Radioactivity

emission of particles by unstable nuclei

1896 Becquerel discovers "uranic rays" from uranium

1898 Marie Curie identifies new elements (Po, Ra) by their distinctive half-lives

1899 Rutherford discovers different types of radioactivity by the curvature of their tracks in a magnetic field (cloud chamber: supersaturated vapor condenses on ions)

[see illustration]

γ = neutral = high energy photons ($E > 1$ keV)

β = negatively charged = electrons

α = positively charged = ^4He nuclei

[1909 Royds, Rutherford showed this in Manchester]

[α particles from radioactive nuclei in crust accumulate in natural gas deposits, from which it can be extracted. Once in atmosphere, it escapes.]

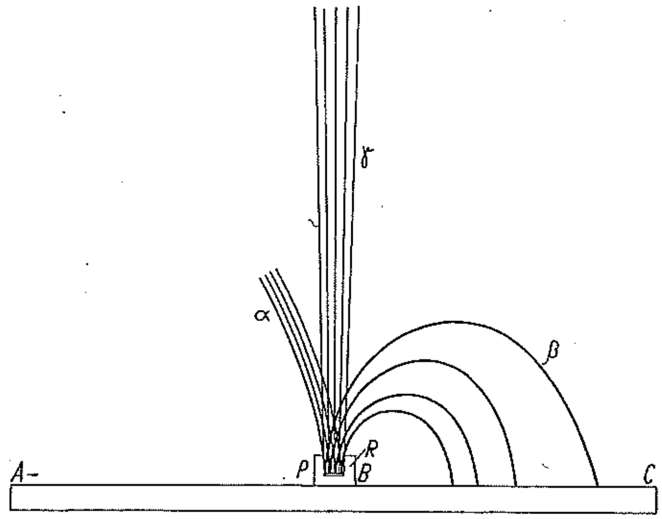


Figure 1-2 Deflection of alpha, beta, and gamma rays in a magnetic field. The nomenclature is due to Rutherford (1899). [Mme Curie, Thesis, 1904.]

Segrè, p. 3

$${}^{A+4}_{Z+2}X \longrightarrow {}^{A}_{Z}Y + {}^{4}_{2}He$$

$$Q = m_{nucl}(X) - m_{nucl}(Y) - m_{nucl}({}^4He)$$

$$= \left[(A+4)u + \Delta(X) + (Z+2)m_e\right] - \left[Au + \Delta(Y) + Zm_e\right] - \left(4u + \Delta(He) - 2m_e\right)$$

$$= \Delta(X) - \Delta(Y) - \Delta({}^4He)$$

[makes sense, since $\frac{B}{A} \uparrow$ as $A \downarrow$, and ^{4}He has large binding energy]

If $Q > 0$, α-decay can occur

Typically $Q \approx 5$ MeV for heavy unstable nuclei (e.g. U)

$$Q = T_Y + T_{He} \quad \text{[for X at rest]}$$

Because $m_{He} \ll m_Y \Rightarrow T_{He} \gg T_Y$

so α particle gets <u>most</u> of the released energy.

In air, an α-particle travels a few cm.

$\Rightarrow$[picture]

Try to estimate energy released in α-decay using semi-empirical (accurate to about 15%)

$$^{238}U \rightarrow {}^{234}Th + \alpha$$

$\Delta = 47.304$ $\quad$ $\Delta = 40.609$ $\quad$ $\Delta = 2.425 \Rightarrow Q = 4.270$ MeV

$\underbrace{\qquad\qquad}_{6.695 \text{ MeV}}$ ← actual difference

use binding energy formula

$$B(^{238}U) = 1778.63 \Rightarrow \Delta = 71.05$$
$$B(^{234}Th) = 1756.03 \Rightarrow \Delta = 62.93$$

} not too close

$$\underbrace{\qquad}_{22.60} \qquad \underbrace{\qquad}_{8.12}$$

← not too bad, but not too good

Now do Taylor expansion

$$B = c_1 A - c_2 A^{2/3} - c_3 \frac{Z^2}{A^{1/3}} - c_4 \frac{(A-2Z)^2}{A}$$

$$B(^A_Z X) - B(^{A-4}_{Z-2} Y) = 4c_1 - \frac{8c_2}{3A^{1/3}} - c_3 \frac{Z^2}{A^{1/3}}\left[\frac{4}{Z} - \frac{4}{3A}\right]$$

$$+ 4c_4 \frac{(A-2Z)^2}{A^2}$$

$$= 62 - \frac{44.8}{A^{1/3}} - \frac{2.88\, Z^2}{A^{1/3}}\left(\frac{1}{Z} - \frac{1}{3A}\right) + \frac{92(A-2Z)^2}{A^2}$$

