A decay (or other transformation) will generally occur provided it does not violate any conservation laws

["what is not forbidden is mandatory"]

Conservation laws

- energy
- momentum

} a consequence of invariance of laws of physics under translations in space and time (Noether's thm)

- electric charge

} a consequence of gauge invariance of electromagnetism

[believed to be absolute]

- baryon number (quark number)
- lepton number

} "accidental" but valid for standard model

[due to no known symmetry but satisfied by the 4 known forces; # quarks + leptons don't convert into each other;

If a process obeys energy & momentum conservation it is said to be "kinematically allowed"

otherwise "kinematically forbidden"

Energy conservation

[types of energy]

- rest energy mc^2
- kinetic energy T

$$E = \gamma m c^2 \quad \text{where} \quad \gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$T = E - mc^2 = (\gamma - 1) mc^2$$

non rel. approx $= \left[\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \ldots\right) - 1\right] mc^2 = \frac{1}{2} m v^2$

- potential energy V
  associated w/ interactions between particles (EM, strong ..)

Well before and well after the process, the particles are well separated
⇒ ignore any potential energies in initial + final state

$$E_{init} = E_{final}$$

$$\sum_{init} (mc^2 + T) = \sum_{final} (mc^2 + T)$$

Neither kinetic energy nor mass is separately conserved
only their sum.

Define the kinetic energy released in a process

$$Q = T_{final} - T_{init}$$

$$= \sum_{init} mc^2 - \sum_{final} mc^2$$

If $Q = 0$, "elastic process" $\Rightarrow$ kinetic energy is conserved
(e.g. scattering of particles which retain their identities)

If $Q < 0$, "inelastic process" $\Rightarrow$ kinetic energy is lost

final state has more mass than initial

(lumps of putty: hot object has slightly more mass than a cool one)

More massive particles can be produced in a collision
e.g. cosmic rays, particle accelerators

If $Q > 0$, "superelastic process"
$\Rightarrow$ mass converted to kinetic energy
(e.g. fission, decay)

If initial state consists of a single particle M_0 at rest ($T_0 = 0$)

then $Q = T_{final} \geq 0$

$$\Rightarrow \quad M_0 c^2 \geq \sum_{final} mc^2$$

$\Rightarrow$ massive particles can only decay into less massive particles
(not vice versa)

Generally, decays w/ larger Q
have larger probability to occur
+ therefore shorter half-lives
though for differing reasons

e.g. α-decay; higher Q $\rightarrow$ small potential barrier to tunnel through

β-decay; higher Q $\rightarrow$ larger phase space

But in some cases, other considerations
cause the relationship to be violated
e.g. parity violation of the weak force
means that $\pi \to \mu$ is more likely
than $\pi \to e$,
despite the smaller phase space available

Conservation laws guarantee stability

e^- is stable because it is least massive pcl w/ electric charge

[if it decayed into lighter neutral particles, violates charge cons.]

Why is p stable?

[not the lightest positively charged pcl : $p \to e^+ \gamma$]

1938 Stuckelberg proposed conservation of baryon # (A)

p is stable because it is lightest pcl w/ baryon #

Baryon # cons. has never been observed to be violated
& is respected by all the forces of standard model
but no deep reason for it.

Extensions of standard model (GUTs) typically violate both
baryon # + lepton #

Proton decay expts since 1970's [Problem]
(Super Kamiokande)
Null results so far

$\tau_p > 2 \times 10^{29}$ yrs [need a lot of protons]

mass / energy conversions

Masses of atoms or nuclei often specified in terms of the unified atomic mass unit (u).

(amu was based on ^{16}O; u is based on ^{12}C)

A mole of particles of mass 1u has a mass of 1g

[PPB for constants]

$$N_A = 6.022141 \times 10^{23}$$

$$\Rightarrow 1\,u = \frac{10^{-3}\,kg}{N_A} = 1.660539 \times 10^{-27}\,kg$$

This amount of mass corresponds to energy ($c = 299792458\ \frac{m}{s}$)

$$mc^2 = 1.492418 \times 10^{-10}\,J$$

using $1\,eV = 1.602176 \times 10^{-19}\,J$ then gives
$1\,MeV = 10^6\,eV$

{chemistry eV
{nucl phys MeV

$$mc^2 = 931.494\,MeV = 0.931494\,GeV$$

$$\boxed{m = 1\,u \Rightarrow mc^2 = 931.494\,MeV}$$

we'll almost always use rest energies

[PPB]

- proton 1.0072765u = 938.272 MeV
- neutron 1.0086649u = 939.565 MeV

Both about 1 GeV

Neutron slightly more massive, so decays

$n \rightarrow p$

[violates charge cons.]

