Wave-particle duality of matter

	Particles	Waves
Light	photon	electromagnetic waves
Matter	electrons, protons, ...	matter waves

In 1926 de Broglie proposed that electrons can behave like waves

Experiments showing interference of electrons soon confirmed this propose

[see 3 OH's]

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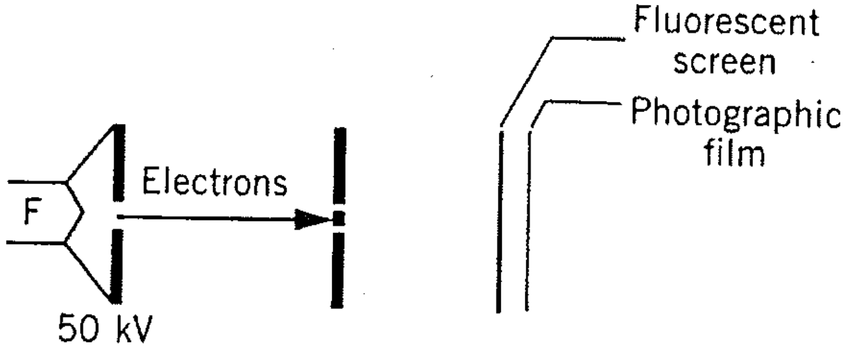
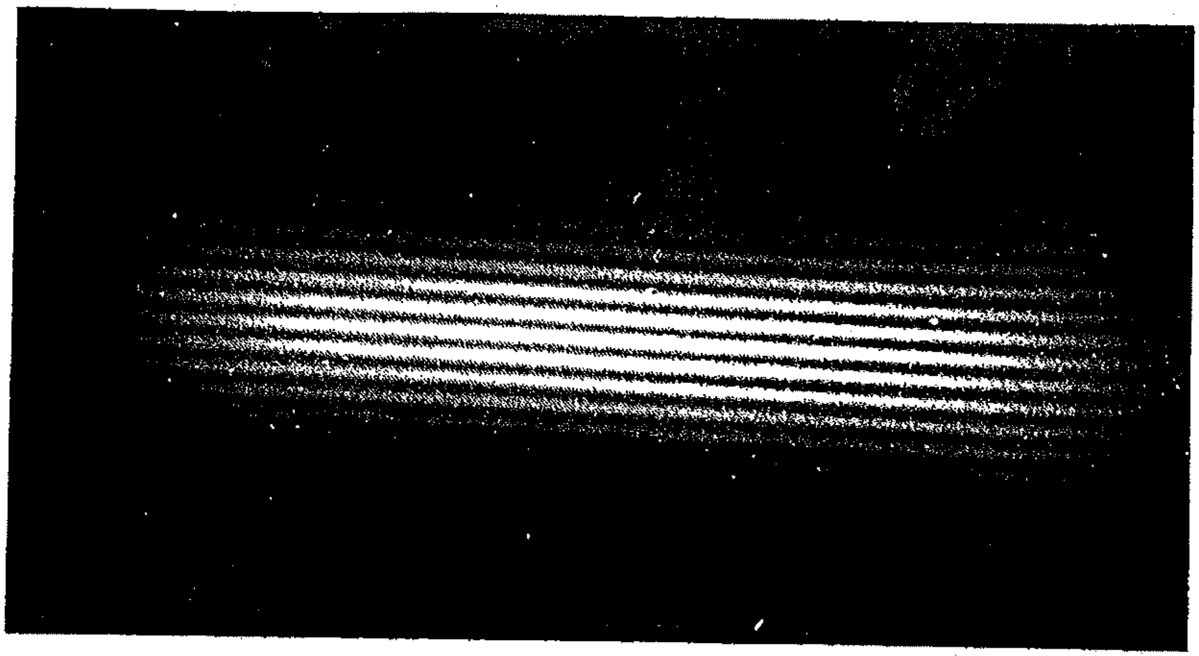


(a)



(b)

Figure 1 (*a*) Apparatus for producing double-slit interference with electrons. A filament F produces a spray of electrons, which are accelerated through 50 kV, pass through the single slit, and strike the double slit. They produce a visible pattern when they strike a fluorescent screen, which can be photographed. (*b*) The resulting electron double-slit interference pattern, showing the interference fringes.

