What is time?

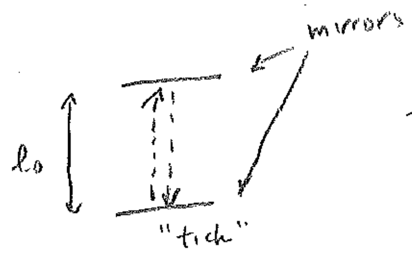
Einstein: time is what clocks measure

[What is a clock? Something that
executes periodic motion,
measuring out equal increments of time.
Pendulum clock.
Pulse (Galileo).
Any kind of harmonic oscillator]

Fundamental assumption

Light travels at const speed c (in vacuum)
in every IRF

Light clock: a gedanken clock



This clock ticks every $\frac{2l_0}{c}$

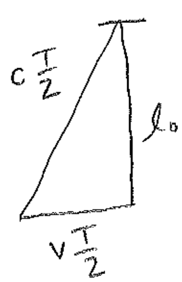
[fig 1-7 Serway]

Moving light clock ticks more slowly because light travels further between ticks

Mirrors moving right at speed v
Clock ticks every T

mirror travels distance vT

light travels distance cT



$$l_0^2 + v^2\left(\frac{T}{2}\right)^2 = c^2\left(\frac{T}{2}\right)^2$$

$$l_0 = \sqrt{c^2 - v^2}\left(\frac{T}{2}\right)$$

$$= \sqrt{1 - \frac{v^2}{c^2}}\left(\frac{cT}{2}\right)$$

$$T = \frac{2l_0}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Moving clock ticks every $\frac{2l_0}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} > \frac{2l_0}{c}$

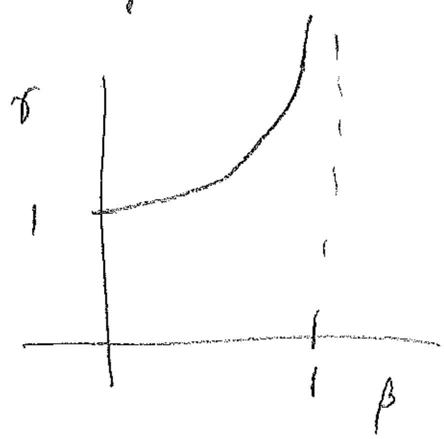
more slowly than stationary clock

Define $\beta = \frac{v}{c}$ (fraction of speed of light)

Define time dilation factor $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

For slow speeds, $\beta \approx 0 \Rightarrow \gamma \approx 1$

For speeds close to light, $\beta \approx 1 \Rightarrow \gamma \to \infty$



Stationary light clock ticks every $\frac{2l_0}{c}$

Moving light clock ticks every $\gamma \cdot \left(\frac{2l_0}{c}\right)$

ie more slowly

principle of relativity: all IRFs are equivalent wrt laws of physics

$\Rightarrow$ no experiment can distinguish between stationary + moving frame

If two clocks tick at same rate in one IRF,
they tick at same rate in all IRF's
otherwise one could use the discrepancy to
distinguish between frames

If moving light clock ticks more slowly,
so do all other clocks.

$\therefore$ Time also flows more slowly,
because time is what clocks measure.

This may seem very theoretical.
Most clocks don't move fast enough for us to notice time dilation.
Need a clock that moves very fast, close to speed of light.

Answer: cosmic rays.

Cosmic rays are very fast moving particles, mostly protons that strike air molecules in the upper atmosphere.
These collisions produce pi mesons,
unstable particles that decay quickly into muons ($T_{\frac{1}{2}} \sim 10^{-8}s$)

Muons are also unstable and decay into electrons + neutrinos.

$\mu^- =$ unstable particle w/ half-life $T_{\frac{1}{2}} = 1.5 \times 10^{-6} s$

half-life: 50% prob. of decay before $T_{\frac{1}{2}}$.
If have large sample of μ^-, half will decay in 1.5 μs
Half of remainder decay in next 1.5 μs.