$n \rightarrow p + e^-$

[violates lepton number]

$n \rightarrow p + e^- + \bar{\nu}_e$

[still obey energy conservation?]

- neutrino $\lesssim 1$ eV
- electron 0.0005486 = 0.511 MeV

About $\frac{1}{2}$ MeV, or $\frac{1}{2000}$ mass of p, n

$$Q = m_n c^2 - m_p c^2 - m_e c^2 - m_{\bar{\nu}} c^2 = 0.782 \text{ MeV}$$

Neutron decay is kinematically allowed.

A nuclear process involving e^- or e^+ is called β-decay

Most unstable light nuclei decay via β-decay

It is generally easier to measure the masses of neutral atoms than of bare nuclei

A <u>neutral atom</u> of Z protons, N neutrons, Z electrons has an approximate mass $(Z+N)\,u \approx A\,u$

ie rest energy

$$mc^2 \approx A\,(931.494\ \text{MeV})$$

It is convenient to characterize the discrepancy as

$$mc^2 \equiv A\,(931.494\ \text{MeV}) + \Delta$$ ← atomic mass (incl. electrons)

Δ = "mass excess" [actually rest energy excess]

$$m = \left(A + \frac{\Delta}{931.494}\right) u$$ [nuclear wallet cards]

The unified mass unit u is <u>defined</u> so that $\Delta \equiv 0$ for $^{12}_{6}C$

ie 1 mole of ^{12}C has mass exactly 12 g

(really a definition of Avogadro's number)

[Using our earlier numbers]

- proton $\quad \Delta = 6.778$ MeV
- neutron $\quad \Delta = 8.071$ MeV

[neutron agrees w/ nuclear wallet cards, but not proton]

- hydrogen = proton + electron $\quad \Delta = 7.289$ MeV ✓

$$n \to p + e^- + \bar{\nu}_e$$

$$Q = m_n - m_p - m_e - \cancel{m_{\bar{\nu}}}^{\,0}$$

[Drop the c's, w/ this freedom comes responsibility]

(need restore units when done calculating ⋮)

$$= m_n - (m_H - m_e) - m_e$$

$$= m_n - m_H$$

$$= (1\,u + \Delta(n)) - (1\,u - \Delta({}^1H))$$

$$= \Delta(n) - \Delta({}^1H)$$

$$= 0.782 \text{ MeV}$$ ✓

Deuteron $d = pn$

[Simplest composite nucleus
p and n bound together by strong force
(residue of color force between quarks

Composite objects hold together due to their binding energy.

Binding energy = difference between the sum of (rest) energies of the components and the (rest) energy of the composite object

Deuteron binding energy:

$$B_d = m_n + m_p - m_d$$

$$= m_n + [m(^1H) - \cancel{m_e}] - [m(^2H) - \cancel{m_e}]$$

$$= [1u + \Delta(n)] + [1u + \Delta(^1H)] - [2u + \Delta(^2H)]$$

$$= \Delta(n) + \Delta(^1H) - \Delta(^2H)$$

$$= 8.071 + 7.289 - 13.136$$

$$= 2.224 \text{ MeV} \quad (\text{about } 0.1\% \text{ of } m_d \sim 2 \text{ GeV})$$

$$\Rightarrow m_d = m_n + m_p - B_d$$

Deuteron mass is less than its constituents

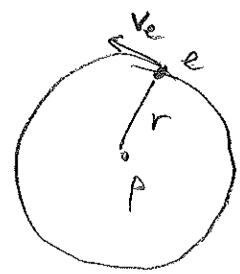
[How to understand B?]

