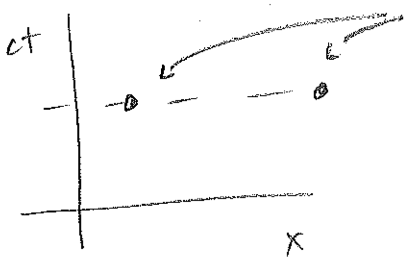
Simultaneity

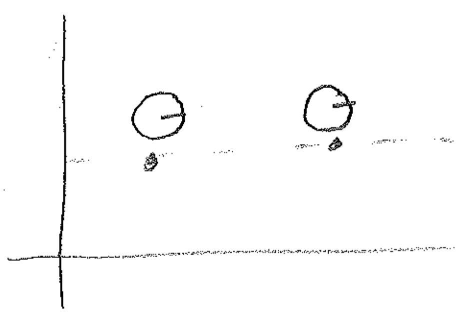


Two events are simultaneous if they occur at the same time,

ie have the same t-coordinate

[Since time is what clocks measure]

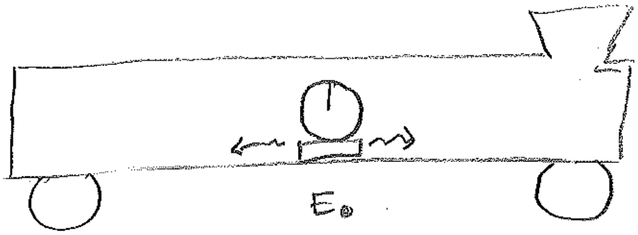
ie. if clocks at each event read the same



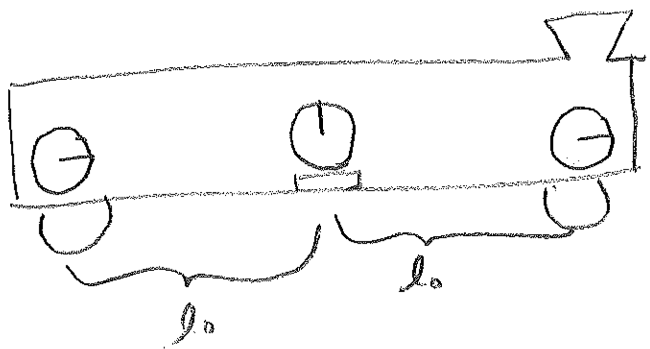
[For this to work, different clocks must be synchronized.]

How to synchronize clocks on a train using light (radio) signals

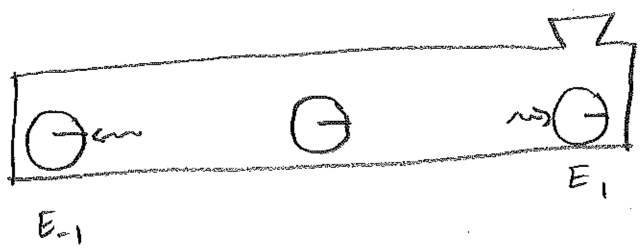
Train frame = S'



(a) send a radio signal from the central "master" clock at $t' = 0$. Call this event E_0



(b) Preset the other clocks to $t' = \frac{l_0}{c}$ where
l_0 = distance from master clock