0.464

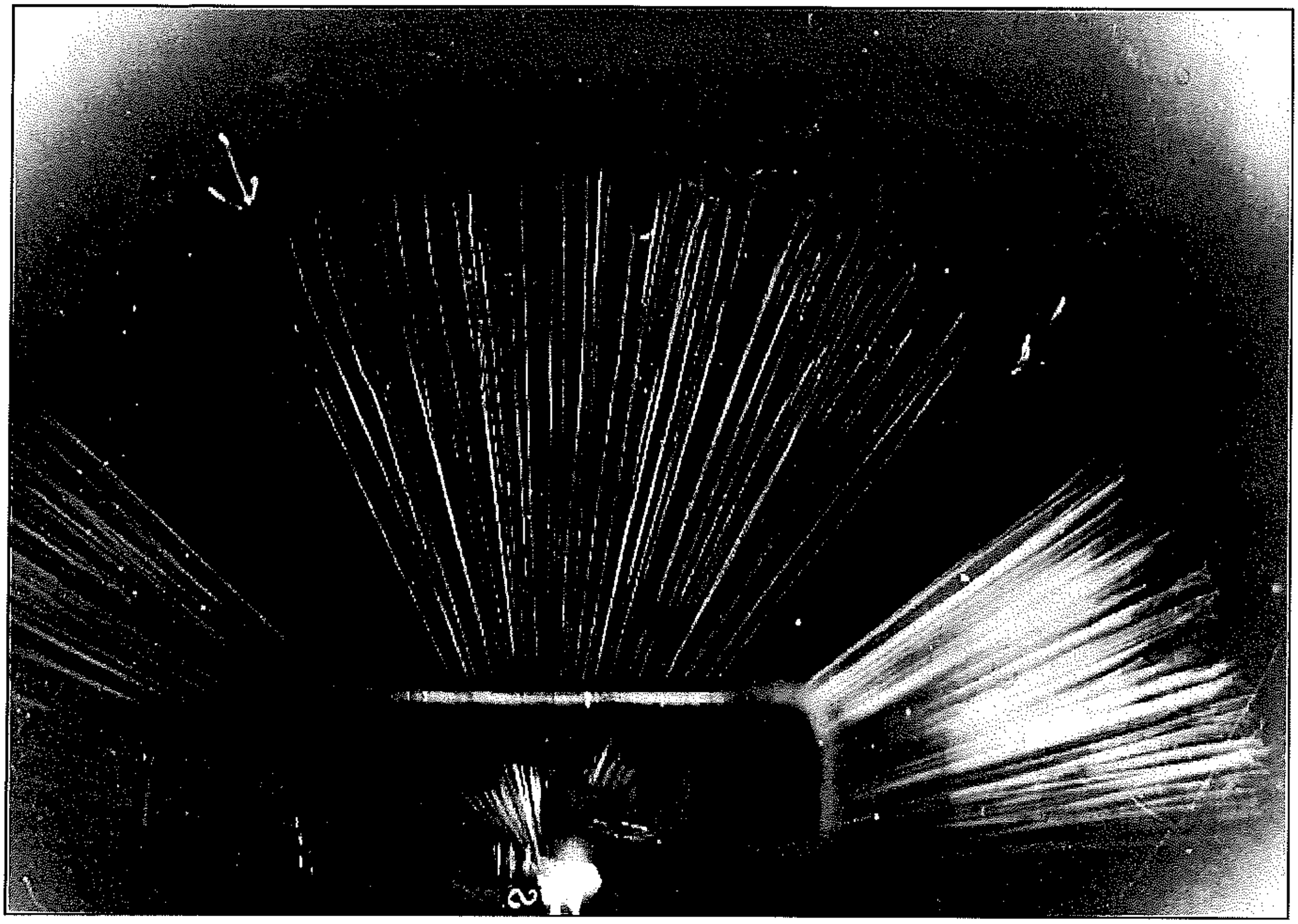
$$^{238}_{92}U \longrightarrow 62 - 7.23 - 37.25 + 4.74 = 22.3$$

Since $\Delta = Z m_H + (A-Z) m_n - A(931.5) - B$

$$\Delta(^A_Z X) - \Delta(^{A-4}_{Z-2} Y) = \underbrace{2m_H + 2m_n - 4(931.5)}_{30.73} - \Delta B$$

$$= -31.3 - \frac{44.8}{A^{1/3}} - 2.88 \frac{Z^2}{A^{1/3}}\left[\frac{1}{Z} - \frac{1}{3A}\right] + 92\frac{(A-2Z)^2}{A^2}$$

$$= -31.3 + 7.2 + 37.2 - 4.7 = 8.4$$



Because all α particles get same kinetic energy they all travel ~ the same distance in air, a few cm

but different nuclei have diff ranges

Asimov: neutrino	
Bragg 1904	
^{232}Th	2.8 cm
^{226}Ra	3.3 cm
^{252}Po	8.6 cm

(see picture →)

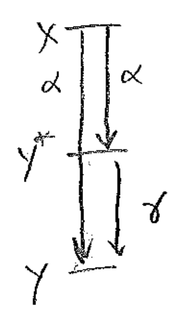
Not strictly true. e.g. see ^{227}Th decay