[Simpler context]

Binding energy of a hydrogen atom.

$$E_H = m_p + m_e + \overbrace{T_p + T_e + V_{ep}}^{-B_{atomic}}$$

Use Bohr model to estimate B_{atomic}



proton $\approx$ at rest $\Rightarrow T_p \approx 0$

$$m_e a_e = F_{ep}$$

$$\frac{m_e v_e^2}{r} = \frac{Ke^2}{r^2}$$

{ virial $\langle T \rangle = \frac{n}{2} \langle V \rangle$ for $V \sim r^n$

$$T_e = \frac{1}{2} m_e v_e^2 = \frac{1}{2} \frac{Ke^2}{r}$$

$$V_{ep} = -\frac{Ke^2}{r}$$

Observe that $T_e = -\frac{1}{2} V_{ep}$ [valid for any inverse sq. force]

$$E_H = m_p + m_e + 0 + \underbrace{\frac{Ke^2}{2r} - \frac{Ke^2}{r}}_{-\frac{1}{2}\frac{Ke^2}{r}}$$

For $r = a_0$ (Bohr radius), $\frac{1}{2}\frac{Ke^2}{r} = 13.6\,\text{eV}$

Hydrogen atom is at rest

$$m_H c^2 = \underbrace{m_p}_{\approx 1\,\text{GeV}} + \underbrace{m_e}_{\approx 0.511\,\text{GeV}} - \underbrace{(13.6\,\text{eV})}_{B_{atomic}}$$

B_{atomic} (about 10^{-8} of total)

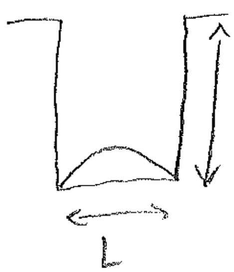
Binding energy of deuteron

strong interaction potential energy

$$m_d = m_p + m_n + \underbrace{T_p + T_n + V_{pn}}_{-B_d} \qquad \text{where } B_d \approx 2\,\text{MeV}$$

Unlike Coulomb force, we have no simple formula for V_{pn}
[nor do we expect one, since p and n are composites]

Use simplistic potential well model to estimate kinetic energy

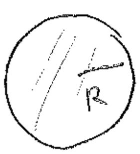


$$T = T_p + T_n = 2 \cdot \frac{\vec{p}^{\,2}}{2m} = 2 \cdot \frac{3}{2m}\left(\frac{h}{\lambda}\right)^2$$ λ = de Broglie wavelength

$$\lambda \simeq 2L$$

$$T = \frac{3}{m}\left(\frac{2\pi\hbar}{2L}\right)^2 = \frac{3\pi^2}{mc^2}\left(\frac{\hbar c}{L}\right)^2$$

$$= \frac{3\pi^2}{(1000\,\text{MeV})}\left(\frac{200\,\text{MeV}\cdot\text{fm}}{L}\right)^2 = 1200\,\text{MeV}\left(\frac{\text{fm}}{L}\right)^2$$



$L = 2R$, $R = 2^{1/3} r_0 \approx 1.5$ fm $\Rightarrow L \sim 3$ fm

$$T \approx 130\,\text{MeV}$$

This is a very crude estimate (probably too high by factor of 2 or 4)

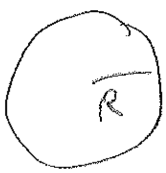
but suggests $V_0 \approx -T \approx -130$ MeV

almost complete cancellation

particle in 3d box

$$T = 3 \cdot \frac{1}{2m}\left(\frac{h}{\lambda}\right) = \frac{3}{2m}\left(\frac{2\pi\hbar}{2L}\right)^2 = \frac{3\pi^2\hbar^2}{2mL^2}$$

particle in a sphere



$$-\frac{\hbar^2}{2m}\frac{1}{r}\frac{d^2}{dr^2}(r\psi) = E\psi$$

$$\frac{d^2}{dr^2}(r\psi) + \frac{2mE}{\hbar^2}(r\psi) = 0$$

$$\psi = \frac{\sin kr}{r} \quad \text{where} \quad \frac{\hbar^2 k^2}{2m} = E$$

$$\psi(R) = 0 \Rightarrow kR = \pi \Rightarrow k = \frac{\pi}{R} \Rightarrow \boxed{E = \frac{\hbar^2\pi^2}{2mR^2}}$$