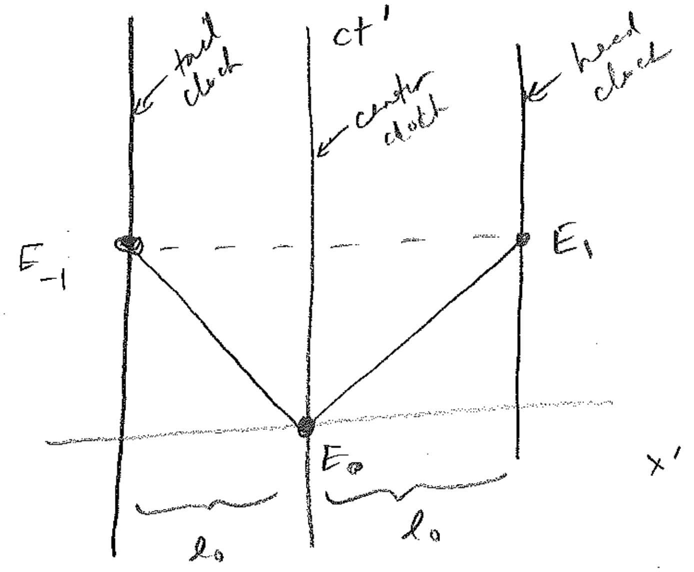


(c) When radio pulse reaches the clock, it starts

E_1 = light reaches clock at head of train

E_{-1} = light reaches clock at tail of train

clocks are synchronized!



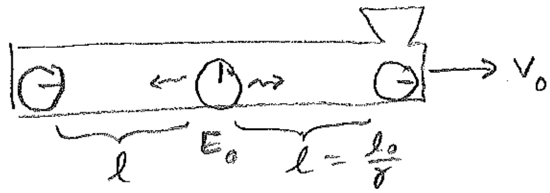
ct' coordinates of E_1 and E_{-1} are both l_0

$\Rightarrow E_1 + E_{-1}$ are simultaneous in S'

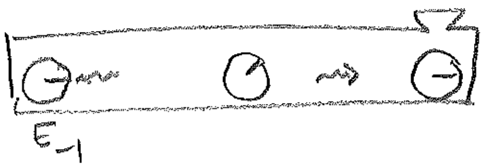
horizontal line = surface of simultaneity in S'

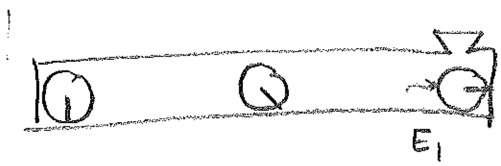
Train is moving at speed v_0 w.r.t. station

View synchronization procedure from station frame S



According to S,
tail clock starts <u>prematurely</u> because
(a) tail clock is moving toward signal
(b) distance between clocks is contracted





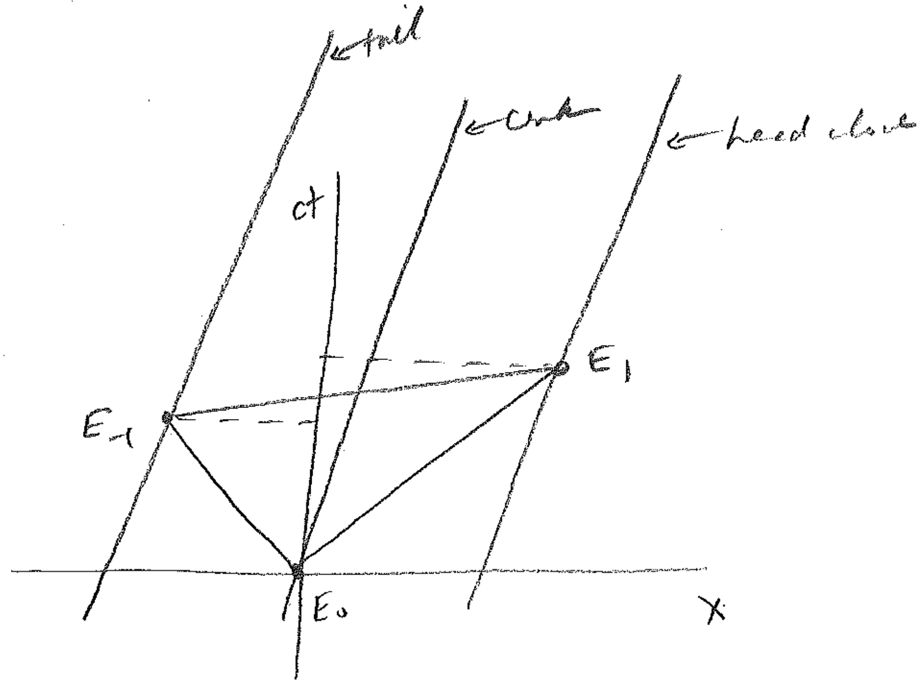
head clocks starts <u>late</u> because
head clock is moving away from signal

[somewhat but not completely
mitigated by the contraction of length]

E_{-1} and E_1 are <u>not</u> simultaneous in S.