$\exists$ maximum $K_\alpha = 6.04$ MeV

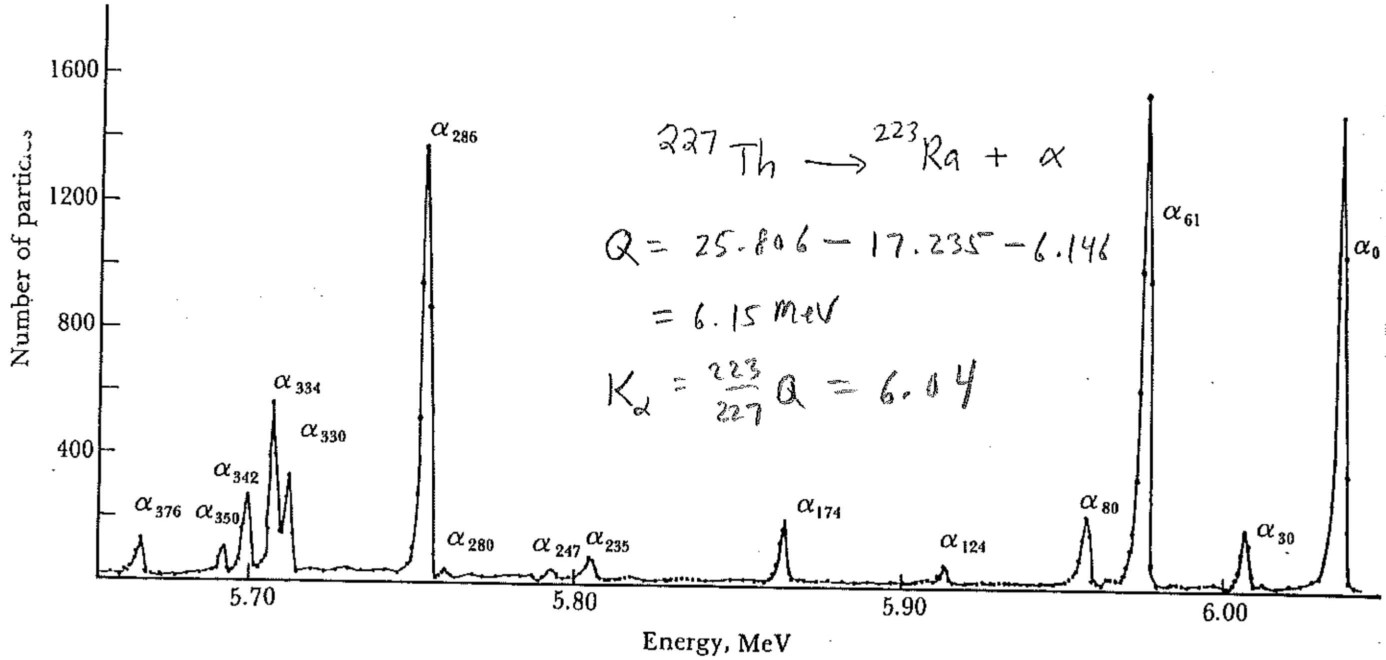
but also smaller K_α, due to decay into an excited state of ^{223}Ra.

These are followed by emission of γ

$$Q = m_{nucl}(X) - m_{nucl}(Y^*) - m_{nucl}(\alpha)$$



Tipler, p. 390-1



$^{227}Th \longrightarrow {}^{223}Ra + \alpha$

$Q = 25.806 - 17.235 - 6.146$

$= 6.15$ MeV

$K_\alpha = \frac{223}{227} Q = 6.04$

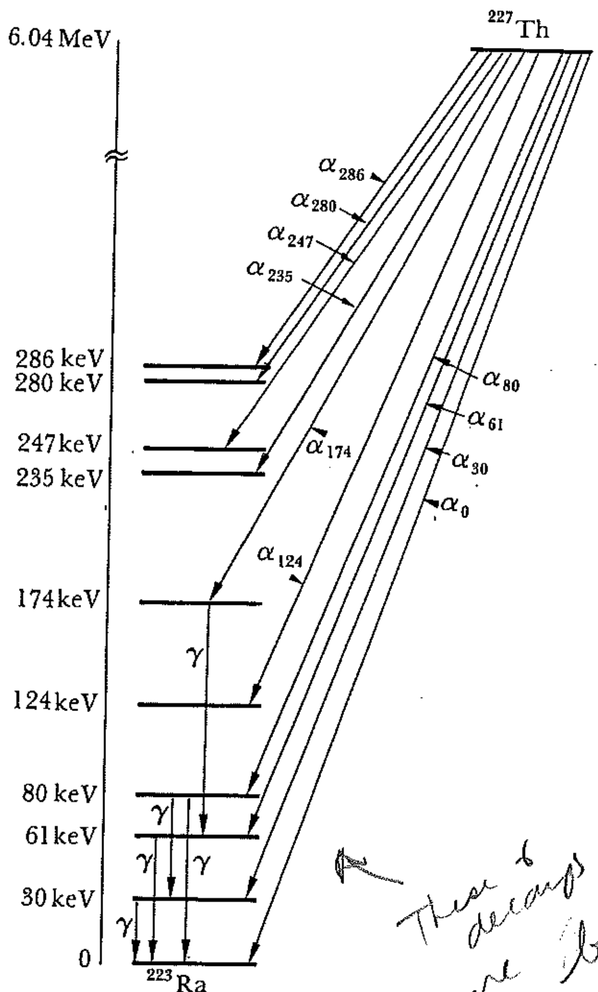
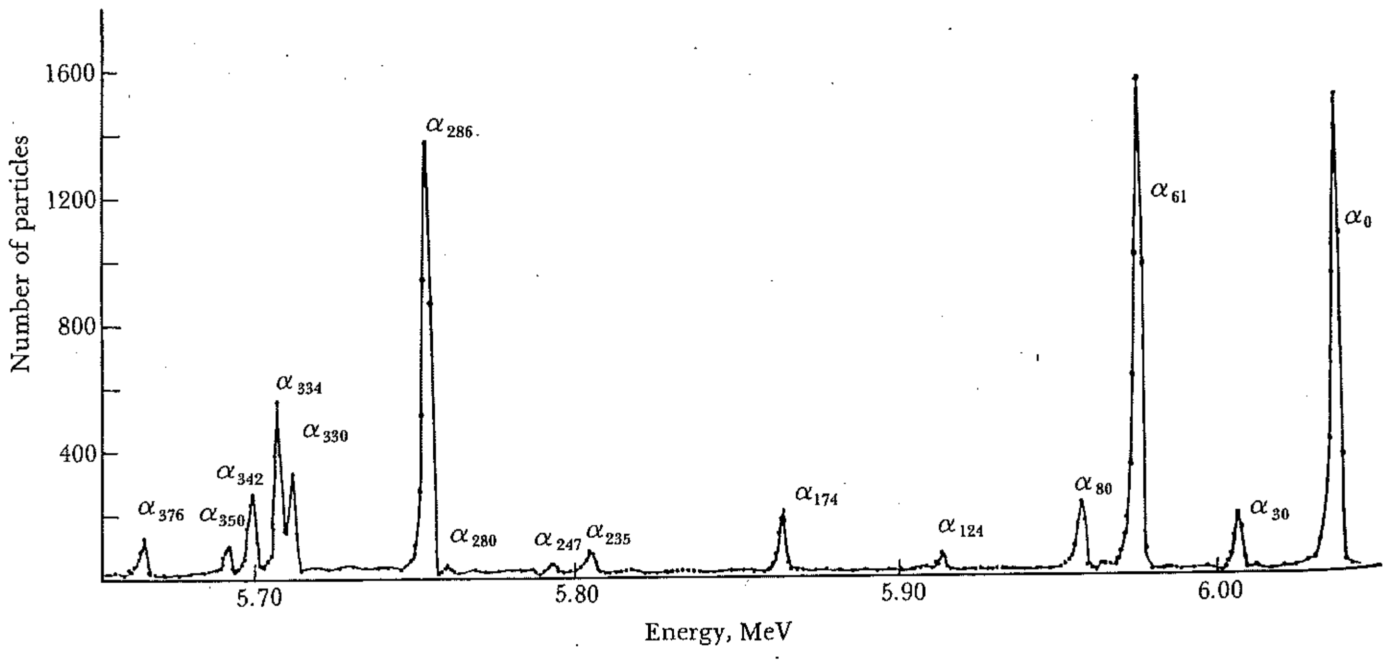


