Unlike Newton's laws, the wave eqn

$$\frac{\partial^2 A}{\partial t^2} - c^2 \left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2} \right) = 0 \qquad (*)$$

is <u>not</u> invariant under Galilean boosts.

Under a boost in the x-direction by v_0, it transforms into

$$\frac{\partial^2 A}{\partial t'^2} - 2v_0 \frac{\partial^2 A}{\partial x' \partial t'} - (c^2 - v_0^2) \frac{\partial^2 A}{\partial x'^2} - c^2 \left(\frac{\partial^2 A}{\partial y'^2} + \frac{\partial^2 A}{\partial z'^2} \right) = 0 \qquad (**)$$

[tedious exercise in chain rule for partial derivatives]

- Therefore (*) only holds in <u>one</u> IRF presumably the frame in which its medium (ether) is at rest.

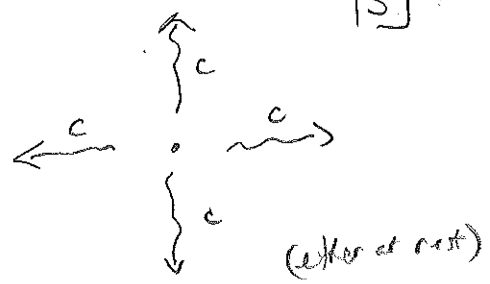
  - In frames in which ether is moving, (**) holds instead.

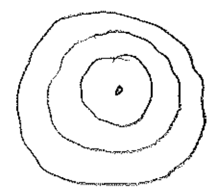
- The wave eqn violates the principle of relativity.

Another way to see that wave eqn is not invariant is by looking at its solutions

In the frame of its medium, (*) predicts that waves travel at speed c in every direction $\Rightarrow$ $|\vec{v}| = c$ (isotropy)

$\boxed{S}$



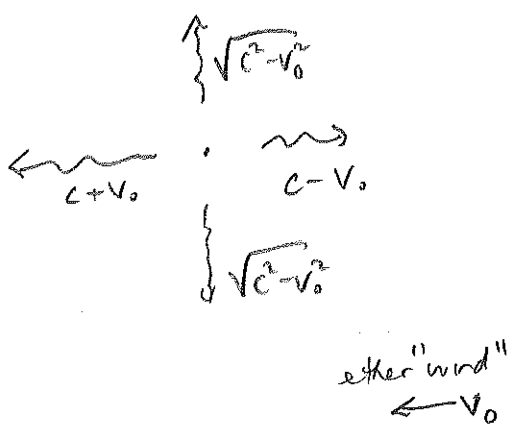


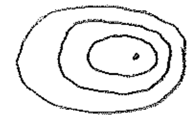
ripples in pond

Under a Galilean boost, the wave speeds transform to

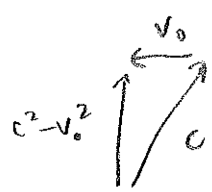
$\vec{v}' = \vec{v} - \vec{v}_0$:

