[Recap]

Symmetry transformation:

= change of coordinates that leaves laws of physics invariant

i.e. the form of the equations is unchanged

e.g. spatial translations $\vec{x}' = \vec{x} - \vec{s}$ (const $\vec{s}$)

temporal translations $t' = t + \tau$ (const τ)

rotations: [we'll do this later]

Principle of relativity:

= laws of physics are the same in all (inertial) ref. frames

Equivalently,

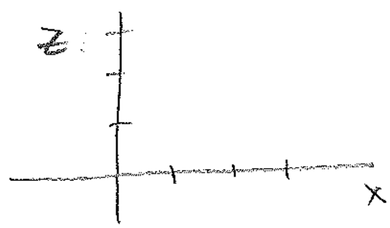
the laws of physics are invariant under <u>boosts</u>

Boost = transformation from a reference frame S to another reference frame S′ moving with constant velocity $\vec{v}_0$ with respect to S.

Reference frame: collection of objects all at rest w.r.t. one another

[E.g. everything in this room.]

[Add meter sticks so we can measure distances]

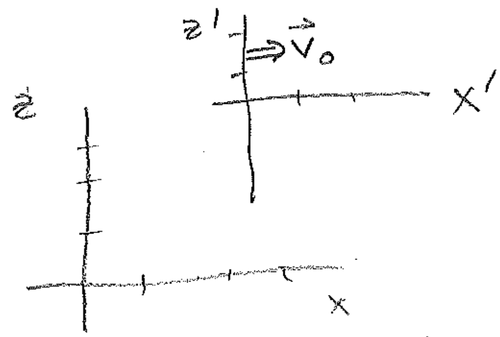


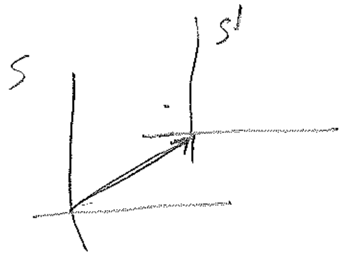
[Add clocks so we can time events.]

[A vast 3d array of meter sticks & clocks.] → see image from TKW

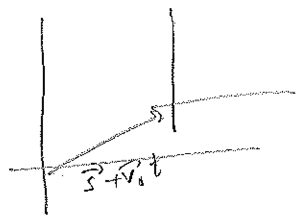
[Now consider another reference frame S′ moving w.r.t. S.]

[E.g. passengers in a plane flying overhead, all w/ own meter sticks & clocks.]

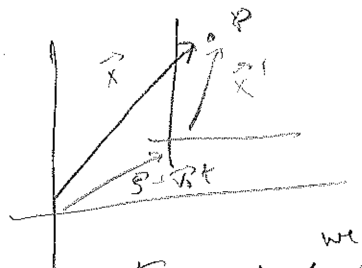




Origin of S' (relative to origin of S) is $\vec{s} + \vec{v}_0 t$



How does position of a point P transform under a boost?



$$\Rightarrow \vec{x} = \vec{s} + \vec{v}_0 t + \vec{x}'$$

For simplicity, we will from now on assume origins of S & S' coincide at $t = 0$ $\Rightarrow \vec{s} = 0$

Then

$$\vec{x}' = \vec{x} - \vec{v}_0 t$$

Time of events

$$t' = t + \tau$$

But assume clocks are synchronized $\Rightarrow \tau = 0$ (both say noon at same time)

Galilean transformation or boost
$$\begin{cases} \vec{x}' = \vec{x} - \vec{v}_0 t \\ t' = t \end{cases}$$

Inverse transformation
$$\begin{cases} \vec{x} = \vec{x}' + \vec{v}_0 t' \\ t = t' \end{cases}$$

↳ obtained by (A) inverting the eqns
or (B) recognizing that S is moving w/ opposite speed wrt S'

How does velocity transform under a Galilean boost?

$$\vec{v} = \frac{d\vec{x}}{dt} = \frac{d(\vec{x}' + \vec{v}_0 t')}{dt'} = \frac{d\vec{x}'}{dt'} + \vec{v}_0 = \vec{v}' + \vec{v}_0$$

$\vec{v} = \vec{v}' + \vec{v}_0$ Galilean law of addition of velocities

[① makes sense [walking on a ~~plane~~ train]
② but what about light? emitted by headlamp of moving train] → 2 possibilities ① speed of source ① speed of medium

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d(\vec{v}' + \vec{v}_0)}{dt'} = \frac{d\vec{v}'}{dt'} = \vec{a}'$$

$\vec{a} = \vec{a}'$ acceleration is invariant under a Galilean boost

Force between 2 objects depends on relative separation

$\vec{x}_1 - \vec{x}_2 = (\vec{x}_1' + \vec{v}_0 t) - (\vec{x}_2' + \vec{v}_0 t) = \vec{x}_1' - \vec{x}_2'$

also invariant, so $\vec{F} = \vec{F}'$ under a boost

$$\therefore \vec{F} = m\vec{a} \Rightarrow \vec{F}' = m\vec{a}'$$

Newton's 2nd law invariant under a Galilean boost

[because it depends on $\vec{a}$ + not on velocity!]

