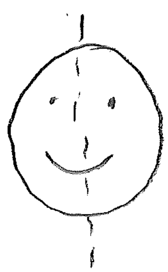
Symmetry is the fundamental principle of modern physics

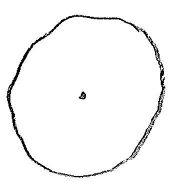
2022

[what is symmetry?]

[same on left and right; mirror reflections]



invariant under reflection



invariant under rotation about center

Symmetry = invariance under some operation

The laws of physics are invariant under certain operations
(eqns of physics have the same form)

- spatial translations
- temporal translations
- rotations
- boosts

[if do expt in room down hall, or tomorrow, or rotated
expect same result.]

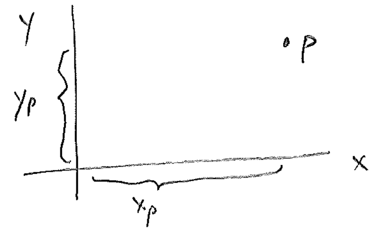
Symmetries imply conservation laws (Emmy Noether theorem)

- cons. of momentum
- cons of energy
- cons of angular momentum

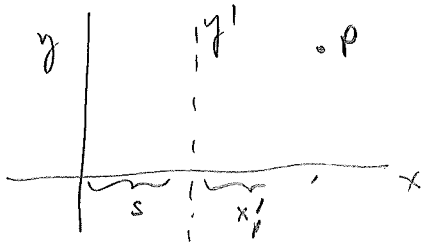
(not obvious!)

[brilliant German female mathematician who is not as well known as she deserves to be. Usually don't hear her name until graduate school.]

SPATIAL TRANSLATIONS

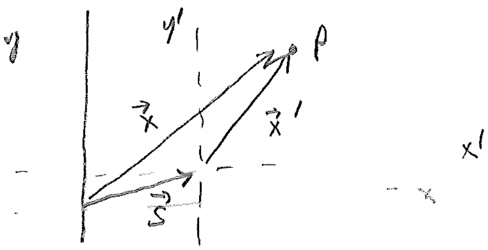


coordinates of P: (x_p, y_p)



shift origin to right by s

$$x_p' = x_p - s$$
$$y_p' = y_p$$



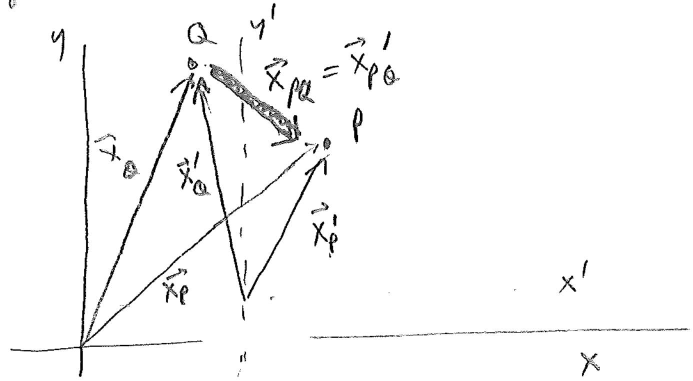
Shift origin by $\vec{s}$

$$\vec{x}' = \vec{x} - \vec{s}$$

coordinates change under a shift
(passive transf.)

Relative positions don't change under a shift

$$\vec{x}_{PQ} = \vec{x}_P - \vec{x}_Q$$
$$= (\vec{x}_P' + \vec{s}) - (\vec{x}_Q' + \vec{s})$$
$$= \vec{x}_P' - \vec{x}_Q'$$
$$= \vec{x}_{PQ}'$$



velocities don't change under a shift

$$\vec{v}' = \frac{d\vec{x}'}{dt} = \frac{d}{dt}(\vec{x} - \vec{s}) = \frac{d\vec{x}}{dt} = \vec{v}$$

Accelerations don't change either

$$\vec{a}' = \frac{d\vec{v}'}{dt} = \frac{d\vec{v}}{dt} = \vec{a}$$

Relative positions, velocities, accelerations are all invariant under spatial translations

Since force between two objects depends on the relative separation (e.g. Coulomb) $\frac{Kq_1q_2}{|\vec{r}_1 - \vec{r}_2|^2}$, forces are also invariant.

laws of physics are invariant under spatial translations

e.g. Newton's 2nd law: $\vec{F} = m\vec{a}$

- force between objects invariant: $\vec{F} = \vec{F}'$
- acceleration is invariant: $\vec{a} = \vec{a}'$

Therefore, $\vec{F}' = m\vec{a}'$

eqn has the same form

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eg cons. of momentum: $\sum_{init} m_j \vec{v}_j = \sum_{final} m_j \vec{v}_j$

- velocities are invariant

$$\Rightarrow \sum_{init} m_j \vec{v}_j' = \sum_{final} m_j \vec{v}_j'$$

[eqns of physics do not depend on choice of origin.]

- similarly for temporal translations + rotations (more later about rotations)