Maxwell (1867) developed equations governing the behavior of electric ($\vec{E}$) and magnetic ($\vec{B}$) fields.

omit

In free space,

$\vec{\nabla} \cdot \vec{E} = 0$ (Gauss)

$\vec{\nabla} \times \vec{B} - \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} = 0$ (Ampere)

$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$ (Faraday)

$\vec{\nabla} \cdot \vec{B} = 0$ (no magnetic monopoles)

ϵ_0 = permittivity of vacuum = $8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$

μ_0 = permeability of vacuum = $1.26 \times 10^{-6} \frac{N \cdot s^2}{C^2}$

An exercise in vector calculus converts these eqns into

$$\frac{\partial^2 \vec{E}}{\partial t^2} - \frac{1}{\mu_0 \epsilon_0} \left(\frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} \right) = 0$$

which is the "classical wave equation"

The constant $\frac{1}{\mu_0 \epsilon_0} = 8.99 \times 10^{16} \frac{m^2}{s^2}$

For convenience let's call this constant c^2 where

$c = 3 \times 10^8$ m/s

more precisely $c = 299{,}792{,}458 \frac{m}{s}$ [look familiar?]

(the speed of light)

$$\frac{\partial^2 A}{\partial t^2} - c^2 \left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2} \right) = 0$$

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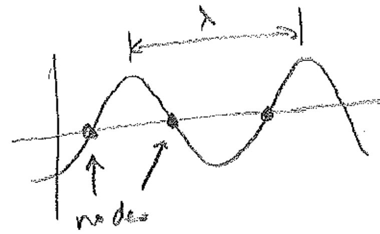
one possible solution to this is the travelling wave

$$A(x,t) = A_0 \sin(kx - \omega t)$$

constants

independent variables



A_0 = amplitude

k = wavenumber $= \frac{2\pi}{\lambda}$

ω = angular frequency $= \frac{2\pi}{T}$

Let's prove this.

[discuss meaning of partial derivatives.
Then evaluate $\frac{\partial A}{\partial x}$, $\frac{\partial^2 A}{\partial x^2}$, $\frac{\partial A}{\partial t}$, $\frac{\partial^2 A}{\partial t^2}$, $\frac{\partial A}{\partial y}$ etc.
and show that :]

$$-A_0 \sin(kx - \omega t) \left[\omega^2 - c^2 k^2 \right] = 0$$

Therefore the travelling wave satisfies the wave equation provided that $\omega^2 = c^2 k^2$ or

$$\boxed{\omega = ck}$$

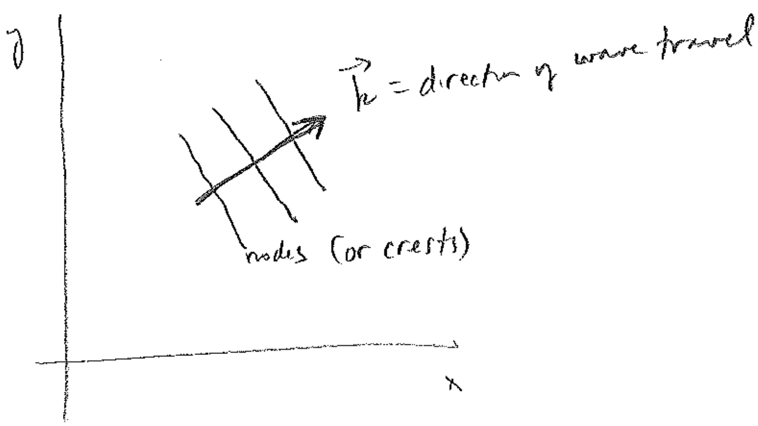
$A_0 \cos(kx - \omega t)$ also works.

More generally, the classical wave eqn has solution

$$A = A_0 \sin(\vec{k} \cdot \vec{x} - \omega t)$$

where $\vec{k} = \begin{pmatrix} k_x \\ k_y \\ k_z \end{pmatrix}$ = wavevector

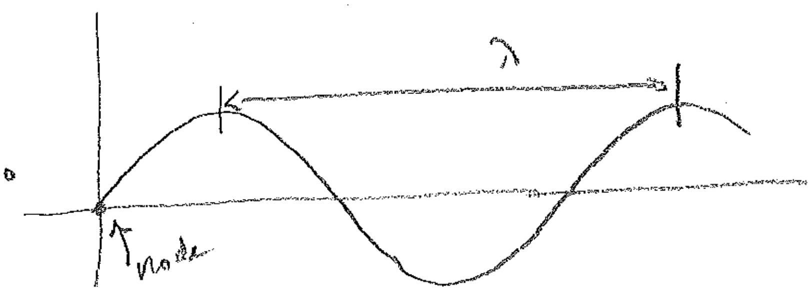
$\vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ = position vector



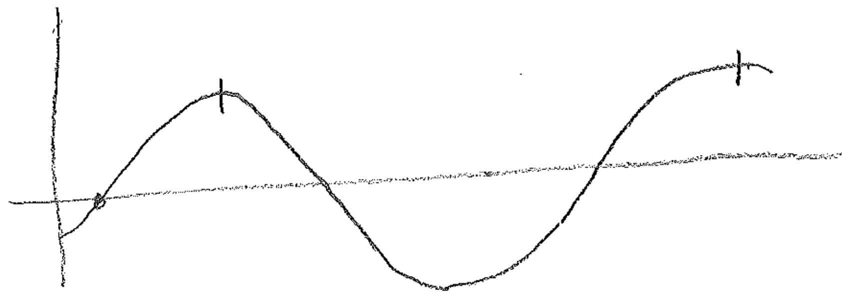
[exercise: prove this, and find condition between ω and $\vec{k}$]

Snapshot of travelling wave

At $t=0$



slightly later



How fast is wave travelling? How fast is node travelling?

$$\sin(kx-\omega t) = \sin\left(k\left[x-\frac{\omega}{k}t\right]\right)$$

Node occurs where $\sin(\;)=0$

Position of node obeys $x_{node} - \frac{\omega}{k}t = 0$

$$x_{node} = \left(\frac{\omega}{k}\right)t$$

(phase) speed of wave = speed of node $= \frac{\omega}{k}$

$$\sin(\vec{k}\cdot\vec{x}-\omega t)$$

More generally speed of wave is $\frac{\omega}{|\vec{k}|}$.

Key observation: speed is independent of direction — (isotropy)

Since $\omega = ck$ from earlier calculation

the speed of electromagnetic waves is c, the speed of light.

That's because light is an electromagnetic wave.

Other waves (sound, water) travel through a medium (air, water)

19th century physicists surmised that light waves travelled through a medium they called "luminiferous ether"