S observer judges that S' clocks are
<u>not</u> properly synchronized

Leading clocks lag and lagging clocks <u>lead</u>.



ct coordinates of E_1 and E_{-1} differ!

E_1 and E_{-1} are not simultaneous in S

They are simultaneous in S'

Slanted line is a surface of simultaneity in S'

<u>Exercise</u>: Use spacetime diagram to show

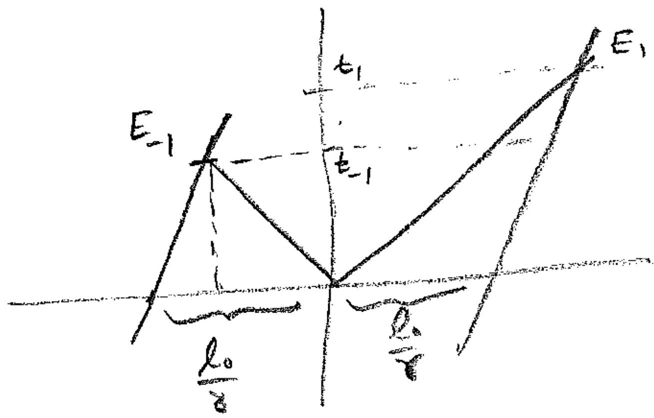
ct-coordinate of E_{-1} is $ct_{-1} = l_0 \sqrt{\frac{c-v_0}{c-v_0}} < ct'_{-1}$

ct-coordinate of E_1 is $ct_1 = l_0 \sqrt{\frac{c+v_0}{c-v_0}} > ct'_1$

<u>Exercise</u>: show that slope of dashed line is $\frac{v_0}{c}$

Exercise answer

What are the coordinates of E_{-1} and E_1?



$$v_0 t_{-1} + c t_{-1} = \frac{l_0}{\gamma} \qquad t_{-1} = \frac{l_0\sqrt{1-\frac{v_0^2}{c^2}}}{c+v_0} = \frac{l_0}{c}\sqrt{\frac{c-v_0}{c+v_0}}$$

$$c t_1 - v_0 t_1 = \frac{l_0}{\gamma} \qquad t_1 = \frac{l_0\sqrt{1-\frac{v_0^2}{c^2}}}{c-v_0} = \frac{l_0}{c}\sqrt{\frac{c+v_0}{c-v_0}}$$

$$\text{slope} = \frac{c\left(\frac{l_0}{c}\sqrt{\frac{c+v_0}{c-v_0}} - \frac{l_0}{c}\sqrt{\frac{c-v_0}{c+v_0}}\right)}{l_0\sqrt{\frac{c+v_0}{c-v_0}} + l_0\sqrt{\frac{c-v_0}{c+v_0}}} = \frac{(c+v_0)-(c-v_0)}{(c+v_0)+(c-v_0)} = \frac{v_0}{c}$$

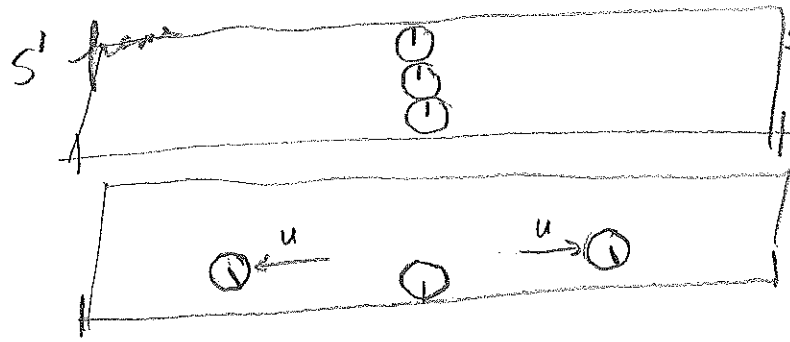
or

$$\frac{c\left(\frac{l_0}{c}\sqrt{\frac{c+v_0}{c-v_0}} - \frac{l_0}{c}\sqrt{\frac{c-v_0}{c+v_0}}\right)}{2l_0\sqrt{1-\frac{v^2}{c^2}}} = \frac{c+v_0 - (c-v_0)}{2c}$$