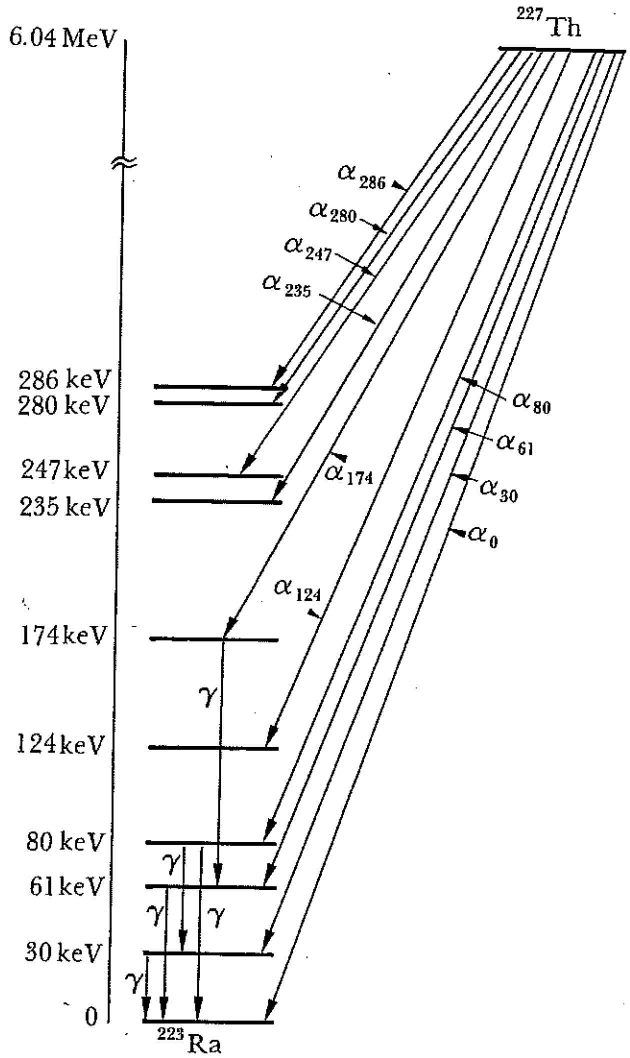
Figure 11-13
Energy levels of ^{223}Ra determined by measurement of α-particle energies from ^{227}Th, as shown in Figure 11-12. Only the lowest-lying levels and some of the γ-ray transitions are shown.

widths due to experimental resolution, not to finite width due to finite lifetime

These 6 decays are between 0.1 ns and 1 ns (see ~~Nuclear~~ Table of Isotopes)



Number of particles
1600
1200
800
400
α286
α61
α0
α334
α330
α342
α376
α350
α280
α247
α235
α174
α124
α80
α30
5.70
5.80
5.90
6.00
Energy, MeV



6.04 MeV
^{227}Th
α_{286}
α_{280}
α_{247}
α_{235}
286 keV
280 keV
α_{80}
α_{61}
247 keV
α_{174}
α_{30}
235 keV
α_{0}
α_{124}
174 keV
γ
124 keV
80 keV
γ
61 keV
γ
γ
30 keV
γ
0
^{223}Ra

Half-life of α-emitters depends strongly on Q

	Q	$T_{1/2}$	
^{227}U	7.2 MeV	1 minute	
^{229}U	6.5 MeV	58 minutes	
^{230}U	6.0 MeV	21 days	
^{232}U	5.4 MeV	69 years	
^{233}U	4.9 MeV	1.6×10^5 yrs	
^{235}U	4.7 MeV	7.0×10^8 years	0.7%
^{238}U	4.3 MeV	4.5×10^9 years	99.3%

As Q ↓, $T_{\frac{1}{2}}$ ↑

[see plot]

1911 Geiger Nuttall rule (empirical): $\ln T_{\frac{1}{2}} = c_1 \frac{Z}{\sqrt{Q}} + c_2$

Can we understand this?