[this realization predates Newton]

[do exercise 1]
↳ mom cons + kin en cons. invt under boosts

The laws of physics are <u>not</u> the same in a frame moving w/ <u>non</u>-constant velocity (i.e. accelerating)

They only hold in a privileged class of frames called inertial reference frames (IRF's)

Define an IRF as one in which Newton's first law holds.
(a body continues in a state of rest or uniform motion in a straight line if no force acts on it)

i.e. if $\vec{F}_{net} = 0$ then $\vec{v} = \text{const}$, i.e. $\vec{a} = 0$

An IRF is one in which unforced objects do not accelerate

[consider object on dashboard as you round a corner]

[surface of the earth is an approximate IRF →]

If S is an IRF (i.e. $\vec{F} = 0 \Rightarrow \vec{a} = 0$)
then S' is also an IRF ($\vec{F}' = \vec{F}$, $\vec{a}' = \vec{a}$, so $\vec{F}' = 0$ and $\vec{a}' = 0$)

There exists an infinite number of IRFs, related by boosts of arbitrary velocity

Laws of physics are the same in all of them.

IF ASKED

Is the earth really an I.R.F?

~~6.1~~

Car rounding a corner

$V = 10 \text{ mph} = 5 \text{ m/s}$

$R = 5 \text{m}$

$a = \frac{V^2}{R} = 5 \frac{m}{s^2}$ (recall $g = 10 \frac{m}{s^2}$)

Rotation of earth

on equator $V = \frac{2\pi(\overbrace{6.4 \times 10^6 \text{ m}}^{(6400 \text{ km})})}{\underbrace{(86400 s)}_{1 \text{ day}}} \approx 500 \text{ m/s}$

$a = \frac{V^2}{R} = 0.03 \text{ m/s}^2$ (max at equator)

This acts as a reduced (& redirected if not at equator) gravitational field

$a = \omega^2 R$

Earth circling sun

$V = \frac{2\pi(1.5 \times 10^{11} \text{ m})}{\underbrace{3 \times 10^7 s}_{1 \text{ year}}} = 30{,}000 \text{ m/s}$

$a = \frac{V^2}{R} = 0.006 \text{ m/s}^2$

But we are in "free fall" so this "cancels out"

Exercise

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} + m_3\vec{v}_3 + \ldots = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} + \ldots$$

$$\vec{v} = \vec{v}\,' + \vec{v}_0$$

$$m_1\vec{v}\,'_{1i} + m_2\vec{v}\,'_{2i} + \ldots + \underbrace{(m_1 + m_2 + \ldots)\vec{v}_0}_{} = m_1\vec{v}\,'_{1f} + \ldots + \underbrace{(m_1 + m_2 + \ldots)\vec{v}_0}_{}$$

cancels out

$$\tfrac{1}{2}m_1 v_{1i}^2 + \tfrac{1}{2}m_2 v_{2i}^2 = \tfrac{1}{2}m_1 v_{1f}^2 + \tfrac{1}{2}m_2 v_{2f}^2$$

$$\tfrac{1}{2}m_1(v'_{1i} + v_0)^2 + \ldots$$

$$\tfrac{1}{2}m_1 v_{1i}'^2 + m_1\vec{v}_{1i}\cdot\vec{v}_0 + \tfrac{1}{2}m_1 v_0^2 + \ldots =$$

$$\tfrac{1}{2}m_1 v_{1i}'^2 + \tfrac{1}{2}m_2 v_{2i}'^2 = \tfrac{1}{2}m_1 v_{1f}'^2 + \tfrac{1}{2}m_2 v_{2f}'^2$$

$$+ (m_1\vec{v}_{1i} + m_2\vec{v}_{2i})\cdot\vec{v}_0 \qquad + (m_1\vec{v}_{1f} + m_2\vec{v}_{2f})\cdot\vec{v}_0$$

— equal by mom cons

$$+ \tfrac{1}{2}(m_1 + m_2)v_0^2 \qquad + \tfrac{1}{2}(m_1 + m_2)v_0^2$$