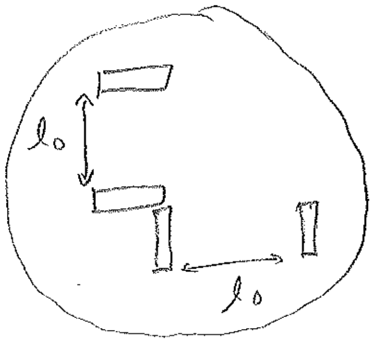
Length contraction: a moving object is observed to contract along the direction of motion.

[Let's calculate how much]

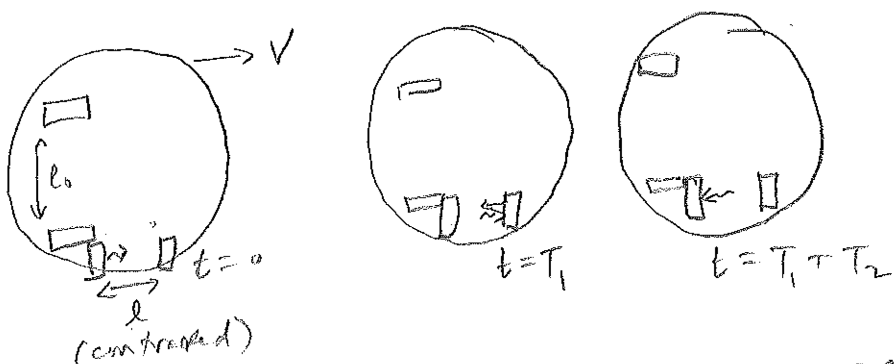
2 stationary light clocks, aligned perpendicular to one another



At rest, both clocks tick at same rate

$$T = \frac{2l_0}{c}$$

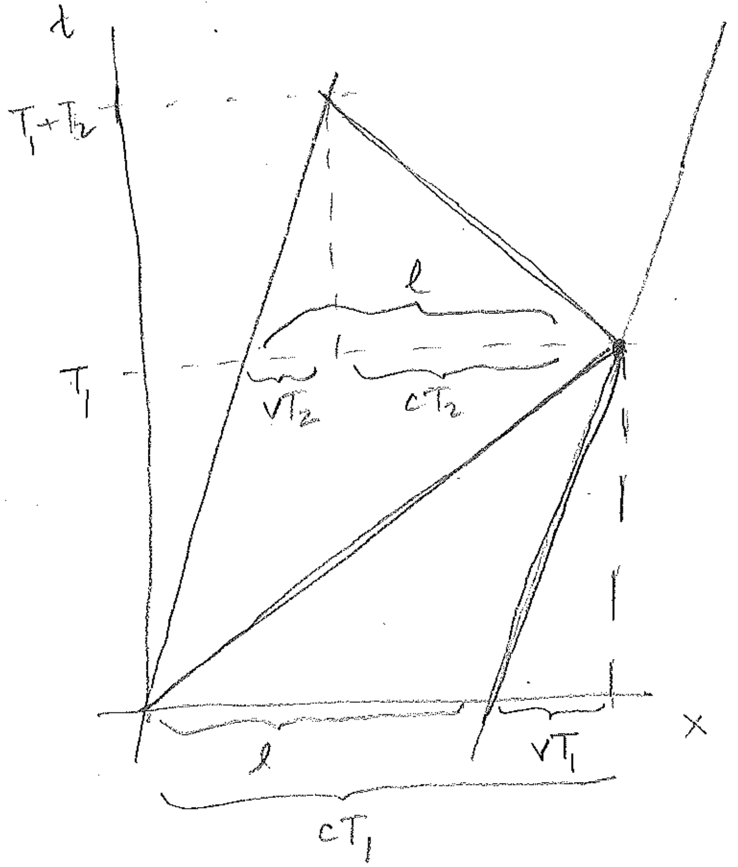
[Bolt them to a frame: Acme dual photochronometer]
[Put them on a train]