(from HW) Recall $T_{\frac{1}{2}} = (\ln 2)\,\tau = \frac{\ln 2}{R}$

$R = \frac{\text{prob. of decay}}{\text{time}}$ ← inherently quantum mechanical (what is the process?)

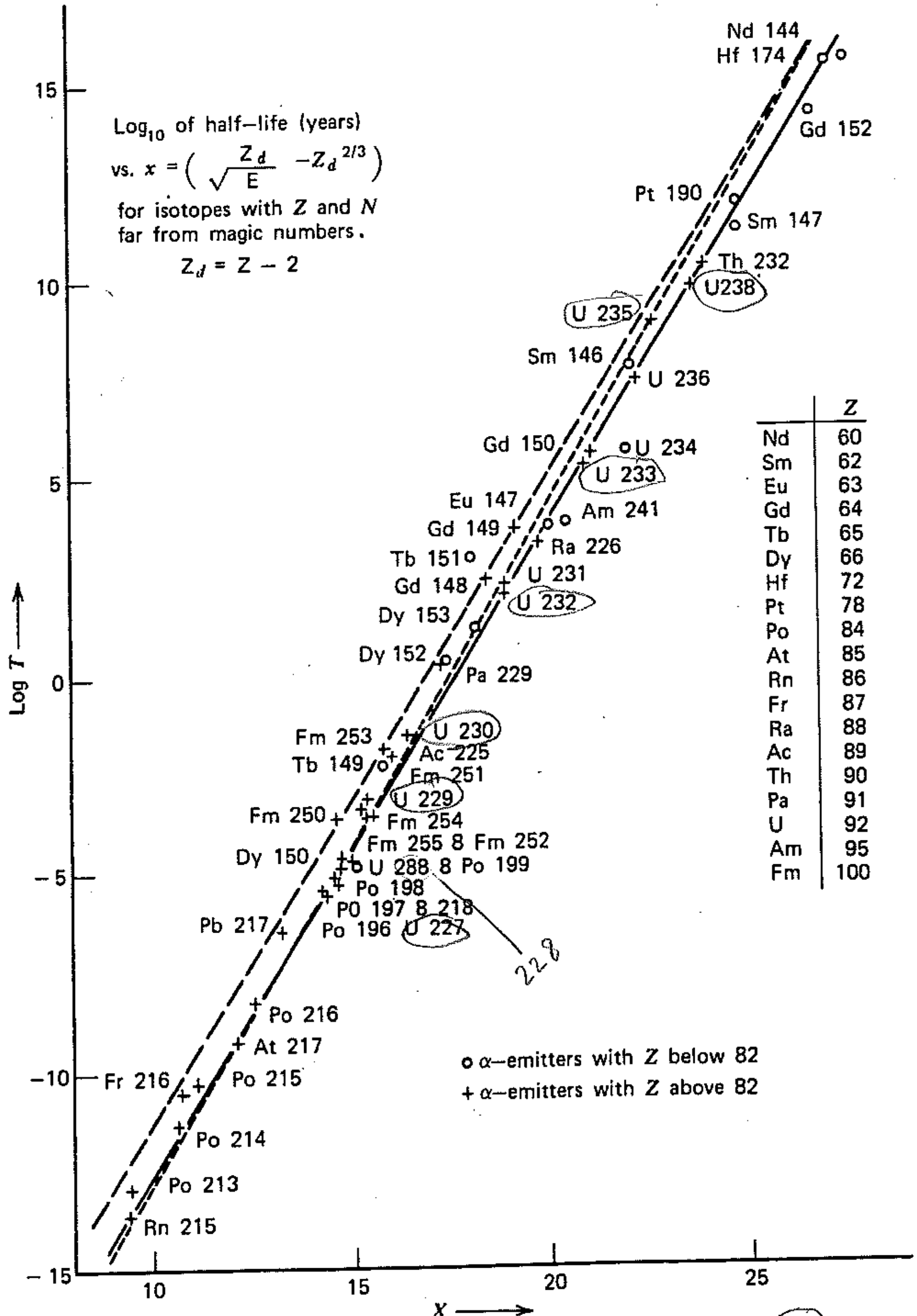
How do we calculate the probability?