If $L = \underbrace{2R}_{\text{diameter}}$ then 3d box $\Rightarrow \frac{3}{8}\frac{\pi^2\hbar^2}{mR^2}$, slightly less

If volumes equal $L^3 = \frac{4\pi}{3}R^3$

$$\text{the 3d box} = \frac{3\pi^2\hbar^2}{2m}\left(\frac{3}{4\pi}\right)^{2/3}\frac{1}{R^2} = 0.577\,\frac{\hbar^2\pi^2}{mR^2}$$

slightly more (by 15%)

[deuteron is not unstable to $d \rightarrow p + n$, but what about]

$$d \xrightarrow{?} p + p + e^- + \bar{\nu}_e$$

$$Q = m_d - 2m_p - m_e$$

$$= (m(^2H) - m_e) - 2(m(^1H) - m_e) - m_e$$

$$= m(^2H) - 2m(^1H)$$

$$= \Delta(^2H) - 2\Delta(^1H)$$

$$= 13.136 - 2(7.289)$$

$$= -1.442 \text{ MeV}$$

Not as much difference as binding energy
but still kinematically forbidden

Free neutron is unstable, but neutron
bound to a proton is stable!

[strong force reduces the energy]

Tritium $t = pnn$

$$t \xrightarrow{?} d + p + e^- + \bar{\nu}_e$$

$$Q = \Delta(^3H) - \Delta(^2H) - \Delta(^1H)$$
$$= 14.95 - 13.136 - 7.289$$
$$= -5.475 \text{ MeV}$$

Not possible

[How does tritium decay?]

$$pnn \longrightarrow ppn + e^- + \bar{\nu}_e$$

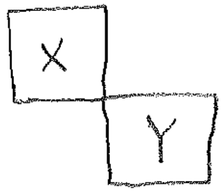
$$Q = [m(^3H) - m_e] - [m(^3He) - 2m_e] - m_e$$
$$= [3u + \Delta(^3H)] - [3u - \Delta(^3He)$$
$$= \Delta(^3H) - \Delta(^3He)$$
$$= 0.019 \text{ MeV}$$

$$\left[\begin{array}{l} m(^3H) = 2809.4496 \\ m(^3He) = 2809.4310 \\ \hline \quad\quad\quad\quad\quad 0.0186 \end{array} \right.$$

just barely unstable

β-decay always connects <u>isobars</u> = nuclei w/ same # nucleons

β^- decay



$${}^{A}_{Z}X \xrightarrow{?} {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e$$

one of neutrons converted to a proton

$$Q = m_{nucl}(X) - m_{nucl}(Y) - m_e$$

$$= [Au + \Delta(X) - Zm_e] - [Au + \Delta(Y) - (Z+1)m_e] - m_e$$

$$= \Delta(X) - \Delta(Y)$$

If $\Delta(X) > \Delta(Y)$, then X is unstable to β-decay

What if $\Delta(X) < \Delta(Y)$?

Can a proton convert to a neutron?

$$e^- + p \rightarrow n + \nu_e$$

kinematically forbidden for e^-, p at rest (but formation of a neutron star)

[conservation laws obeyed]

$$e^- + {}^{A}_{Z+1}Y \xrightarrow{?} {}^{A}_{Z}X + \nu_e$$

one of the atomic electrons — one of protons converted to a neutron

$$Q = m_e + m_{nucl}(Y) - m_{nucl}(X)$$

$$= m_e + [A u + \Delta(Y) - (Z+1) m_e] - [A u + \Delta(X) - Z m_e]$$

$$= \Delta(Y) - \Delta(X)$$

If $\Delta(Y) > \Delta(X)$, then Y is unstable to EC (electron capture)

EC is followed by X-ray emission, as one of outer shell electrons fills the vacancy created by the s-shell electron

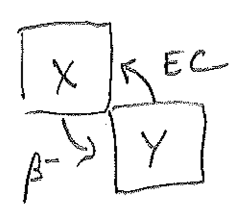
[first observed 1938 Alvarez : cf Krane, p. 272]