$\boxed{S'}$

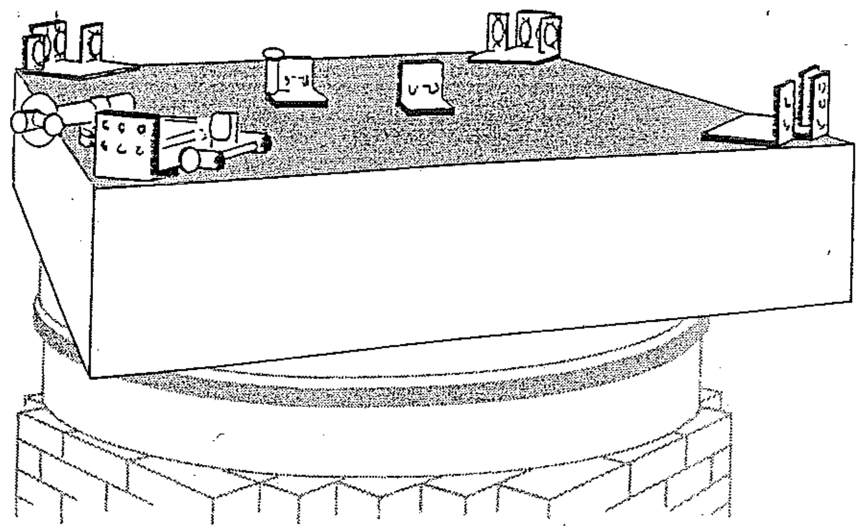




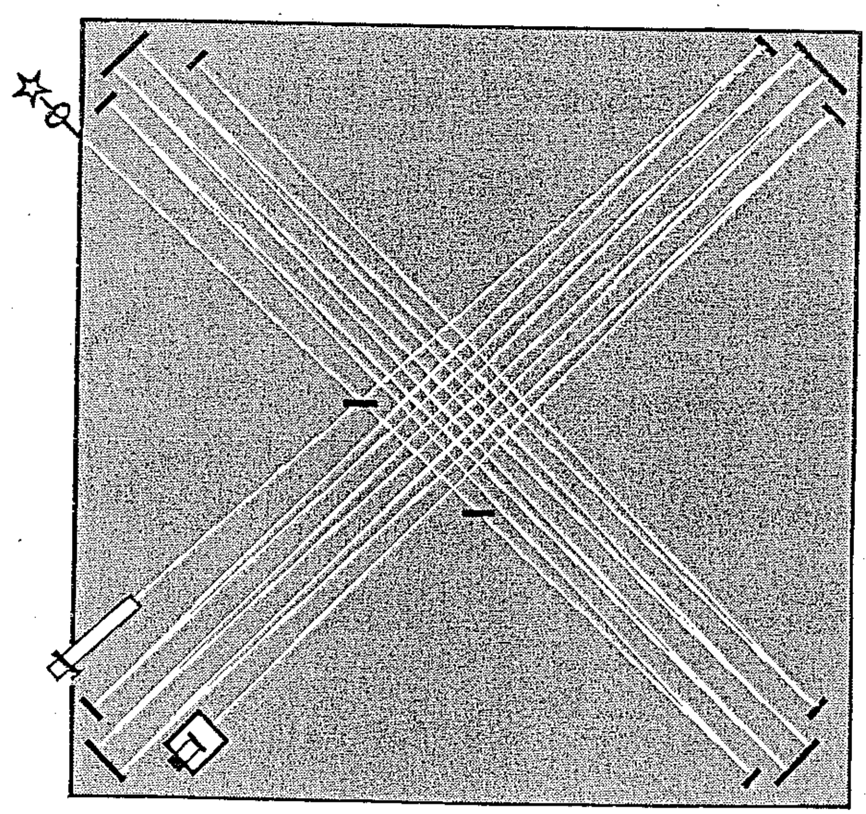
Isotropy is violated in S'
Wave speed depends on the direction (w.r.t wind)



