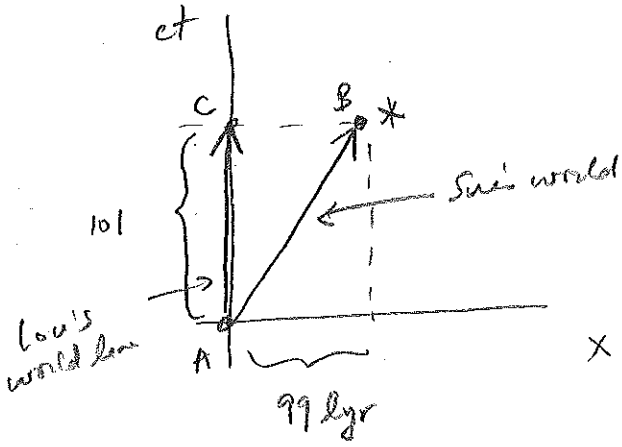
Twin paradox

[Sue & Lou separated from birth.
Sue "volunteers" for mission to distant star * 99 ly from earth.
Trip takes 101 year earth time.]



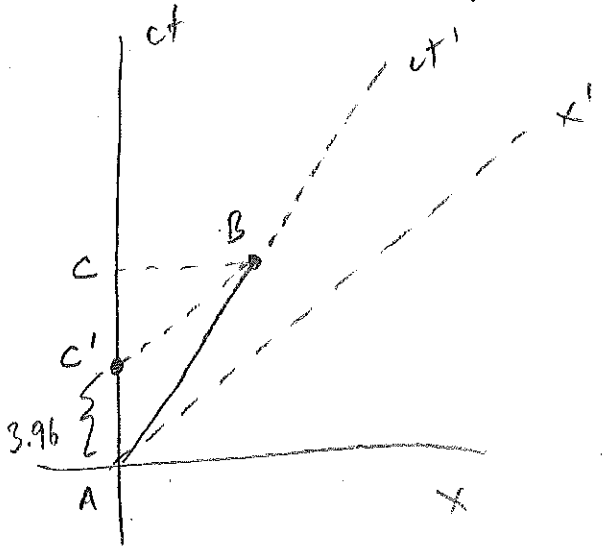
• Speed of rocket $\beta = \frac{99}{101}$

• $\gamma = \frac{1}{\sqrt{1-\left(\frac{99}{101}\right)^2}} = \frac{101}{\sqrt{101^2-99^2}} = \frac{101}{20} \sim 5$

Lou says: "Sue, when you reach *, you'll be $\frac{101}{\gamma} = 20$ year old, and I'll be 101."

[$ct' = \gamma(ct - \beta x) = \frac{101}{20}\left(100 - \frac{99}{101}\cdot 99\right) = \frac{101^2-99^2}{20} = 20$]

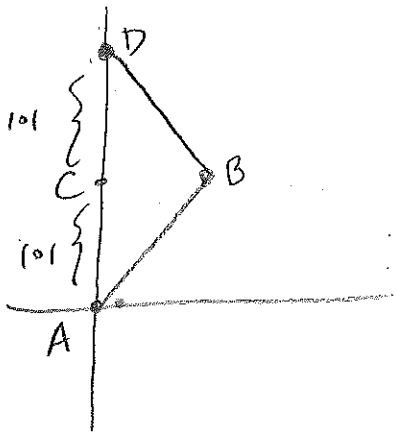
Sue says: "I agree that I will be 20 when I reach *, but you'll only be $\frac{20}{\gamma} \approx 3.96$ year old"



[Simultaneity is relative]

[$ct' = \gamma(ct - \beta x) \Rightarrow 20 = \frac{101}{20}(ct - 0) \Rightarrow ct = \frac{20^2}{101} \approx 3.96$]

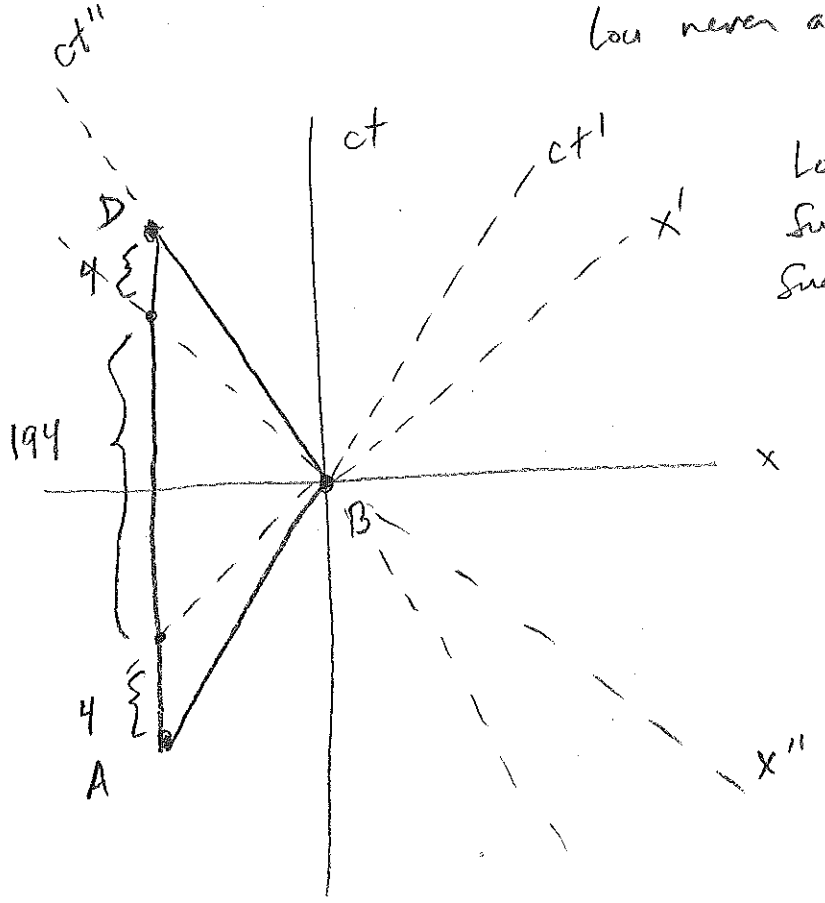
But who will be older when Sue comes home?



Lou will be 202
but Sue will be 40.

Scenario is not symmetric because Sue has to reverse velocity at B, ∴ accelerate.
During this process, she is in a noninertial frame.

Lou never accelerates!



Lou's frame = S
Sue's outgoing frame = S'
Sue's returning frame = S''
[use 3 colors]

Lou ages slowly on Sue's trip out and back (4 yrs each)
but he appears to age 194 yrs while Sue is accelerating

To the right of B time seems to run in during acceleration

[Twin who experienced no acceleration ages the most]

[cf MTW "accel ref frames: limit in size"]