Given two adjacent isobars, one will always be unstable (but next to adjacent isobars can exist)

[Segrè chart]

${}^{113}_{48}Cd \xrightarrow{\beta^-} {}^{113}_{49}In$ 9.3×10^{15} yrs

${}^{123}_{52}Te \xrightarrow{EC} {}^{123}_{51}Sb$ 1.3×10^{13} yrs



(but shows as stable on Segrè chart)

${}^{113}_{48}Cd$, $\Delta = -89.051$, $T_{\frac{1}{2}} = 9.3E15$ yrs β^- decay $J = \frac{1}{2}+$

12.22%

${}^{113}_{49}In$ $\Delta = -89.367$ 4.3% $J = \frac{9}{2}+$

${}^{123}_{51}Sb$ $\Delta = -89.223$ 42.7% $J = \frac{7}{2}+$

${}^{123}_{52}Te$ $\Delta = -89.172$ 1.3×10^{13} yrs EC $J = \frac{1}{2}+$

0.908%

β^+ decay

$$p \longrightarrow n + e^+ + \nu_e$$ not kinematically possible

$$ {}^{A}_{Z+1}Y \xrightarrow{?} {}^{A}_{Z}X + e^+ + \nu_e$$

proton converted to neutron

$$Q = m_{nucl}(Y) - m_{nucl}(X) - m_e$$

$$= [Au + \Delta(Y) - (Z+1)m_e] - [Au + \Delta(X) - Zm_e] - m_e$$

$$= \Delta(Y) - \Delta(X) - 2m_e$$

If $\Delta(Y) - \Delta(X) \geq 2m_e = 1.022$ MeV

then Y is unstable to β^+ decay (also EC).

β^+ decay is followed by

$$e^+ + e^- \longrightarrow \gamma + \gamma$$ (pair annihilation)

$$Q = m_{e^+} + m_{e^-} = 1.022 \text{ MeV}$$

Each γ carries of 0.511 MeV (tell-tale sign of e^+e^- annihilation)

[first observed 1934 Joliot-Curie; cf Krane p.272]

Binding energy of a nucleus $^{A}_{Z}X$

$$B = Z m_p + N m_n - m_{nucl}(X)$$
$$= Z[m(^1H) - m_e] + N m_n - [m(X) - Z m_e]$$
$$= Z[1u + \Delta(^1H)] + N[1u + \Delta(n)] - [Au + \Delta(X)]$$
$$= Z \underbrace{\Delta(^1H)}_{7.289} + N \underbrace{\Delta(n)}_{8.071} - \Delta(X)$$

Rewrite this in terms of $N+Z$ and $N-Z$

$$B = (N+Z)(7.680 \text{ MeV}) + (N-Z)(0.391 \text{ MeV}) - \Delta(X)$$

For ^{12}C, $N=Z$ and $\Delta = 0$ ~~so~~

[B and Δ not the same]

$$B(^{12}C) = 12\,(7.680 \text{ MeV})$$

If we assume ^{12}C is "typical" ie $N \approx Z$ and $\Delta \approx 0$

$$B = (7.7 \text{ MeV})\, A$$

ie binding energy per nucleon is approximately constant

[show next page] ⇒ "curve of binding energy"

[need to explain deviations]

$$m_{nucl}(X) = Z m_p + N m_n - B$$
$$\approx (939 \text{ MeV})\, A - (8 \text{ MeV})\, A$$
$$\approx (931 \text{ MeV})\, A$$

≈ 1u, chosen so Δ ≈ 0

Nuclei are ~1% less massive than sum of constituents

[recall deuteron about 0.1% → relatively weak binding]