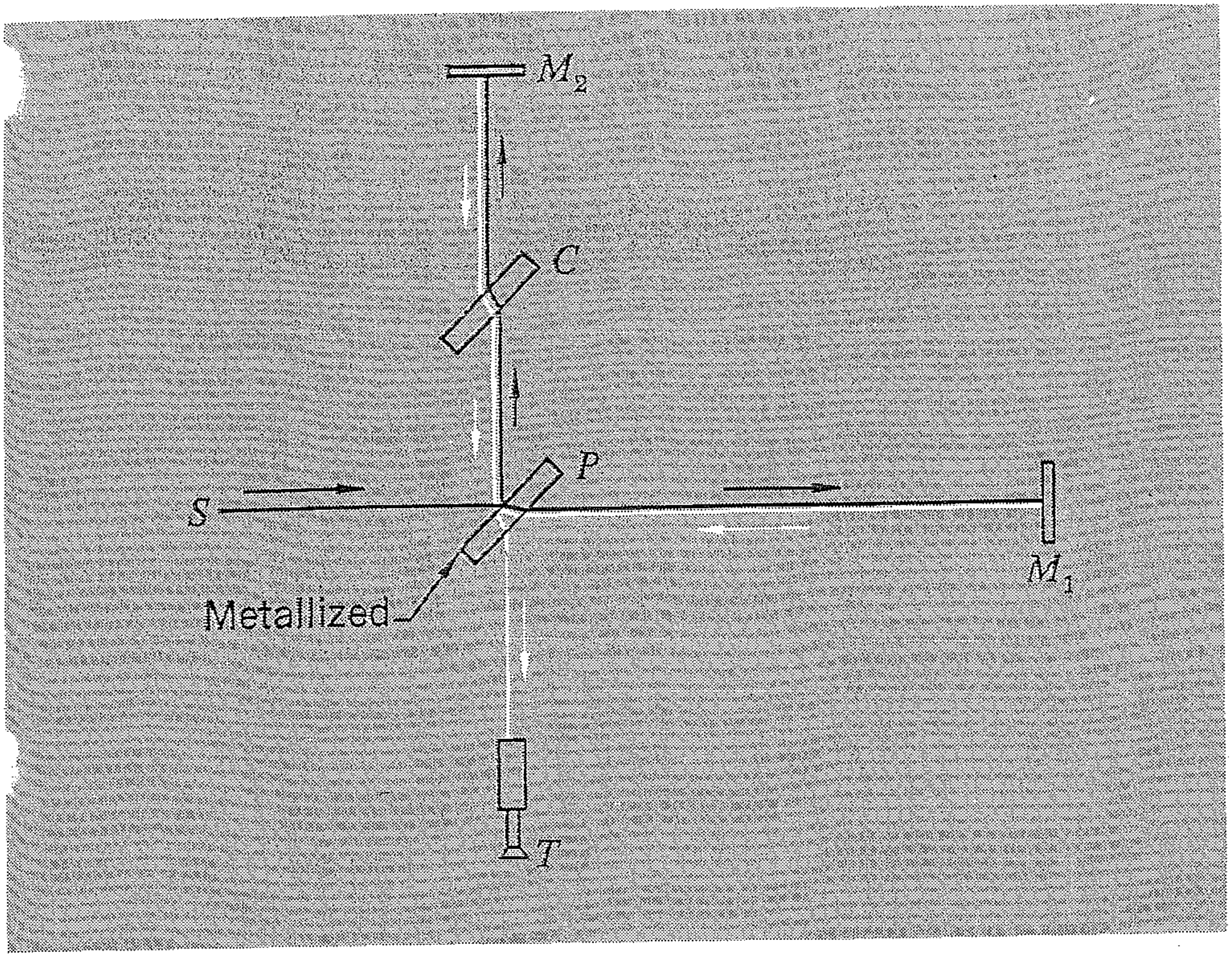
The Michelson-Morley experiment



(a)

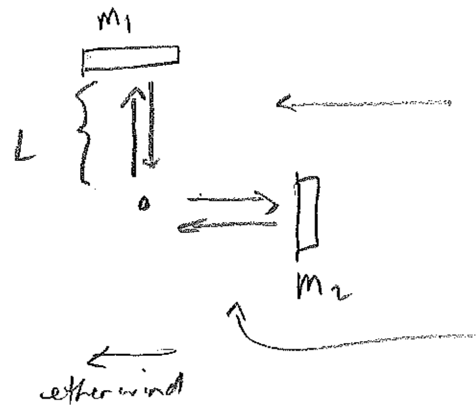


(b)



The Michelson-Morley experiment

Michelson - Morley experiment (1887) tried to measure speed of the "ether wind"

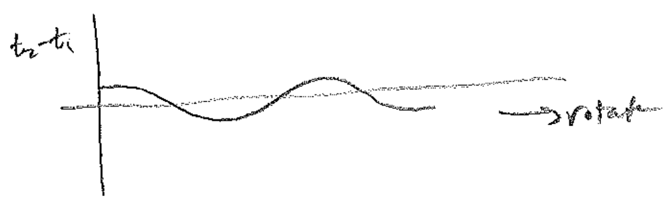


Round trip travel time of light

$$t_1 = \frac{2L}{\sqrt{c^2 - v_0^2}}$$

$$t_2 = \frac{L}{c - v_0} + \frac{L}{c + v_0} = \frac{2Lc}{c^2 - v_0^2}$$

$t_2 > t_1$ but if rotate apparatus $t_2 < t_1$



Time difference produces a phase difference → interference

→ expect shift of interference fringes as rotate apparatus

→ no such shift was found

[dog in night]

"Is there any point to which you would wish to draw attention?"

"To the curious incident of the dog in the night-time."

"The dog did nothing in the night-time."