$U \rightarrow Th + \alpha$

Imagine a uranium nucleus as an α particle trapped inside a thorium nucleus

7

Gasiorowicz



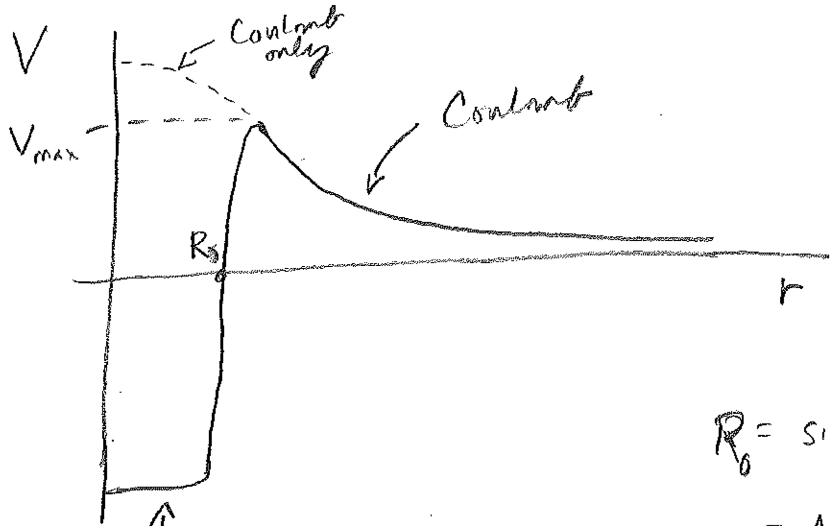
	Z
Nd	60
Sm	62
Eu	63
Gd	64
Tb	65
Dy	66
Hf	72
Pt	78
Po	84
At	85
Rn	86
Fr	87
Ra	88
Ac	89
Th	90
Pa	91
U	92
Am	95
Fm	100

Figure 5-9. Plot of $\log_{10} 1/\tau$ versus $C_2 - C_1 Z_1/\sqrt{E}$ with $C_1 = 1.61$ and a slowly varying $C_2 = 28.9 + 1.6 Z_1^{2/3}$. (From E. K. Hyde, I. Perlman, and G. T. Seaborg, *The Nuclear Properties of the Heavy Elements,* Vol. 1, Prentice-Hall, Englewood Cliffs, N.J. (1964), reprinted by permission.)

$\ln \frac{1}{T} \approx -3.71 \frac{Z_1}{\sqrt{E}}$

$${}^{A+4}_{Z+2}X \longrightarrow {}^{A}_{Z}Y + \alpha$$

Consider the potential energy between Y and α as a function of separation r



R_0 = size of Y nucleus

$= A^{1/3} r_0$

Rough estimate of height of barrier

$$V_{max} \sim \frac{K q_1 q_2}{R} = \frac{K(Ze)(2e)}{A^{1/3} r_0} = 2\left(\frac{Ke^2}{r_0}\right)\frac{Z}{A^{1/3}}$$

$$\sim 2 \frac{(1.44 \text{ MeV fm})}{(1.2 \text{ fm})} \frac{Z}{A^{1/3}}$$

$$\sim (2.4 \text{ MeV}) \frac{Z}{A^{1/3}}$$

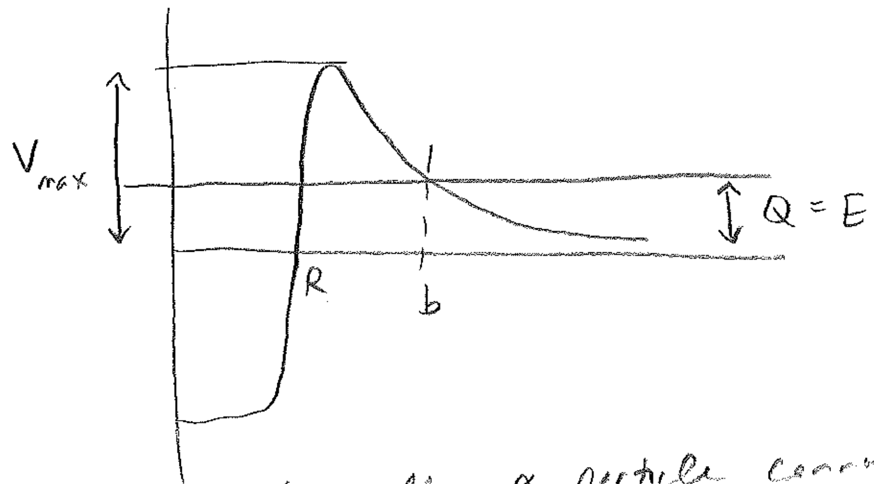
For ^{238}U decay, $Z = 90$, $A = 234 \Rightarrow V_{max} \sim 35$ MeV

[actual $V_{max} \sim 25$ MeV]

What is energy of α particle?

After it escapes, $E = T + V$

$\approx Q + 0$



Classically, α particle cannot escape

Quantum mechanically, it can "tunnel" through barrier

[Gamow (1929)
Gurney-Condon]

Tunnelling rate

$$R = (\text{const})\, e^{-2\int_R^b dr \sqrt{\frac{2m(V(r)-E)}{\hbar^2}}}$$

[HW] calculate this to find $R \approx e^{-c_1 \frac{Z}{\sqrt{Q}}}$

[HW: const = 3.96
expt: 3.68 (Gasiorowicz)]

$$T_{\frac{1}{2}} \sim \frac{\ln 2}{R}$$

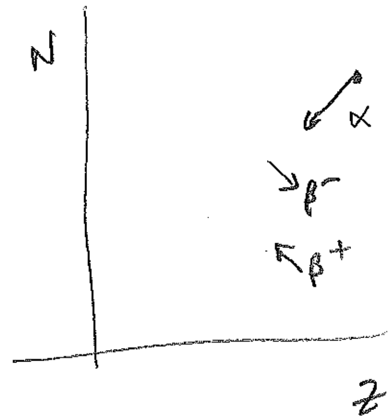
$$\ln T_{\frac{1}{2}} \sim -\ln R + \text{const}$$