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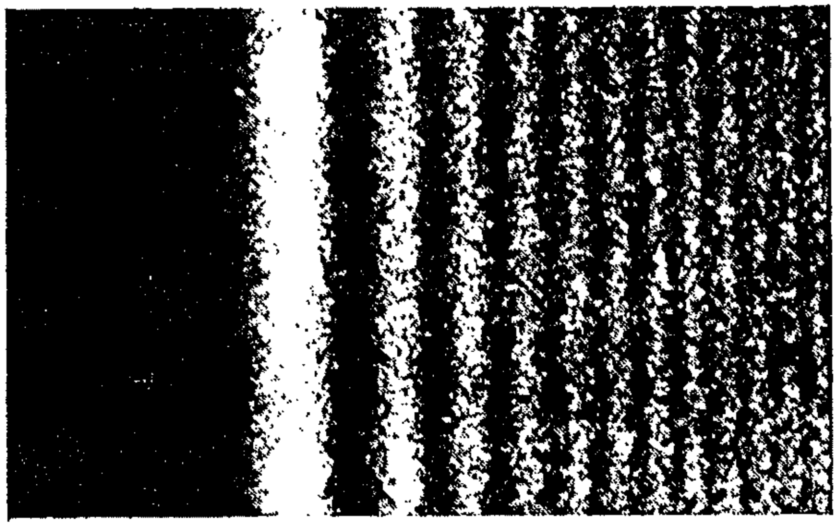
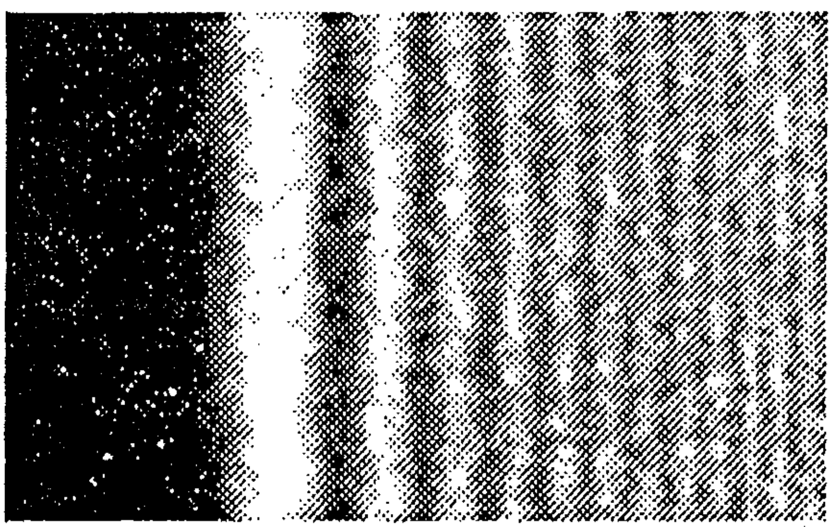
Figure 2 An electron diffraction pattern using a circular aperture of diameter 30 μm and 100-keV electrons. Compare with the optical pattern (Fig. 2 of Chapter 46).



(*a*)



(*b*)

Figure 3 Diffraction of (*a*) light and (*b*) electrons from a straight edge.

If an electron is described by a wave,

what is its wavelength λ?

Depends on its momentum p.

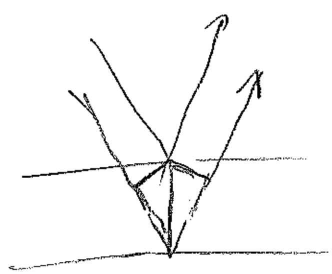
[analogous to light, where E depends on f]

Louis de Broglie proposed

$$\boxed{\lambda = \frac{h}{p}}$$ (de Broglie wavelength)

Davisson & Germer (1926) confirmed this experimentally

using Bragg scattering off a nickel crystal.



