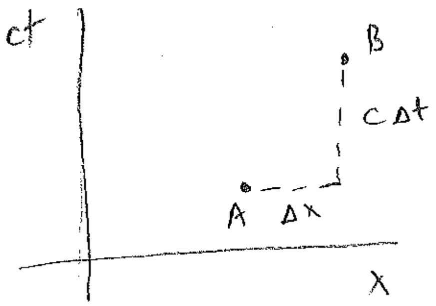
Consider a pair of events A + B
w/ spacetime separation $(c\Delta t, \Delta x, \Delta y, \Delta z)$.



The invariant interval between the events is

$$\Delta S^2 = -(c\Delta t)^2 + |\Delta \vec{x}|^2$$

There are 3 possibilities:

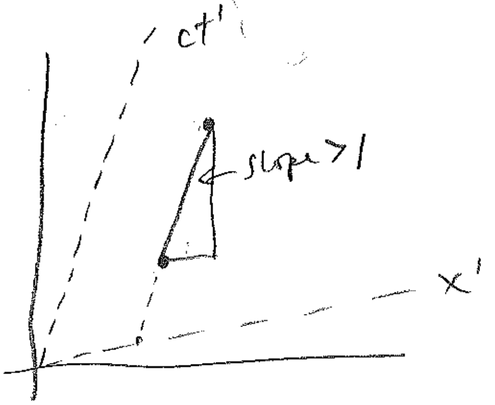
① $c\Delta t > |\Delta \vec{x}| \Rightarrow \Delta S^2 < 0$ time-like separation

② $c\Delta t < |\Delta \vec{x}| \Rightarrow \Delta S^2 > 0$ space-like separated

③ $c\Delta t = |\Delta \vec{x}| \Rightarrow \Delta S^2 = 0$ light-like separated

This division into cases is independent of reference frame

- If 2 events are time-like separated, $\Rightarrow \frac{c\Delta t}{|\Delta x|} > 1$
  $\exists$ an IRF S' in which they occur at same place
  (they are separated only by time).



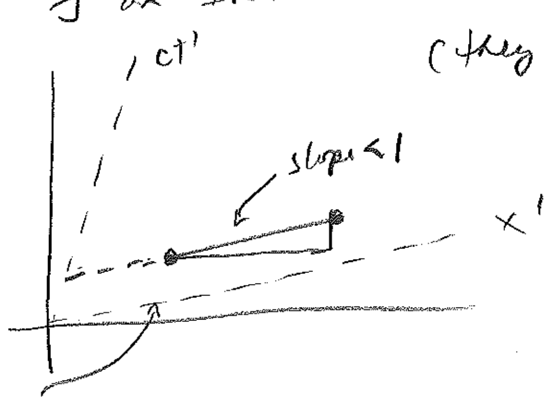
In this frame, separation is

$$(c\Delta\tau, 0, 0, 0)$$

where $\Delta\tau$ = proper time between the events.

$$\Delta s^2 = -(c\Delta\tau)^2 < 0.$$

- If 2 events are space-like separated, $\frac{c\Delta t}{|\Delta x|} < 1$
  $\exists$ an IRF S' in which they occur at the same time
  (they are separated by pure space)



Then

$$\Delta s^2 = +(\Delta\sigma)^2 > 0$$

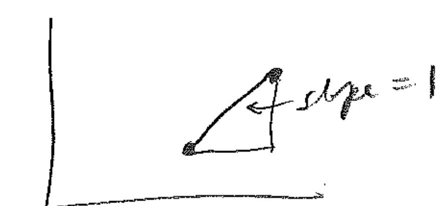
where

$\Delta\sigma$ = proper distance
= distance in frame in which they are simultaneous

$\left[\frac{V_0}{c} = \frac{c\Delta t}{\Delta x} \right]$

- If 2 events are light-like separated,
  a light signal can travel between them.

