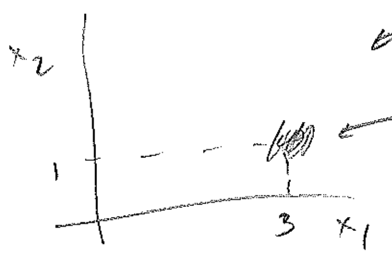
[In 2019, I had 2 half-classes left. So on the first day I presented this:] G1

<u>Multiparticle systems</u> → (after rest of the course)

State of 2 particles described by

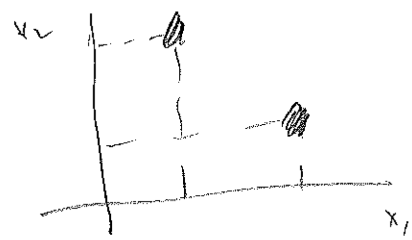
$\psi(x_1, x_2, t)$

$|\psi(x_1,x_2,t)|^2 =$ probability density of finding particle 1 at x_1 & particle 2 at x_2



← (density plot)

particle 1 at $x_1 = 3$
particle 2 at $x_2 = 1$



← if particles identical must have

$$|\psi(x_1,x_2)|^2 = |\psi(x_2,x_1)|^2$$

<u>2 possibilities</u>

(a) $\psi(x_1,x_2) = \psi(x_2,x_1)$ → particles obeying this are bosons ($W^\pm$, Z, γ, Higgs)

(b) $\psi(x_1,x_2) = -\psi(x_2,x_1)$ → particles obeying this are fermions (e^-, ν, q, p, n)

What is prob of finding 2 fermions at same place?

$\psi(x,x) = -\psi(x,x) \Rightarrow \underline{|\psi(x,x)|^2 = 0}$ (Pauli)

$2\psi(x,x) = 0$

[on the second half-class I presented this]

A single particle in a potential of well defined energy ~~E~~ is described by $\psi(x,t) = u(x) e^{-iEt/\hbar}$

where $-\frac{\hbar^2}{2m}\frac{d^2u}{dx^2} + V(x)u = Eu$

This has solutions $u_n(x)$ of energy E_n

A system of 2 particles both in external potential $V(x)$ + of possible interparticle potential $V_{12}(x_1 - x_2)$ is described by

$\psi(x_1, x_2, t) = u(x_1, x_2) e^{-iEt/\hbar}$ where

$$-\frac{\hbar^2}{2m}\frac{\partial^2 u}{\partial x_1^2} + V(x_1)u - \frac{\hbar^2}{2m}\frac{\partial^2 u}{\partial x_2^2} + V(x_2)u + \cancel{V_{12}(x_1 - x_2)}u = Eu$$

assume we can neglect this

[eg. helium atom of 2 electrons]

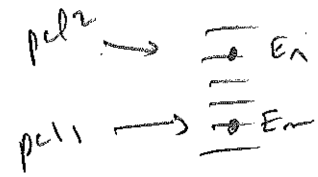
This has a solution $u_m(x_1)\, u_n(x_2)$

Proof:

$$\underbrace{\left[-\frac{\hbar^2}{2m}\frac{d^2 u_m}{dx_1^2} + V(x_1) u_m(x_1)\right]}_{E_m u_m(x_1)} u_n(x_2) + u_m(x_1)\underbrace{\left[-\frac{\hbar^2}{2m}\frac{d^2 u_n}{dx_2^2} + V(x_2)u_n(x_2)\right]}_{E_n u_n(x_2)} = E\, u_m(x_1)\, u_n(x_2)$$

$(E_m + E_n)\, u_m u_n = E\, u_m u_n$ so

Energy of system $= E_m + E_n$



We cd describe this by saying
pcl 1 has energy E_m
pcl 2 " " E_n

But there is another solution w/ same energy

$$u_n(x_1)\,u_m(x_2) \qquad \text{w/ } E = E_n + E_m$$

pcl 1 → E_n
pcl 2 → E_m

we say "pcl. 1 has en E_n
+ pcl 2 " " E_m"

In QM, we can also describe some w/ linear combinations

$$u_1(x_1,x_2) = \alpha\, u_m(x_1)\,u_n(x_2) + \beta\, u_n(x_1)\,u_m(x_2)$$

harder to describe: pcl 1 has prob. $|\alpha|^2$ of having E_m
+ prob $|\beta|^2$ of having E_n
while pcl 2 ...

"entangled state"

If the pcls are identical, they <u>must</u> be entangled

Bosons. $u(x_1,x_2) = \frac{1}{\sqrt{2}}\left[u_m(x_1)\,u_n(x_2) + u_n(x_1)\,u_m(x_2)\right]$
because must have $u(x_1,x_2) = u(x_2,x_1)$

Fermions $u(x_1,x_2) = \frac{1}{\sqrt{2}}\left(u_m u_n - u_n u_m\right)$
because must be antisymmetric

E_n ~~Observe~~ one pcl w/ en E_n
+ one pcl w/ en E_m
E_m

But if fermions; $m = n$ not allowed
⇒ can't be both in same state

Pauli exclusion principle