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wave number $k = \frac{2\pi}{\lambda} = \frac{2\pi}{h} p \quad \Rightarrow \quad p = \frac{h}{2\pi} k$

For convenience, define $\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}\ \mathrm{J \cdot s}$

Then $\boxed{p = \hbar k}$ de Broglie relation

Calculate λ for an electron w/ kinetic energy 1.0 eV.

$$E_{kin} = \frac{1}{2} m v^2 \quad \Rightarrow \quad v = \sqrt{\frac{2E_{kin}}{m}}$$

$$p = mv = \sqrt{2mE_{kin}}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_{kin}}} = \frac{hc}{\sqrt{2mc^2 E_{kin}}}$$

$hc = 1240$ nm·eV

$mc^2 = 0.511$ MeV, for an electron

$E_{kin} = 1.0$ eV

$\Rightarrow \lambda = 1.2$ nm (atomic dimensions)

Beware: for light $\lambda = \frac{c}{f} = \frac{hc}{E_{photon}}$

but for (non relativistic) particles $\lambda = \frac{h}{\sqrt{2mE_{kin}}}$

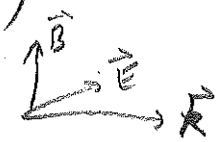
$v \ll c$

Not the same!

Light is described by an electromagnetic wave consisting of two real vector fields $\vec{E}$ + $\vec{B}$

A travelling EM wave w/ wave number $k = \frac{2\pi}{\lambda}$ takes the form

$$\vec{E} = \begin{pmatrix} 0 \\ A\sin(kx-\omega t) \\ 0 \end{pmatrix} \qquad \vec{B} = \begin{pmatrix} 0 \\ 0 \\ \frac{A}{c}\sin(kx-\omega t) \end{pmatrix}$$



Intensity $I = \epsilon_0 c \langle |\vec{E}|^2 \rangle = \frac{\epsilon_0 c}{2} A^2$

An electron is described by a matter wave which is a complex scalar field

$\psi = \psi_R + i\,\psi_I$ where $i = \sqrt{-1}$

complex number
$z = x + iy$
where x = real part
y = imag part

A travelling matter wave w/ wave number $k = \frac{2\pi}{\lambda}$ takes the form

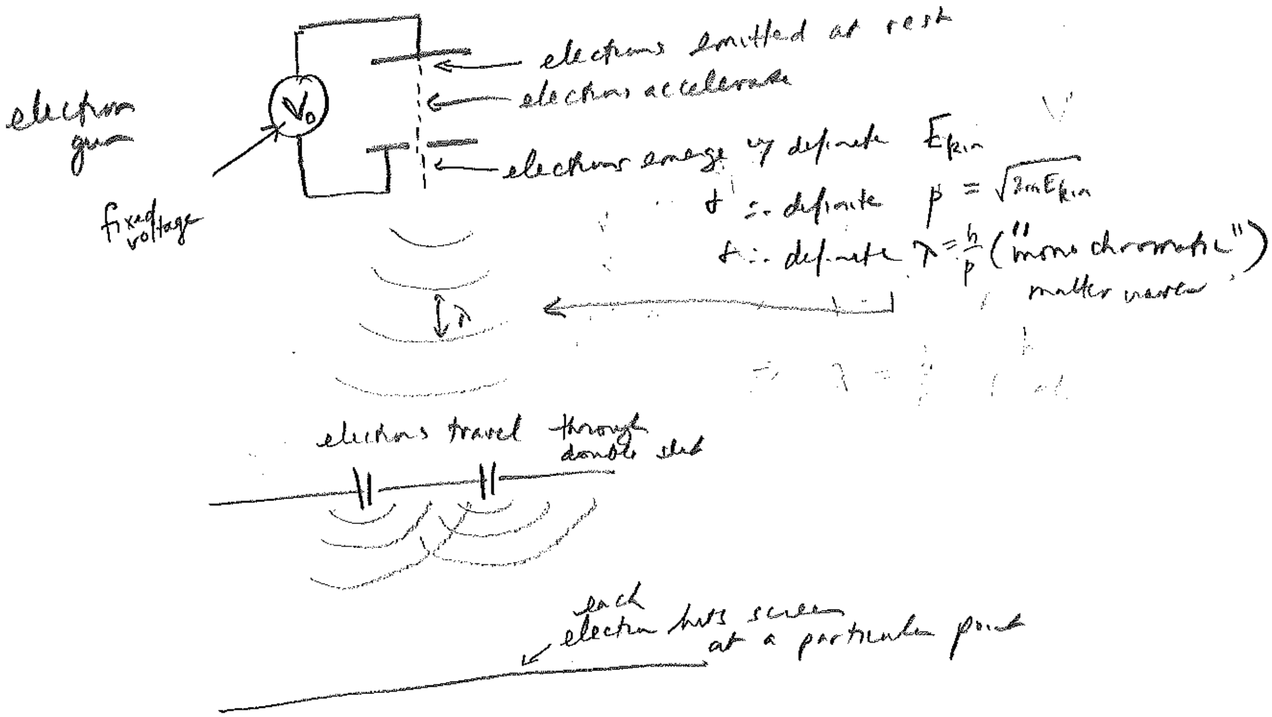
$\psi = A\cos(kx-\omega t) + i\,A\sin(kx-\omega t)$

