Thermal radiation

Any object with $T > 0$ emits electromagnetic radiation
(if it's hot enough, radiation will be visible
but even objects at room temperature emit IR.)

Temperature is proportional to internal kinetic energy
(random motion of atoms)

Roughly, $E_{atom} \sim k_B T$

k_B = Boltzmann's constant
$= 1.38 \times 10^{-23} \frac{J}{K}$ or $8.62 \times 10^{-5} \frac{eV}{K}$

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e.g. in a gas, $E_{atom} = \frac{3}{2} k_B T$

diatomic molecule $E_{molec} = \frac{5}{2} k_B T$

At room temperature ($T \sim 300K$), $k_B T \sim \frac{1}{40}$ eV

(typical kinetic energy per atom/molecule)

Random motion of atoms
$\Rightarrow$ random acceleration of charges (electrons)
$\Rightarrow$ emission of EM waves at <u>all</u> frequencies
$\Rightarrow$ <u>continuous</u> thermal spectrum for any substance with lots of interactions among atoms

1) heated solids (e.g. tungsten filament in bulb)

2) heated liquids (e.g. molten steel)

3) high pressure gas (e.g. sun)

[can show images in 1140scans]

(Low pressure gases, on the other hand, emit <u>discrete</u> <u>spectra</u>.
We'll study them later.)

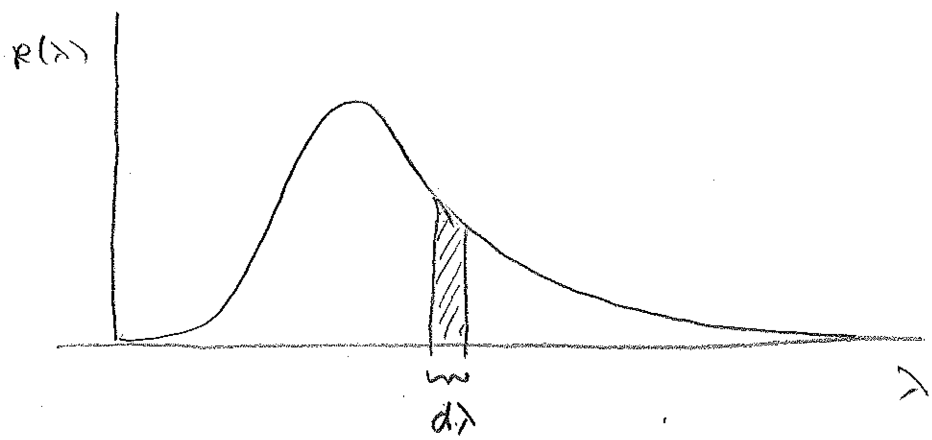
Thermal radiation $\Rightarrow$ loss of energy (radiant cooling)
- thermoses
- cloudless nights

[How to describe a continuous spectrum?]

Intensity = (time average of) power/area $\left(\frac{J}{s \cdot m^2}\right)$

Radiancy $R(\lambda) = \frac{\text{intensity}}{\text{unit wavelength}}$ $\left(\frac{J}{s \cdot m^3}\right)$

Thermal radiation radiancy curve (a.k.a. "blackbody spectrum")

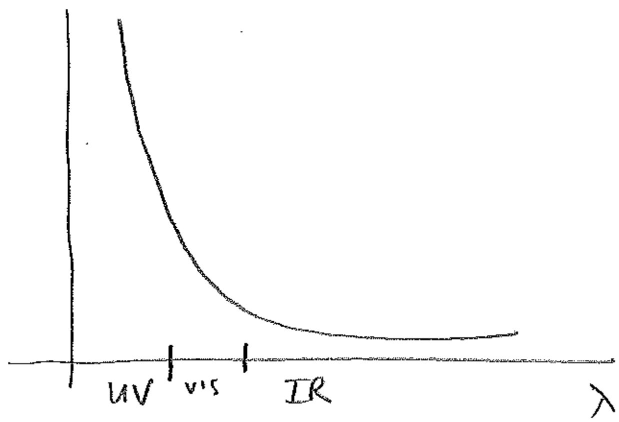


Area of strip = $R(\lambda)\, d\lambda$ = intensity emitted between λ and $\lambda + d\lambda$

Total intensity emitted I = area under curve = $\int_0^\infty R(\lambda)\, d\lambda$

Classical theory of electromagnetism predicts that hot objects should emit thermal radiation w/ radiancy

$$R(\lambda) = \frac{2\pi c\, k_B T}{\lambda^4} \qquad \text{(Rayleigh-Jeans law)}$$



[More visible than IR
UV than visible
X-rays than UV
Lethal!]

