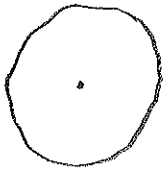
Symmetry is the fundamental principle of modern physics

[what is symmetry?]

2019

[some on left and right; mirror reflections]

invariant under reflection



invariant under rotation about center

Symmetry = invariance under some operation

The laws of physics are invariant under certain operations
(eqns of physics have the same form)

- spatial translations
- temporal translations
- rotations
- boosts

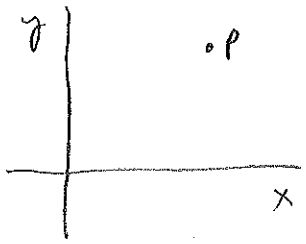
[if do expt in room down hall, or tomorrow, or rotated expect same result.]

Noether's theorem: Symmetries imply conservation laws

- cons. of momentum
- cons of energy
- cons of angular momentum

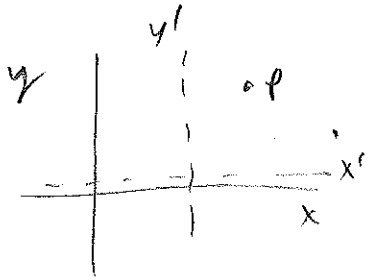
SPATIAL TRANSLATIONS

Define a Cartesian coord. system



coordinates of P: (x_P, y_P)

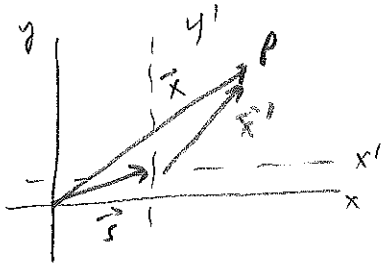
Shift origin to right by s



$x_P' = x_P - s$

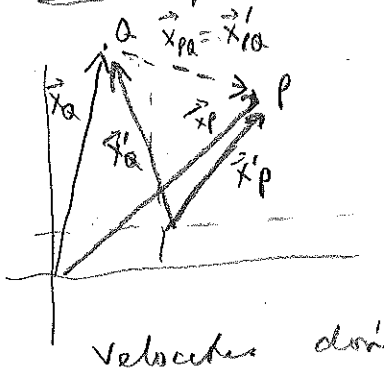
$y_P' = y_P$

More generally, shift origin by $\vec{s}$



$\vec{x}' = \vec{x} - \vec{s}$

Relative positions don't change under a shift [do this first, then diagram]



$\vec{x}_{PQ} = \vec{x}_P - \vec{x}_Q$

$= (\vec{x}_P' + \vec{s}) - (\vec{x}_Q' + \vec{s})$

$= \vec{x}_P' - \vec{x}_Q' = \vec{x}'_{PQ}$

Velocities don't change under a shift $\vec{v} = \lim_{\Delta t \to 0} \frac{\vec{x}(t+\Delta t) - \vec{x}(t)}{\Delta t}$

$$\vec{v}' = \frac{d\vec{x}'}{dt} = \frac{d}{dt}(\vec{x} - \vec{s}) = \frac{d\vec{x}}{dt} = \vec{v}$$

Also $\vec{a}' = \frac{d\vec{v}'}{dt'} = \frac{d\vec{v}}{dt} = \vec{a}$

Velocities and accelerations are invariant under a general Spatial translation

Laws of physics are invariant under spatial translations

e.g. Newton's 2nd law: $\vec{F} = m\vec{a}$

- force between 2 objects depends on relative separation $\Rightarrow \vec{F} = \vec{F}'$
- acceleration is invariant: $\vec{a} = \vec{a}'$

Therefore, $\vec{F}' = m\vec{a}'$

eqn has the <u>same form</u>

eg cons. of momentum: $\sum_{\text{init}} m_j \vec{v}_j = \sum_{\text{final}} m_j \vec{v}_j$

- velocities are invariant

$$\Rightarrow \sum_{\text{init}} m_j \vec{v}_j' = \sum_{\text{final}} m_j \vec{v}_j'$$

eqns of physics do not depend on choice of origin.

- similarly for temporal translations & rotations