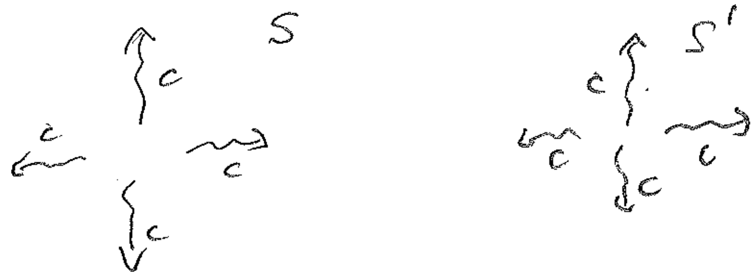
"That was the curious incident," remarked Sherlock Holmes.

A. Conan Doyle, 1893

Michelson-Morley showed that,
if ether wind exists, its speed $V_0 < 5 \frac{km}{s}$
(at all times of the year)

How explain this null result?

Light apparently travels at same speed
in every direction and in every frame



There is no preferred (ether) frame in which wave eqn holds
There is no ether.
Wave eqn holds in every frame
and therefore obeys the principle of relativity

This picture is inconsistent with Galilean transformation

$$\vec{x}' = \vec{x} - \vec{V}_0 t \qquad \Rightarrow \qquad \vec{v}' = \vec{v} - \vec{V}_0$$
$$t' = t$$

Therefore we must discard Galilean transformations.
and adopt a new conception of space & time.

This new conception leads to 3 effects

1. time dilation
2. length contraction
3. relativity of simultaneity

phase + group velocities differ!

not for class!!

$$\frac{\partial^2 A}{\partial t'^2} - 2v_0 \frac{\partial^2 A}{\partial x' \partial t'} - (c^2 - v_0^2)\frac{\partial^2 A}{\partial x'^2} - c^2\left(\frac{\partial^2 A}{\partial y'^2} - \frac{\partial^2 A}{\partial z'^2}\right) = 0$$

$$A = \sin(\vec{k}' \cdot \vec{R}' - \omega' t')$$

$$-\omega'^2 - 2v_0 k_x' \omega' + (c^2 - v_0^2) k_x'^2 + c^2 (k_y'^2 + k_z'^2) = 0$$

$$\omega'^2 = c^2(k_x'^2 + k_y'^2 + k_z'^2) - v_0^2 k_x'^2 - 2v_0 k_x' \omega'$$

$$\omega'^2 + \omega'(2v_0 k_x') - c^2 k'^2 - v_0^2 k_x'^2 = 0$$

$$\omega' = -v_0 k_x' \pm \sqrt{v_0^2 k_x'^2 + c^2 k'^2 - v_0^2 k_x'^2}$$

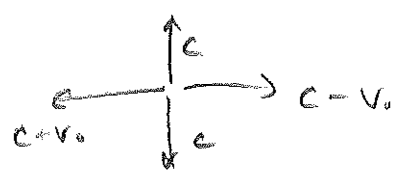
$$\omega' = -v_0 k_x' \pm c k'$$ choose + $\Rightarrow \boxed{\omega' = ck' - vk_x'}$

so if $\vec{k}'$ in yz plane: $\omega' = ck'$

if $\vec{k}' = (k_x', 0, 0) \Rightarrow$ ~~$\omega' = (c \pm v_0) k_x'$~~

$$\omega' = c|k_x'| - v_0 k_x' = \begin{cases} (c - v_0) k_x' \\ (c + v_0)|k_x'| \end{cases}$$

so phase velocities are



but group velocities

$$\left(\frac{d\omega'}{dk_x'}, \frac{d\omega'}{dk_y'}, \frac{d\omega'}{dk_z'}\right) = \left(c\frac{k_x'}{k'} - v,\ c\frac{k_y'}{k'},\ c\frac{k_z'}{k'}\right)$$

so for $\vec{V}_g'$ in y direct, need $ck_x' = vk'$

so $c^2 k_x'^2 = v^2(k_x'^2 + k_y'^2)$

$(c^2 - v^2) k_x'^2 = v^2 k_y'^2$

so $(V_g')_y = c\frac{k_y'}{k'} = \frac{c}{\sqrt{1 + \frac{k_x'^2}{k_y'^2}}} = \frac{c}{\sqrt{1 + \frac{v^2}{c^2 - v^2}}} = \sqrt{c^2 - v^2}$ agrees w/ usual results