What is the nature of matter?

Particle:

electrons (Thomson 1897)
protons (Rutherford 1911)
neutrons (Chadwick 1932)

Wave:
de Broglie (1922) proposed $p = \hbar k = \frac{h}{\lambda}$ is also true for particle
$\lambda = \frac{h}{p}$ = de Broglie wavelength of a particle

Davisson & Germer (1925) observed interference of electrons

Light = waves of electric & magnetic fields ($\vec{E}, \vec{B}$)
Matter = waves of ?? ψ = matter wave
= wavefunction

Suppose that the ~~matter~~ wave describing a massive particle w/ energy E and momentum $\vec{p} = p\hat{x}$ has the form

$$\psi = \sin\left(\frac{px}{\hbar} - \frac{Et}{\hbar}\right)$$ Similar to travelling EM wave

What equation does this obey? [quote from Bloch]

Once at the end of a colloquium I heard Debye saying something like: "Schrödinger, you are not working right now on very important problems anyway. Why don't you tell us some time about that thesis of de Broglie, which seems to have attracted some attention?" So, in one of the next colloquia, Schrödinger gave a beautifully clear account of how de Broglie associated a wave with a particle, and how he could obtain the quantization rules of Niels Bohr and Sommerfeld by demanding that an integer number of waves should be fitted along a stationary orbit.

When he had finished, Debye casually remarked that he thought this way of talking was rather childish. As a student of Sommerfeld he had learned that, to deal properly with waves, one had to have a wave equation.

–Felix Bloch

My colleague Debye suggested that one should have a wave equation; well, I have found one.

–Erwin Schrödinger

debye.pi

Does ψ obey the classical wave eqn?

$$\frac{\partial^2 \psi}{\partial t^2} \stackrel{?}{=} c^2 \frac{\partial^2 \psi}{\partial x^2}$$

$$-\frac{E^2}{\hbar^2}\psi \stackrel{?}{=} -c^2 \frac{p^2}{\hbar^2}\psi$$

$$E^2 = c^2 p^2$$

only works for massless particles, such as the photon

Electrons have mass & move inside atoms at nonrelativistic speeds.

Use nonrelativistic approximation

$$p = mv$$
$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

[should do it relativistically but much more complicated so start with this.]

Start w/ a free electron (no forces) & ignore rest energy, all of its energy is kinetic

$$E = \frac{p^2}{2m}$$

Try to find a wave equation that yields this

$$E = \frac{p^2}{2m}$$

$$\psi = \sin\left(\frac{px}{\hbar} - \frac{Et}{\hbar}\right)$$

To get p^2, try $\frac{\partial^2}{\partial x^2}$ $\qquad \frac{\partial^2 \psi}{\partial x^2} = -\frac{p^2}{\hbar^2} \sin\left(\frac{px}{\hbar} - \frac{Et}{\hbar}\right)$

To get E, try $\frac{\partial}{\partial t}$ $\qquad \frac{\partial \psi}{\partial t} = -\frac{E}{\hbar} \cos\left(\frac{px}{\hbar} - \frac{Et}{\hbar}\right)$

Try a more general travelling wave

$$\psi = A \cos\left(\frac{px}{\hbar} - \frac{Et}{\hbar}\right) + B \sin\left(\frac{px}{\hbar} - \frac{Et}{\hbar}\right)$$

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} = -\frac{p^2}{\hbar^2}\psi \implies p^2\psi = -\hbar^2 \frac{\partial^2 \psi}{\partial x^2} \quad (*)}$$

Can we satisfy $\frac{\partial \psi}{\partial t} \sim \frac{E}{\hbar}\psi$?

$$\frac{E\psi}{\hbar} \stackrel{?}{=} D \frac{\partial \psi}{\partial t}$$

$$\frac{EA}{\hbar}\cos + \frac{EB}{\hbar}\sin \stackrel{?}{=} \frac{DAE}{\hbar}\sin(\,) - \frac{DBE}{\hbar}\cos$$

Match coeffs of sin & cos.

$$\left.\begin{aligned} B &= DA \\ A &= -DB \end{aligned}\right\} \longrightarrow \begin{aligned} B &= -D^2 B \\ D^2 &= -1 \\ D &= i \end{aligned}$$

Thus $\quad \frac{E\psi}{\hbar} = i \frac{\partial \psi}{\partial t} \implies E\psi = i\hbar \frac{\partial \psi}{\partial t} \quad (**)$

Also $B = DA = iA$ so

$$\boxed{\psi = A\left[\cos\left(\frac{px}{\hbar} - \frac{Et}{\hbar}\right) + i \sin\left(\frac{px}{\hbar} - \frac{Et}{\hbar}\right)\right]}$$

We want

$$E\psi = \frac{p^2}{2m}\psi$$

Use (**) and (*) to find

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}}$$

time-dependent Schrodinger eqn for a free particle

Schrodinger generalized this to a particle subject to external forces (e.g. Coulomb for electron in atom)

$$E = \frac{p^2}{2m} + V(x)$$

$V(x)$ = potential energy
[use V not U in QM]

$$E\psi = \frac{p^2}{2m}\psi + V(x)\psi$$

(*) and (**) are no longer strictly valid,
nonetheless Schrodinger postulated that [Jan 1926]

$$i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(x)\psi$$

t.d.S.e for a particle in a potential