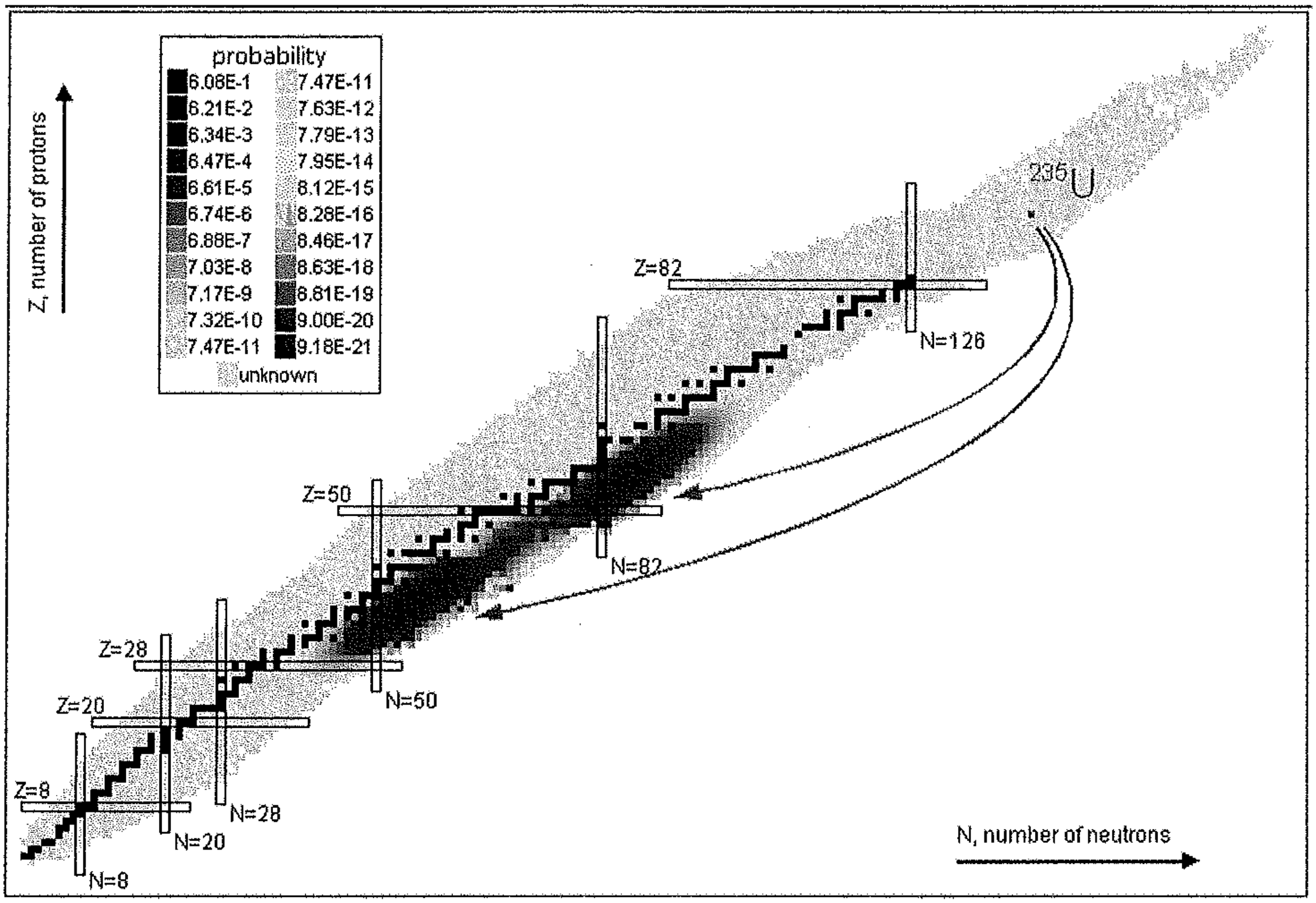
$$n + {}^{235}U \longrightarrow {}^{236}U$$