Define probability density $\rho = |\psi|^2 \equiv \psi_R^2 + \psi_I^2$

$= A^2\cos^2(kx-\omega t) + A^2\sin^2(kx-\omega t)$

$= A^2$

Two slit experiment of electrons



Electrons, like classical particles, are emitted (eg from cathode) and detected (eg at screen) one at a time in specific locations but unlike classical particles, they do not follow a definite trajectory. Each electron (wave) goes through both slits. (see transparency)

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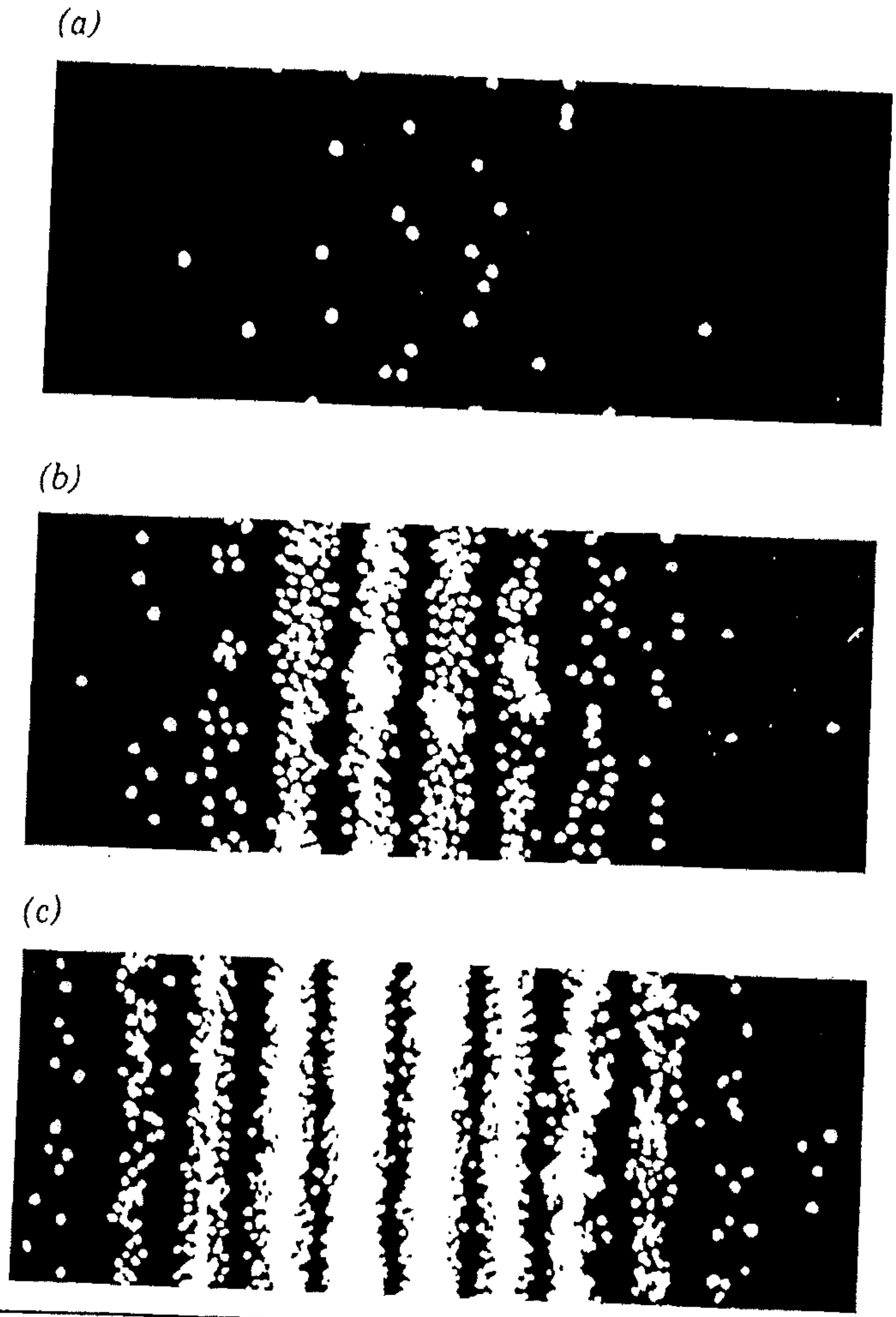
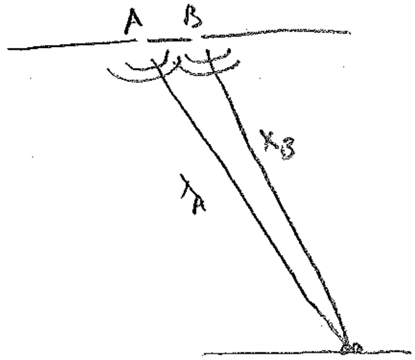