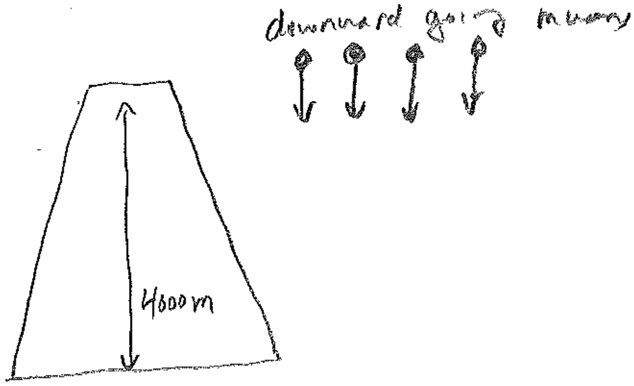
How far can they travel in $T_{\frac{1}{2}}$?

If $v \approx c$, then $d = c T_{\frac{1}{2}} = 450$ m.

Created in upper atmosphere, many are observed at ground level. How is this possible? Time dilation

They last 1.5 μs in their own frame.
To us, they tick (decay) much more slowly.

[MIT experiment → see blackboard.]



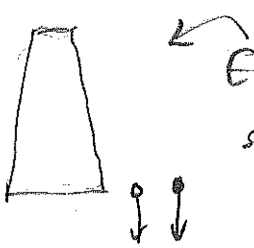
Suppose there are N_0 muons [per m² per sec] at top of mtn

How many survive trip to ground level?

If $v \approx c$, then travel time $t = \frac{4000\,m}{c} = 1.3 \times 10^{-5}\,s = 9\,T_{\frac{1}{2}}$

Nine half lives: expect $\left(\frac{1}{2}\right)^9 N_0 \approx 0.002\, N_0$

[2 for every thousand]



Experimentally, half survive,
so trip takes only one half life in muon frame, $t' = T_{\frac{1}{2}}$

$\gamma = 9 \quad \Rightarrow \beta = 0.994, \qquad v = 0.994\,c$

Atomic clocks placed on aircraft also tick more slowly

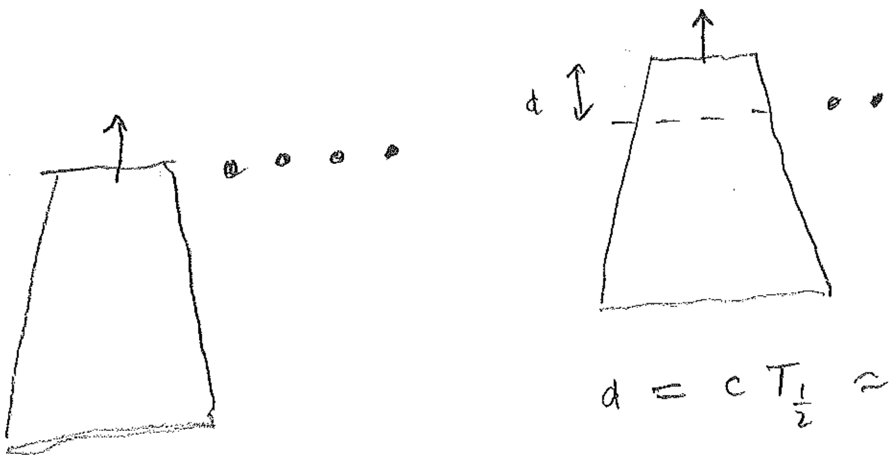
[exercise 4]

Wolfram α still
does not do this 2017

The muon and the mountain paradox

In μ rest frame, mountain is moving upward at $v \approx .994c$

How far does mountain travel in one half-life?

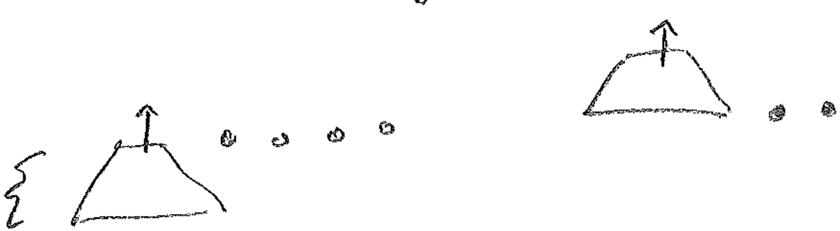


$$d = cT_{\frac{1}{2}} \approx 450 \text{ m}$$

But we know that half of muons reach bottom!

Resolution: Moving mountain is length contracted by factor of $\gamma \approx 9$ and is only $\frac{4000\text{m}}{\gamma} \approx 450$ m tall

450 m



[Paul Simon: one man's ceiling is another man's floor]

One observer's time dilation is another observer's length contraction.