$Q = 6.5$ MeV $\Longrightarrow$ why difference? even $\to$ odd vs odd $\to$ even

$$\begin{bmatrix} 40.916 \\ 8.071 \\ -42.442 \\ \hline 6.545 \end{bmatrix}$$

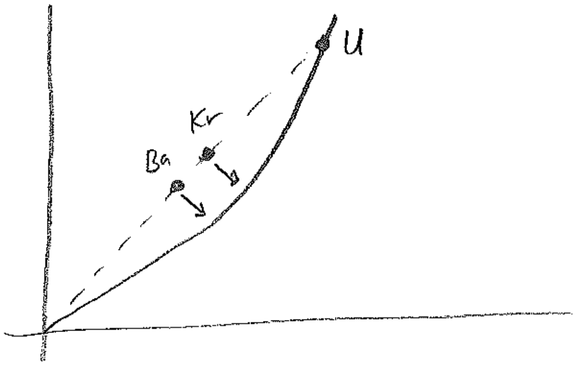
and activation energy of ^{236}U is 6.2 MeV
no additional kinetic energy required!

[? 5.2 in HRK]



labDemmingLounge ~~blogspot~~ blogspot.com

[Why are Ba + Kr radioactive? Because they are neutron rich]



Fission often releases 2 or 3 neutrons in addition to fission fragments

For example

$$n + {}^{235}_{92}U \xrightarrow{e.g.} {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3n$$

$$\xrightarrow{e.g.} {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2n$$ [note: asymmetric]

Released neutrons can initiate further fissions

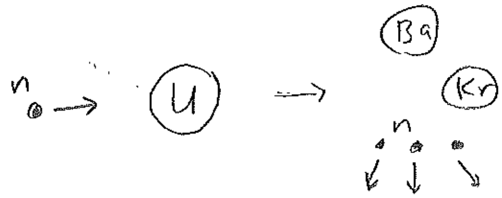
$\Rightarrow$ chain reaction

controlled $\Rightarrow$ nuclear reactor
uncontrolled $\Rightarrow$ weapons

about 200 MeV released per fission
vs 1-10 eV released in chemical reactions [$\Rightarrow$ factor of 10^7 to 10^8]

First controlled nuclear reaction
12/2/1942 U of Chicago squash courts (Fermi)

$$n + {}^{235}U \longrightarrow {}^{236}U \xrightarrow{e.g.} {}^{141}Ba + {}^{92}Kr + 3n$$



Secondary neutrons
avg # ~ 2.47/fission
avg T ~ 2 MeV
(some are delayed)

escape through surface w/o causing fission
→ solution: Increase volume to surface ratio (critical mass)
→ depends on mean free path

fast n fission of ^{238}U
[possible since $\bar{E} > 1.8$ MeV]
occurs < 20% of time
[because of low cross-section for fast n absorption]
→ not sustainable
[need at least 40%]

slowed by inelastic collisions w/ ^{238}U
(> 80% of time)
→ captured: re-emitted w/ E < 1 MeV
→ slow neutrons absorbed by $^{238}U \to {}^{239}U$ w/o subsequent fission
(Between 1-100 eV) capture resonance

Slowed by a moderator (water, heavy water, ultrapure graphite) below 1 eV
→ Slow n fission of ^{235}U w/ thermal neutrons (0.04 eV)

[Need: critical mass, and a pure moderator]

k = multiplication factor = # secondary neutrons that go on to produce subsequent fission

$k < 1$ subcritical
$k = 1$ critical (self-sustaining)
$k > 1$ supercritical

$$P(t) = P_0 k^{\frac{t}{t_{gen}}}$$

Use neutron absorbers to keep reaction from going supercritical
eg Cadmium control rods

[Richard Rhodes, Making of the Atomic Bomb]

Just talk about this :

fi-13

[What if want a supercritical rxn, ie a bomb?]