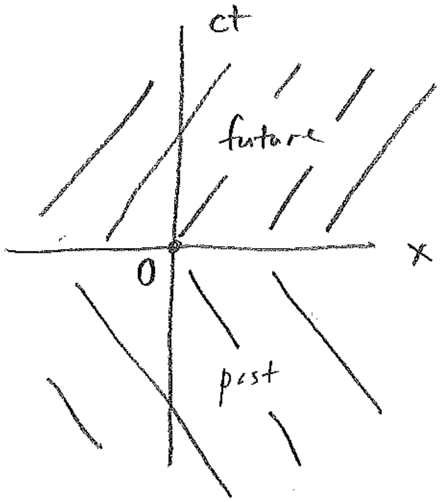
$$\Delta s^2 = 0$$



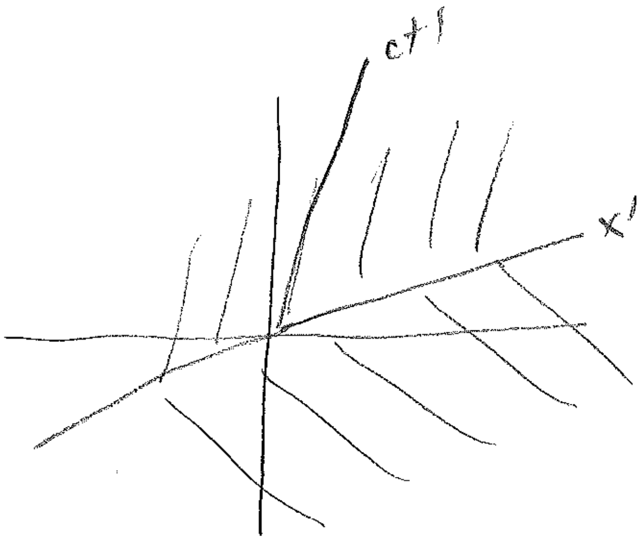
[cannot define proper time or proper distance between them because $\exists$ no IRF in which they occur at same place or same time]

Future and past

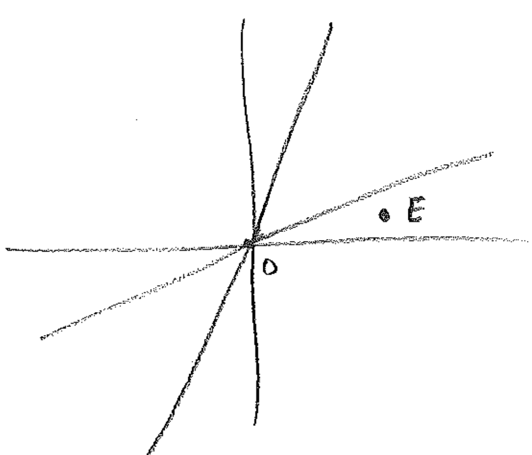
Conventional view: future of O = all events occurring after O, ie $t > 0$
past of O = " " " before O, ie $t < 0$



[But just as simultaneity is relative, so is future + past]