Figure 26 The buildup of interference fringes as electrons fall on screen B of Fig. 25. In (*a*), about 30 electrons have landed on the screen, in (*b*) about 1000, and in (*c*) about 10,000. The probability density of the wave describing the electron determines where the electrons will land on the screen.

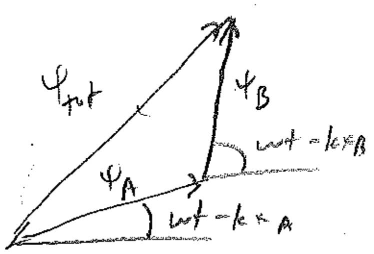
Two slit experiment



Superposition principle for matter waves

$$\psi_{tot} = \psi_A + \psi_B$$

Represent waves by phasors



Phase difference $\Delta\phi = kx_A - kx_B$

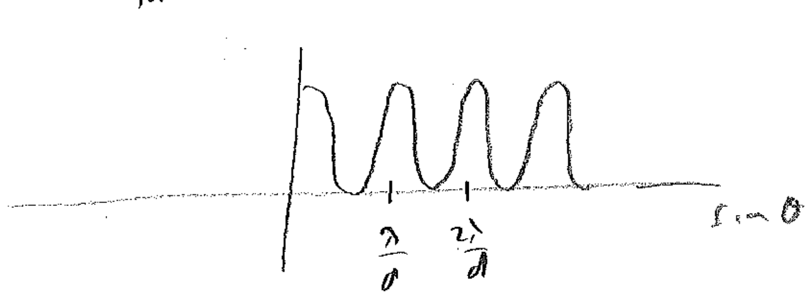
$$= \frac{2\pi}{\lambda}\Delta x$$

just as for light, but using de Broglie wavelength $\lambda = \frac{h}{p}$

For light, intensity $I \sim |A_{tot}|^2$

For electrons, probability density $P = |\psi_{tot}|^2$

$|\psi_{tot}|^2$ exhibits maxima (const. interf.) when $d\sin\theta = m\lambda$



The interference pattern represents the probability density of electrons, i.e. the statistical distribution of electrons reaching the screen. Intensity is proportional to number of electrons or photons

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