- slow n fission not suitable, energy release causes U to expand & ∴ n's will all escape
- fast n fission of ^{238}U impossible because $k<1$.
- fast n fission of ^{235}U __is__ possible

- enrichment: electromagnetic separation } Oak Ridge
  gaseous diffusion } ($2 Billion)

Little Boy ~ 80% ^{235}U (about 50 kg)

(critical mass ~10 cm)

$$[\; n + {}^{238}U \rightarrow {}^{239}U' \rightarrow {}^{239}Np \rightarrow {}^{239}Pu \;]$$

^{239}Pu is fissionable

[Fat Man]

Nuclear physics

Meitner biography (Sime)
(also Philip Ball: serving the Reich)

1919 Rutherford induced nuclear rxns w/ α

$$\alpha + {}^{14}N \rightarrow {}^{17}O + p$$

but not until 1924 was it realized that α was absorbed, not scattered. (Blackett). These rxns ⇒ existence of a strong attractive force that could overcome Coulomb repulsion.

Nuclei heavier than K never absorbed α's, however, nor did Li, Be, C, O.

→ ~~But~~ (1930) Bothe & Becker discovered a new penetrating type of radiation from α (from Po) bombardment of Be, Li, B. Joliot & Irene Curie (1932) found that the radiation ejected p from paraffin, & interpreted it as γ, but Chadwick proved it was really neutrons (1932)

$$\alpha + {}^{9}Be \rightarrow {}^{12}C + n$$

→ Then $n + {}^{14}_{7}N \rightarrow {}^{11}_{5}B + \alpha$ was observed by Norman Feather

$n + {}^{16}_{8}O \rightarrow {}^{13}_{6}C + \alpha$ " " " Meitner & Philipp

1934 Fermi theory of β-decay, which used Pauli's ν idea

1934 Joliot & Curie produced artificial radioactivity

$$\alpha + {}^{27}Al \rightarrow {}^{30}P + n$$

$$\underset{3\text{ minutes}}{\hookrightarrow} {}^{30}Si + e^{+} + \nu$$

1934 Fermi ~~[illegible]~~ & Rasetti bombarded elts w/ n producing artificial radioactivity of F, Al, Si, P, Cu, Zn ~~all~~ up thru U

$$n + {}^{238}U \rightarrow {}^{239}U \rightarrow {}^{239}Np \rightarrow {}^{239}Pu$$

this was hypothesized but not proved

Fermi observed (n, α) rxns & (n, p) rxns any lighter elts

$n + {}^{27}Al \to {}^{24}Na + \alpha$

$n + {}^{28}Si \to {}^{28}Al + p$

and (n, γ) rxns any heavier

$n + {}^{127}I \to {}^{128}I + \gamma \to {}^{128}Xe + e^- + \bar{\nu}$

and that the radioactive nuclei always emitted e^-

~~Meitner + Hahn decided to start studying transuranics~~

Szilard + Chalmers observed

$\gamma + {}^{9}Be \to \quad + n$ followed by $n + {}^{127}I \to {}^{128}I$

~~that~~ Meitner found that the (lower energy) n could not be absorbed by Al, Na, Si which required higher energies to split off an α or p's but only in capture processes by heavier nuclei Ag, I, Au

She suggested that slower neutrons are more easily absorbed than fast neutrons

1934 Fermi showed that paraffin, etc. slowed the n, after which they were more easily absorbed by Ag, Cu, I, ~~…~~

Uranium enrichment

Manh. project { → electromagnetic separation
→ gaseous diffusion

→ ~~large gas~~ gas centrifuge (most common & economical)

3–5% ^{235}U ⇒ power reactor

70–90% ^{235}U ⇒ weapons

Hiroshima ⇒ 64 kg of ^{235}U ⇒ 15 kilotons
could do it w/ 25 kg (size of a melon)

Trinity ⇒ 6 kg of ^{239}Pu ⇒ 20 kilotons
could do it w/ 5 kg or less (size of a plum)