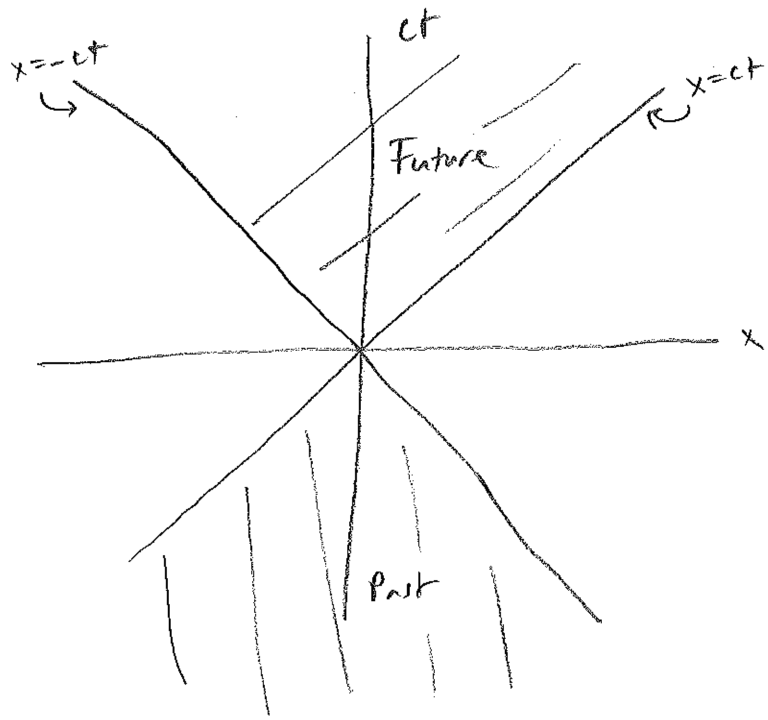


How can E be in O's future (in S) + also in O's past (in S')?



New definition:

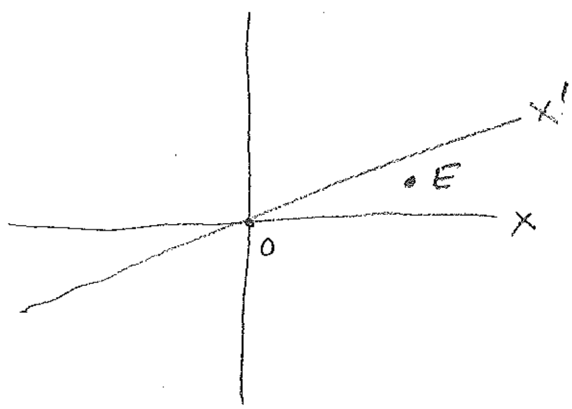
Future of O = all events occurring after O in all IRFs
Past " " = " " " before " " " "



Events in O's future or past are time-like or light-like separated from O ($\Delta s^2 \leq 0$).

Events spacelike separated from O are neither in future nor past
("elsewhere")

If we accept that the future cannot affect the past, then there can be no causal connection between space-like separated events (such as O and E)



In S, O precedes E
so E cannot affect O

In S', E precedes O
so O cannot affect E

$\Rightarrow$ No causal connection possible

Why can't a signal travel faster than c?

If $v > c$, then could send a signal from O to E
& $\therefore$ O could affect E, but not possible

[There was a lady from Wright]

optional

Since this clock is moving, it runs slow.

$$\Delta t = \gamma \Delta \tau$$

$$(\Delta t)^2 = \frac{(\Delta \tau)^2}{\left(1 - \frac{V_0^2}{c^2}\right)}$$

$$(\Delta \tau)^2 = (\Delta t)^2 - \frac{(V_0 \Delta t)^2}{c^2}$$ but $V_0 \Delta t = \Delta x$

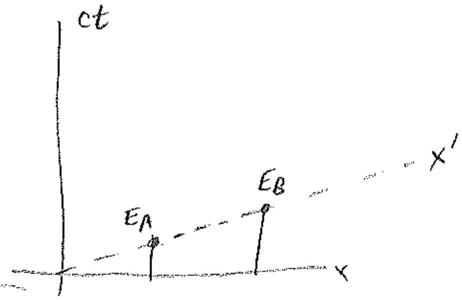
$$(\Delta \tau)^2 = (\Delta t)^2 - \frac{(\Delta x)^2}{c^2}$$

$$(c \Delta \tau)^2 = (c \Delta t)^2 - (\Delta x)^2$$

Since $(c\Delta t)^2 = (c \Delta \tau)^2 + (\Delta x)^2 \geq (c \Delta \tau)^2$

the proper time is the minimal coordinate time

separation between time-like separated events.