$$= \frac{v_0}{c}$$

$$\frac{l_0 2c}{\sqrt{c^2-v_0^2}}$$

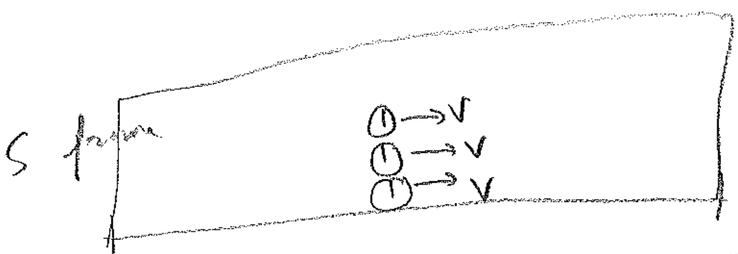
Alternative method of synchronization [just talk this through]



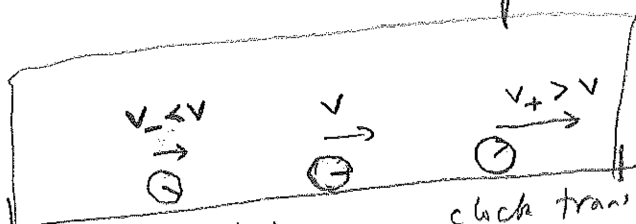
synchronize all clocks at the same place

carry them to the ends of the train

During transport, clocks lose some time due to their motion but they'll both lose same amount so they'll still be synchronized with one another in S' [can correct for this]



all clocks initially synchr. & all run slow by factor $\gamma = 1/\sqrt{1-v^2/c^2}$



clock transported to rear of train runs slow by a smaller fact

$$\frac{1}{\sqrt{1-\frac{v_-^2}{c^2}}} < \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

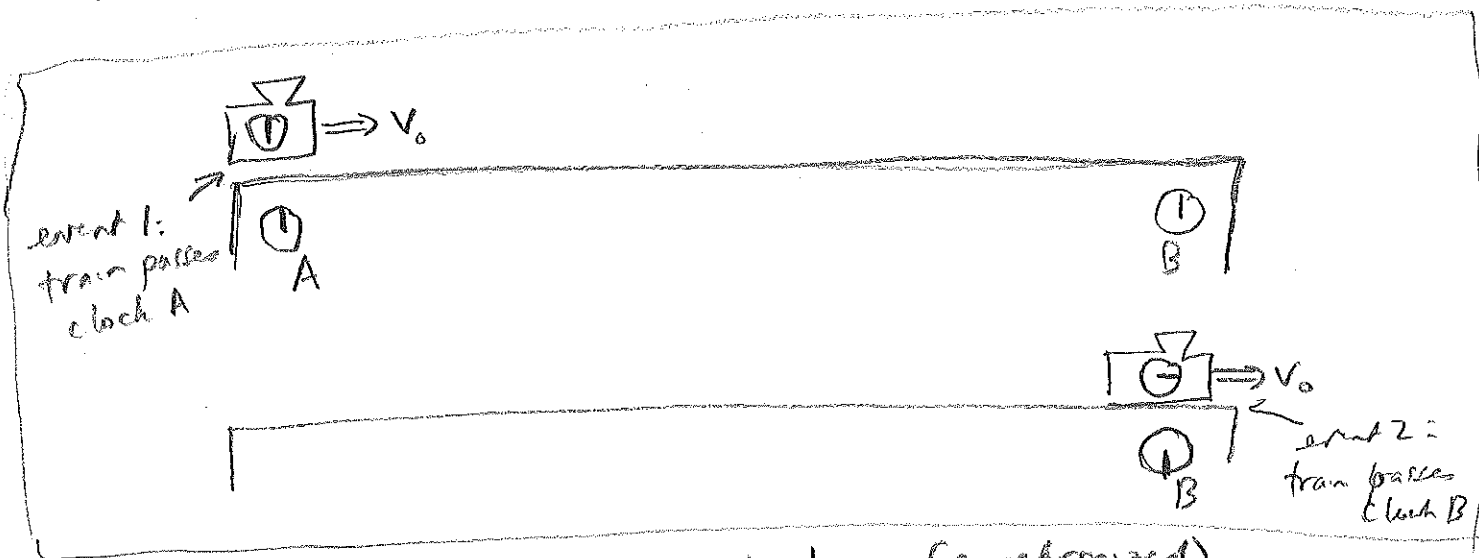
clock transported to front of train runs slow by a larger factor $\frac{1}{\sqrt{1-\frac{v_+^2}{c^2}}} > \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