The "transverse" clock ticks w/ period $T_{tr} = \frac{2l_0}{c}\gamma$ $\qquad \gamma = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}}$

The "longitudinal" clock ticks w/ period $T_{long} = T_1 + T_2$

(takes longer to reach right mirror because it is moving away from the light)



$$l + vT_1 = cT_1 \Rightarrow T_1 = \frac{l}{c-v}$$

$$l = cT_2 + vT_2 \Rightarrow T_2 = \frac{l}{c+v}$$

$$\therefore T_{lng} = \frac{l}{c-v} + \frac{l}{c+v} = \frac{2lc}{c^2-v^2} = \frac{2l}{c\left(1-\frac{v^2}{c^2}\right)} = \frac{2l}{c}\gamma^2$$

Both moving clocks tick at same rate [why?]

$$T_{tr} = T_{lng}$$

$$\frac{2l_0}{c}\gamma = \frac{2l}{c}\gamma^2$$

$$l_0 = l\gamma$$

$\boxed{l = \frac{l_0}{\gamma}}$ contracted by factor of γ from its rest length l_0

Rest length of an object is its length measured in its own rest frame (& is a maximum)

There was once a young man named Fisk
Whose fencing was exceedingly brisk.
So fast was his action
That Lorentz contraction
Reduced his rapier to a disk.

[optional]

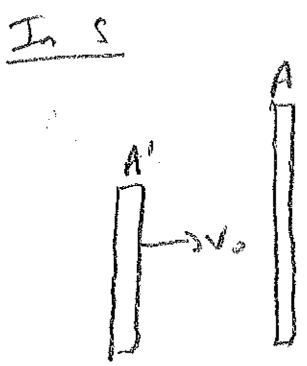
Do objects contract along directions transverse (perpendicular) to their motion? No.

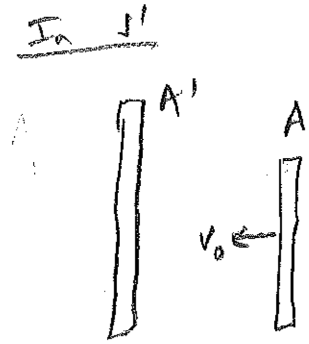
Suppose they did.

Let A be a meter stick at rest in S

Let A' be a meter stick at rest in S'

S' moves to right w/ speed v_0 w.r.t. S





When they pass, either $A > A'$ or $A < A'$, not both

[put a blade on end of A'. Either A gets shortened or not.]

Contradiction. Therefore $A = A'$.

No transverse length contraction

Exercise Solution

$(1+\epsilon)^2 = 1 + 2\epsilon + \epsilon^2$

$(1+\epsilon)^3 = 1 + 3\epsilon + 3\epsilon^2 + \epsilon^3$

$(1+\epsilon)^n = 1 + n\epsilon + \dots$

$$V = \frac{5\,\mathrm{km}}{\mathrm{hr}} = 1.39\ \frac{\mathrm{m}}{\mathrm{s}}$$

$$\frac{v}{c} = 4.6\,E{-9}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \approx 1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 = 1 + 1.07 \times 10^{-17}$$

Not the way to do it

$$= \frac{1}{\sqrt{(1+\frac{v}{c})(1-\frac{v}{c})}} = \frac{1}{\sqrt{1+\frac{v}{c}}}\ \frac{1}{\sqrt{1-\frac{v}{c}}} =$$

$$= \frac{1}{\sqrt{1+5E{-9}}}\ \frac{1}{\sqrt{1-5E{-9}}} =$$

$$\left(1 - \frac{1}{2}\frac{v}{c} - \frac{1}{8}\frac{v^2}{c^2}\right)\left(1 + \frac{v}{2c} - \frac{1}{8}\frac{v^2}{c^2}\right)$$

$$= 1 - \underbrace{\left(\frac{1}{8} + \frac{1}{8} + \frac{1}{4}\right)}_{\frac{1}{2}}\frac{v^2}{c^2}$$