optional

$$\Delta x' = \Delta\sigma \quad \Rightarrow \quad \Delta x = \gamma \, \Delta\sigma$$

$$(\Delta x)^2 = \frac{(\Delta\sigma)^2}{1 - \frac{v_0^2}{c^2}}$$

$$(\Delta\sigma)^2 = (\Delta x)^2 \left(1 - \frac{v_0^2}{c^2}\right)$$

$$= (\Delta x)^2 \left[1 - \frac{(c\Delta t)^2}{(\Delta x)^2}\right]$$

$$\boxed{(\Delta\sigma)^2 = (\Delta x)^2 - (c\,\Delta t)^2}$$

Since $(\Delta x)^2 = (\Delta\sigma)^2 + (c\Delta t)^2 \geq (\Delta\sigma)^2$

the proper distance is the <u>minimal</u> spatial coordinate separation between space-like separated events

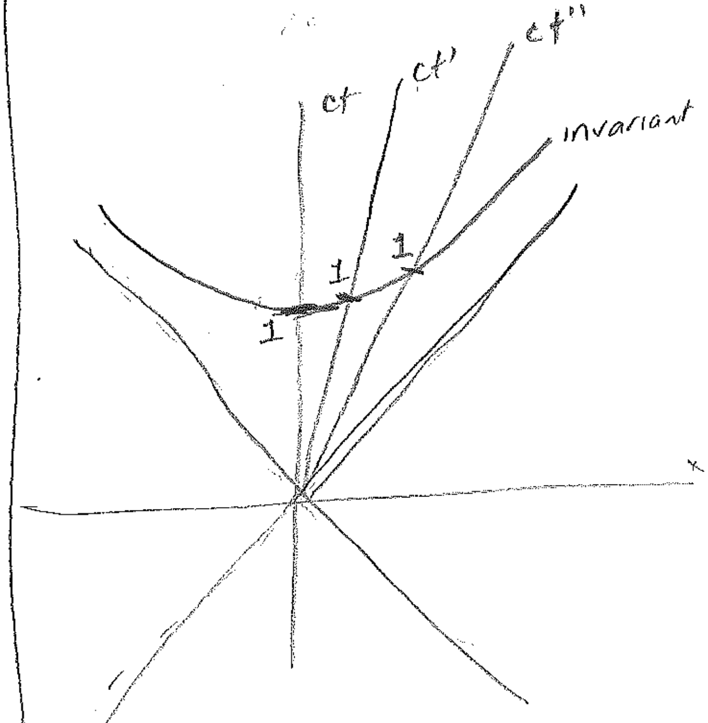
[NB. not to be confused w/ the <u>proper length</u> or <u>rest length</u> of an object, which is <u>maximal</u>.]

optional

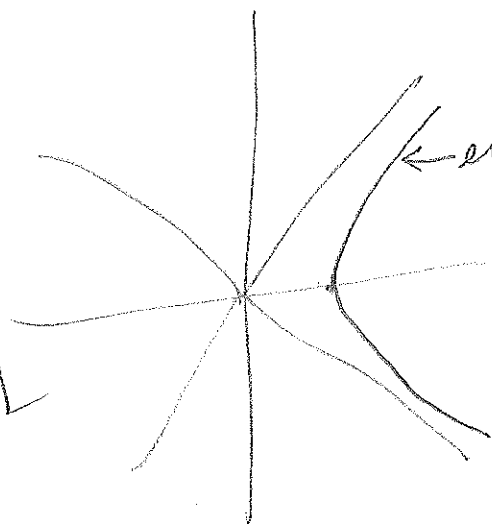
Consider all events separated from origin by a fixed time-like interval

$$\Delta s^2 = x^2 + y^2 + z^2 - (ct)^2 = \text{const} < 0$$

↑ eqn for hyperbola



This fixes the scale in the t' axis.
since $\Delta s^2 = -(\text{proper time})^2$



← events separated from origin by fixed space-like interval