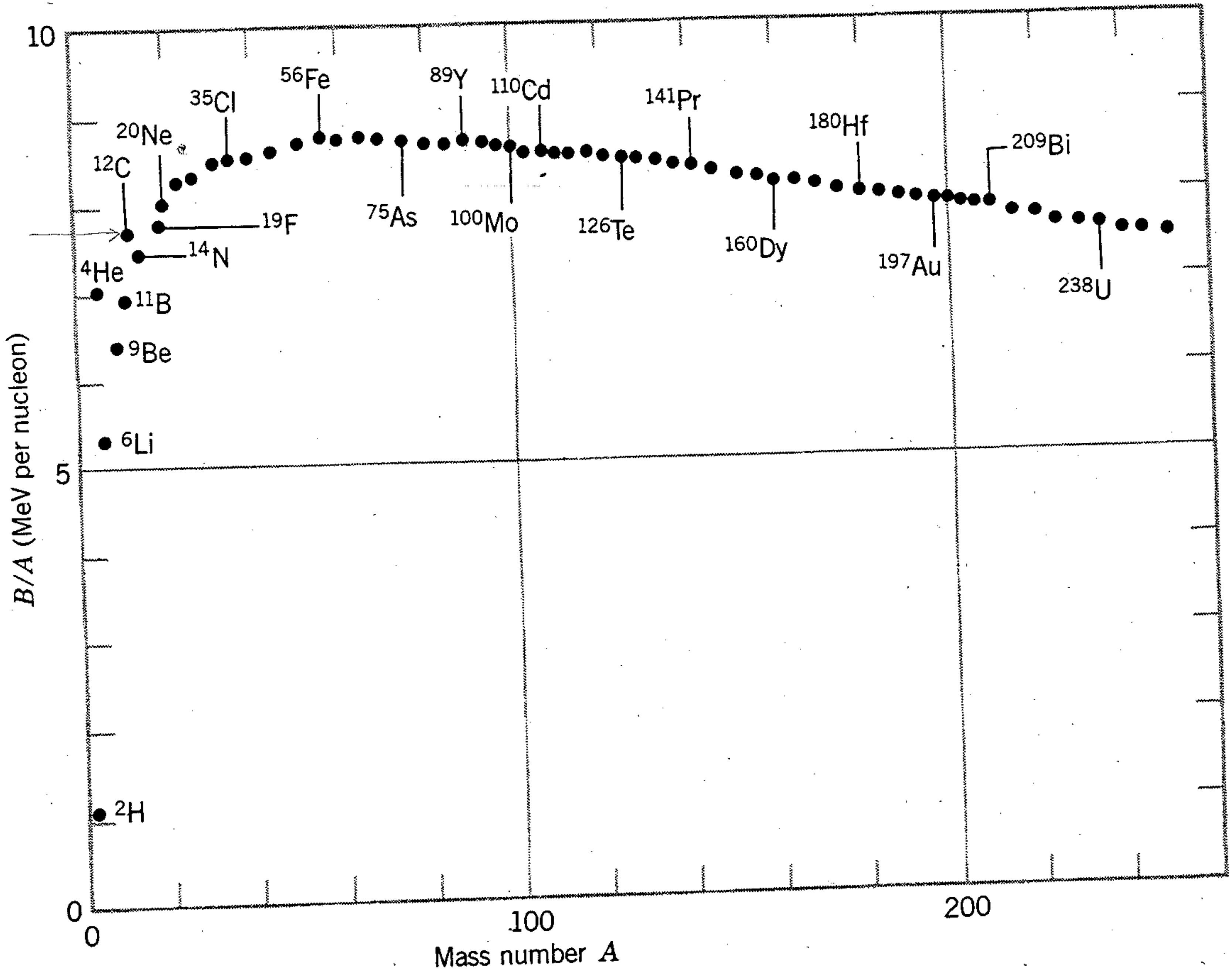
