Recall non-relativistic (Galilean) addition of velocities

$$\vec{V} = \vec{V}' + \vec{V}_0$$

For a Galilean boost in the $+x$ direction by v_0

$$v_x = v_x' + v_0$$

$$v_y = v_y'$$

These are accurate for velocities much less than c.
What happens more generally? [race car on a train]

Lorentz boost: $$x = \gamma(x' + v_0 t')$$
$$t = \gamma(t' + \frac{v_0}{c^2} x')$$

Let $\Delta x = x_1 - x_2$ and $\Delta t = t_1 - t_2$ then

$$\Delta x = \gamma(\Delta x' + v_0 \Delta t')$$
$$\Delta t = \gamma(\Delta t' + \frac{v_0}{c^2}\Delta x')$$

Consider $$\frac{\Delta x}{\Delta t} = \frac{\gamma(\Delta x' + v_0 \Delta t')}{\gamma(\Delta t' + \frac{v_0}{c^2}\Delta x')} = \frac{\frac{\Delta x'}{\Delta t'} + v_0}{1 + \frac{v_0}{c^2}\frac{\Delta x'}{\Delta t'}}$$

But $v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$ and $v_x' = \lim_{\Delta t' \to 0} \frac{\Delta x'}{\Delta t'}$ so

$$v_x = \frac{v_x' + v_0}{1 + \frac{v_0 v_x'}{c^2}}$$ (relativistic addition of velocities)

If $v \ll c$, then
$v_x \approx v_x' + v_0$

Also, $$v_x' = \frac{v_x - v_0}{1 - \frac{v_0 v_x}{c^2}}$$

eg. race car on a train

$$v_0 = \frac{3}{4}c, \quad v_x' = \frac{3}{4}c \Rightarrow v_x = \frac{\frac{3}{2}c}{1+\frac{9}{16}} = \frac{24}{25}c < c$$

flashlight on a train

$$v_0 = \frac{3}{4}c, \quad v_x' = c \Rightarrow v_x = \frac{c + \frac{3}{4}c}{1+\frac{3}{4}} = c \quad \checkmark$$

All velocities are relative, except for c, which is absolute

[There is no privileged IRF, only a privileged velocity]

Exercise: $$v_y = \frac{v_y'}{\gamma\left(1 + \frac{v_0 v_x'}{c^2}\right)}$$

If $v \ll c$, $\gamma \to 1 \Rightarrow v_y \approx v_y'$