clocks at ends of train do not appear synchronized w/ each other in S.

Leading clocks lag + lagging clocks lead

Time dilation paradox: each observer correctly claims that the other's clocks run slow. How can both be right?

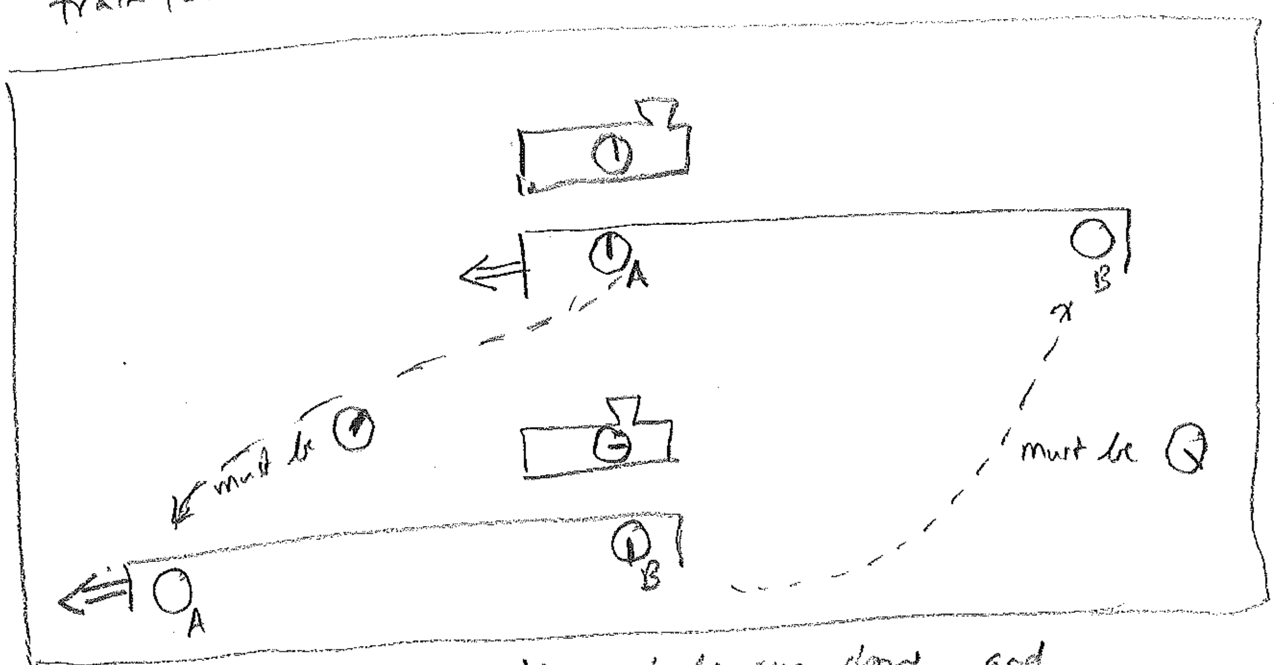
station frame S



Train clock is compared against two (synchronized) station clocks

station to train: your clock runs slow!

train frame S'



Train to station: Your clocks run slow and they are also unsynchronized!