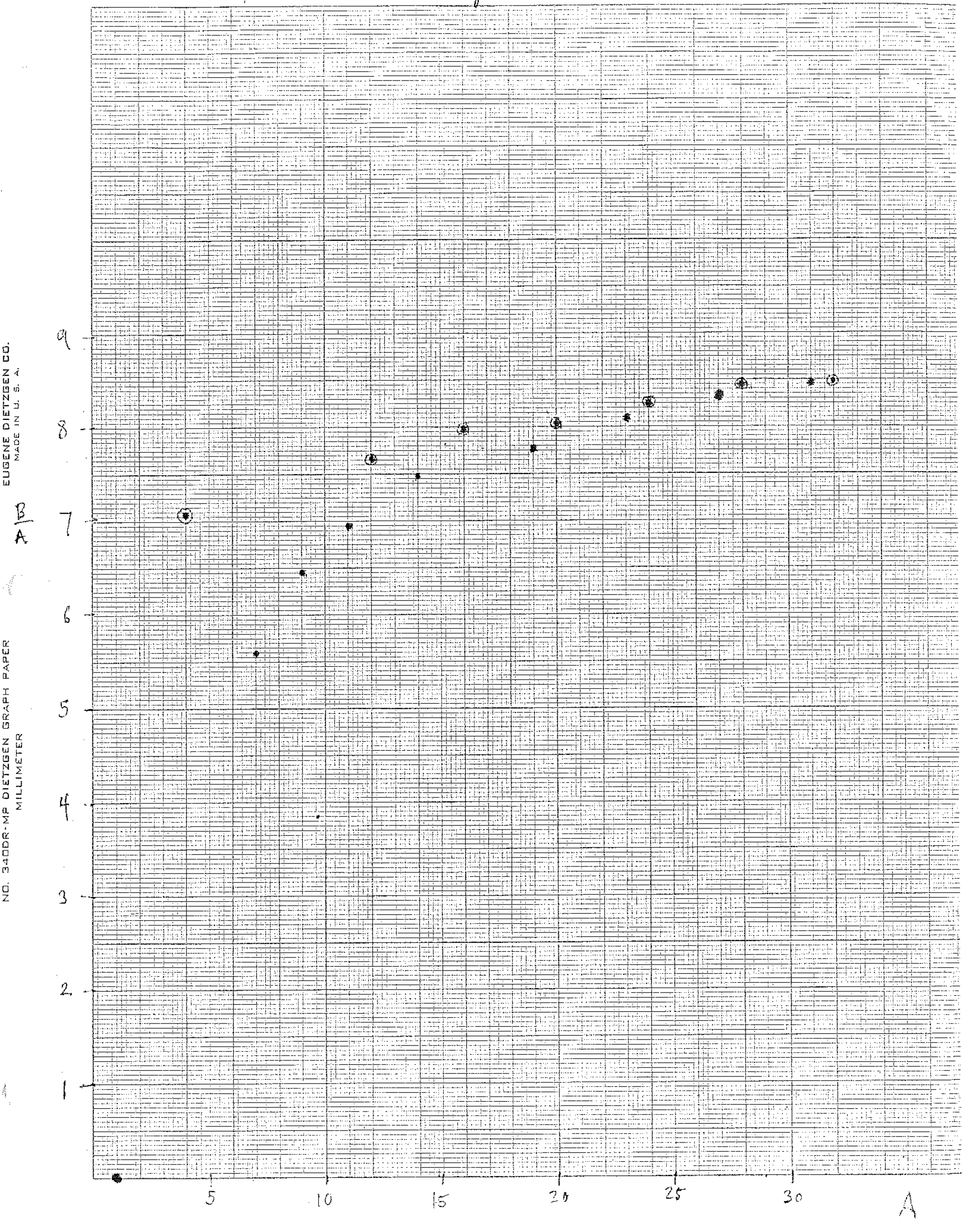


