Part II: Quantum physics

Light travels at const speed c in vacuum (Roemer: 1676)

What is the nature of light?

Newton (1666) corpuscles [explains straight line motion]
vs.
Huygens (1690) waves [explains const speed]

Young (1801) 2-slit experiment demonstrates interference

Maxwell (1864) electromagnetic waves travel at c

$$\vec{E} = \vec{E}_0 \sin(\vec{k}\cdot\vec{x} - \omega t) \quad \text{with} \quad \omega = c|\vec{k}|$$

ω = angular frequency (rad/sec)

$f = \frac{\omega}{2\pi}$ = frequency ($\frac{\text{cycles}}{\text{sec}}$ = Hz)

Planck (1900) explains blackbody spectrum
Einstein (1905) explain photoelectric effect
by asserting that energy of light waves is quantized.
Light consists of particles (photons) of energy

$$E = hf = \frac{h\omega}{2\pi} = \hbar\omega$$

where h = Planck's const, $\hbar = \frac{h}{2\pi}$

wave-particle duality [not getting rid of wave picture]

Compton scattering (1923) demonstrated that photons have momentum $p = \frac{E}{c}$

i.e. $E = cp$. Since $E^2 = (cp)^2 + (mc^2)^2$, then $\Rightarrow$ <u>photons are massless</u>

$$p = \frac{E}{c} = \frac{\hbar \omega}{c} = \hbar |\vec{k}| \quad \Rightarrow \quad \vec{p} = \hbar \vec{k}$$

$$\begin{tikzcd}
E \arrow[r, leftrightarrow, "E = cp"] \arrow[d, leftrightarrow, "E = \hbar\omega"'] & \vec{p} \arrow[d, leftrightarrow, "\vec{p} = \hbar \vec{k}"] \\
\omega \arrow[r, leftrightarrow, "\omega = ck"'] & \vec{k}
\end{tikzcd}$$

$$\left.\begin{array}{l} E = \hbar\omega \\ \vec{p} = \hbar \vec{k} \end{array}\right\} \Rightarrow p^\mu = \hbar k^\mu$$

$$\vec{E} = \vec{E}_0 \sin(\vec{k}\cdot\vec{x} - \omega t)$$
$$= \vec{E}_0 \sin\left(\frac{\vec{p}\cdot\vec{x}}{\hbar} - \frac{Et}{\hbar}\right)$$ describes a travelling wave.

The wave equation is a linear equation

$$\frac{\partial^2 A}{\partial t^2} - c^2 \frac{\partial^2 A}{\partial x^2} = 0$$

[each term contains only one factor of A]

$\Rightarrow$ linear combinations of solutions are also solutions

If A_1 + A_2 are solutions, so is $\alpha A_1 + \beta B_1$
for const α, β

Travelling wave solutions $\sin(kx - \omega t)$
can be written as linear combinations of
standing wave solutions $\sin(kx)\cos(\omega t)$,
$\sin(kx)\sin(\omega t)$

and vice versa..

Exercise use trig identities to express a standing wave

$$\sin(kx)\cos(\omega t)$$

as a superposition (linear comb.) of travelling waves

[Answer. $\sin(kx)\cos(\omega t) = \frac{1}{2}\left(\sin(kx+\omega t) + \sin(kx-\omega t)\right)$]

[After answer is presented in class: go over this:

Comment: ~~because~~ the wave eqn is linear (in A), any linear comb. of solutions is also a solution

Math tricks: You should $\boxed{\text{memorize}}$ these!

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$

Then use $\cos(-B) = \cos B$ (even) and $\sin(-B) = -\sin B$ (odd)

to recall (don't memorize)

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$
$$\sin(A-B) = \sin A\cos B - \sin B\sin A$$

Also $\boxed{\text{know}}$ special cases: $A = B$

$$\Rightarrow \cos 2A = \cos^2 A - \sin^2 A$$
$$\sin 2A = 2\sin A\cos A$$

In the second set of formulas, $A = B$ gives

$$\cos 0 = 1 = \cos^2 A + \sin^2 A \checkmark$$
$$\sin 0 = 0 = 0$$

already recalled in earlier exercise

on board

Note to instructor. [~~These will reappear soon when introduce complex exps!~~]