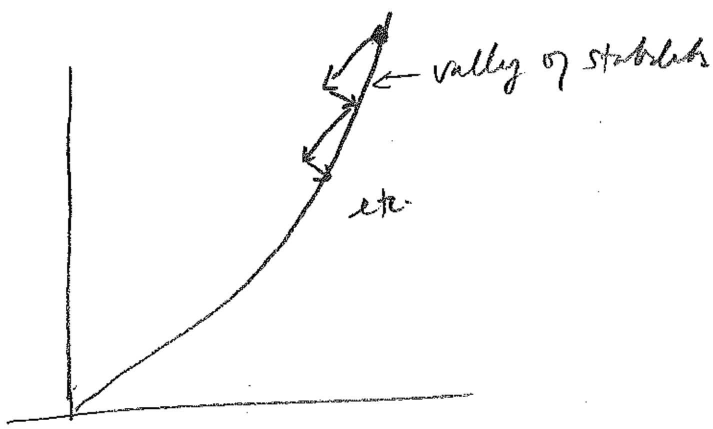
$$\sim c_1'' \frac{Z}{\sqrt{Q}} + \text{const} \quad \text{(Geiger-Nuttall)}$$

α: $Z \to Z-2$
$A \to A-4$

β^-: $Z \to Z+1$
$A \to A$

β^+, EC: $Z \to Z-1$
$A \to A$





Since α, β conserve A mod 4,
there are 4 distinct radioactive series

$A = 4n+2$ ^{238}U (4.5E9 yrs) $\longrightarrow$ ^{206}Pb

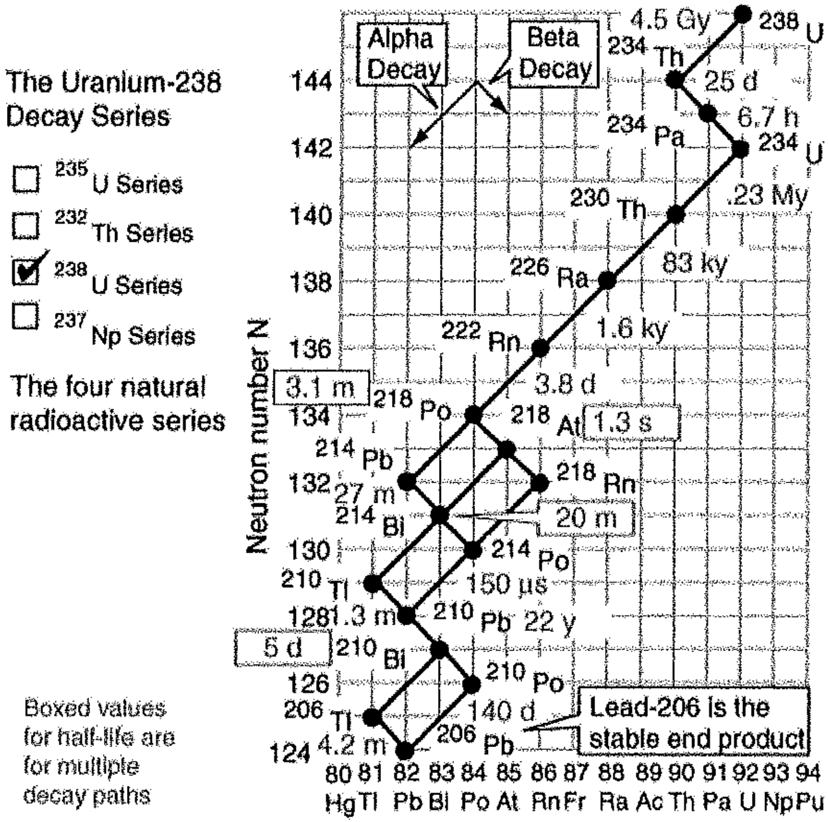
$A = 4n+3$ ^{235}U (7.1E8 yrs) $\longrightarrow$ ^{207}Pb

$A = 4n$ ^{232}Th (1.4E10 yrs) $\longrightarrow$ ^{208}Pb

$A = 4n+1$ ^{233}U (1.6E7 yrs) $\longrightarrow$ ^{209}Bi $\xrightarrow{\text{1.9E19 yrs}}$ ^{205}Tl

hyperphysics.phy-astr.gsu.edu/hbase/nuclear

(georgia state u.)