Figure 3.16 The binding energy per nucleon.

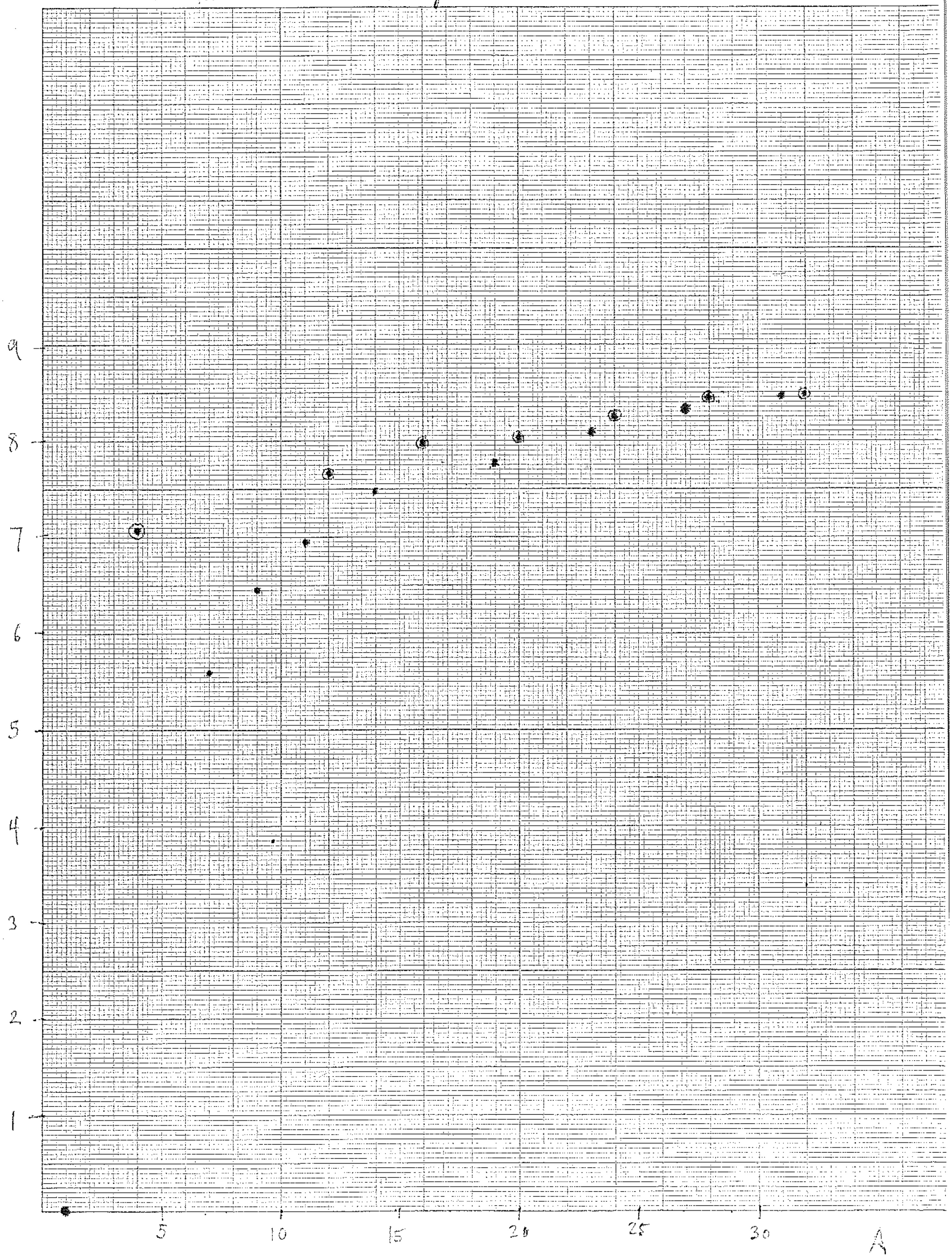
⊙ = even-even nuclides

B/A for most abundant isotopes



B/A for most abundant isotopes

⊙ = even-even nuclides



	Z	A	Δ	$A-2Z$	$\frac{B}{A} = 7.68 + \frac{(A-2Z)(0.39) - \Delta}{A}$
H	1	1	7.29	−1	0
He	2	4	2.42	0	$\boxed{7.07}$
Li	3	7	14.91	1	5.61
Be	4	9	11.35	1	6.46
B	5	11	8.67	1	6.93
C	6	12	0	0	$\boxed{7.68}$
N	7	14	2.86	0	7.48
O	8	16	−4.74	0	$\boxed{7.98}$
F	9	19	−1.49	1	7.78
Ne	10	20	−7.05	0	$\boxed{8.03}$
Na	11	23	−9.53	1	8.11
Mg	12	24	−13.93	0	$\boxed{8.26}$
Al	13	27	−17.20	1	8.33
Si	14	28	−21.49	0	$\boxed{8.45}$
P	15	31	−24.44	1	8.48
S	16	32	−26.02	0	$\boxed{8.49}$