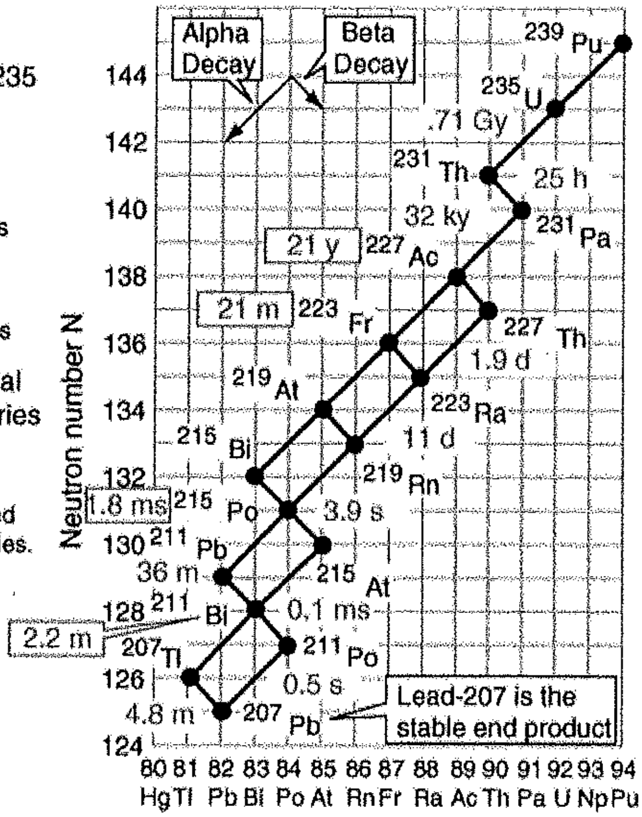


The Uranium-235 Decay Series

- [x] ^{235}U Series
- [ ] ^{232}Th Series
- [ ] ^{238}U Series
- [ ] ^{237}Np Series

The four natural radioactive series

This series is traditionally called the Actinium series.

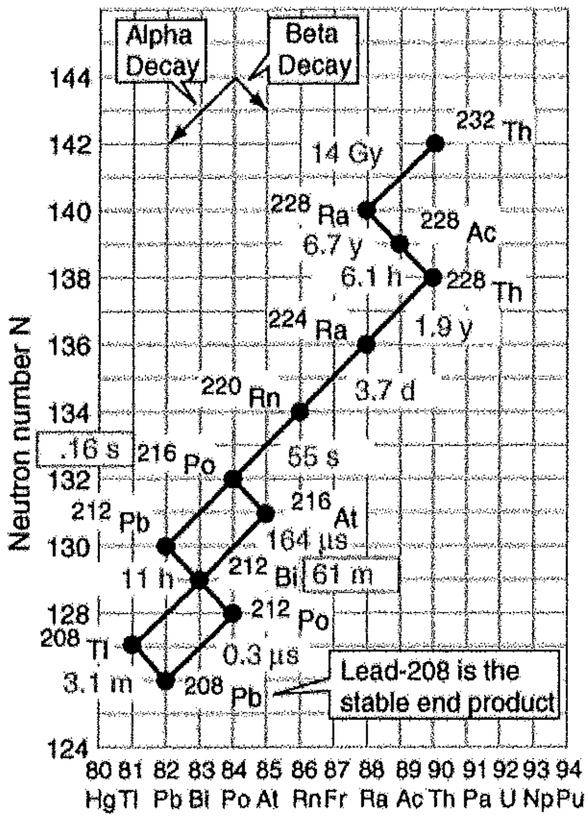


Boxed values for half-life are for multiple decay paths

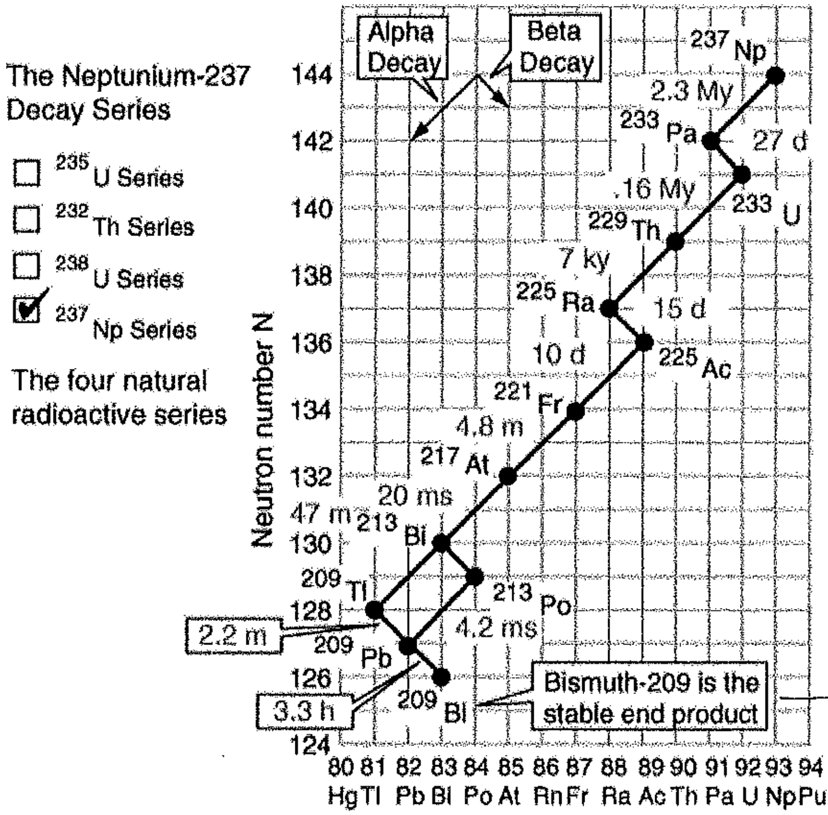
The Thorium-232 Decay Series

- ☐ ^{235}U Series
- ☑ ^{232}Th Series
- ☐ ^{238}U Series
- ☐ ^{237}Np Series

The four natural radioactive series



Boxed values for half-life are for multiple decay paths



$\xrightarrow{\alpha}$ ^{205}Tl

$T_{\frac{1}{2}} = 1.9 \times 10^{19}$ yrs

[2003]