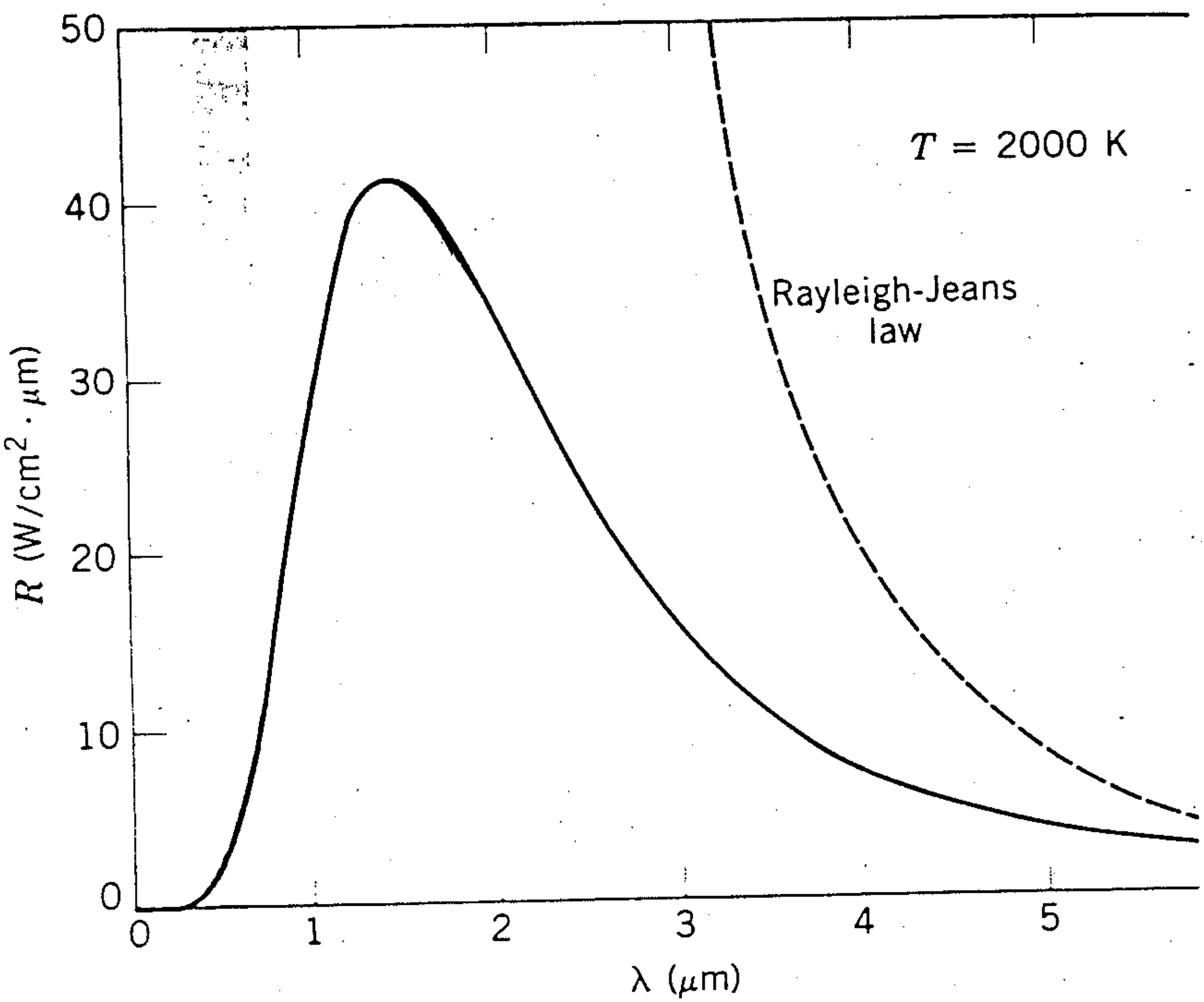
Total intensity

$$I = \int_0^\infty R(\lambda)\,d\lambda \sim \int_0^\infty \frac{d\lambda}{\lambda^4} \sim -\frac{1}{3\lambda^3}\Big|_0^\infty = \infty$$

ultraviolet catastrophe!

classical theory is a failure

[actual thermal radiancy curve shown next →]



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OH

Max Planck invoked his quantum hypothesis

$$E_{photon} = hf$$

to calculate a different radiancy

$$\boxed{R(\lambda) = \frac{2\pi c^2 h}{\lambda^5 \left(e^{\frac{hc}{\lambda k_B T}} - 1\right)}}$$

(Planck distribution)

which agreed extremely well with the observed blackbody spectrum

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This does not suffer an ultraviolet catastrophe, but rather peaks at a wavelength where $\frac{dR}{d\lambda} = 0$

You will solve this eqn. in HW to find

$$\boxed{\lambda_{peak} \approx \frac{1}{5} \frac{hc}{k_B T} \approx \frac{(2.9 \times 10^6 \text{ n.m.K})}{T}}$$

(Wien's displacement law)

Observe: as $T \uparrow$, $\lambda_{peak} \downarrow$

[see OH →]

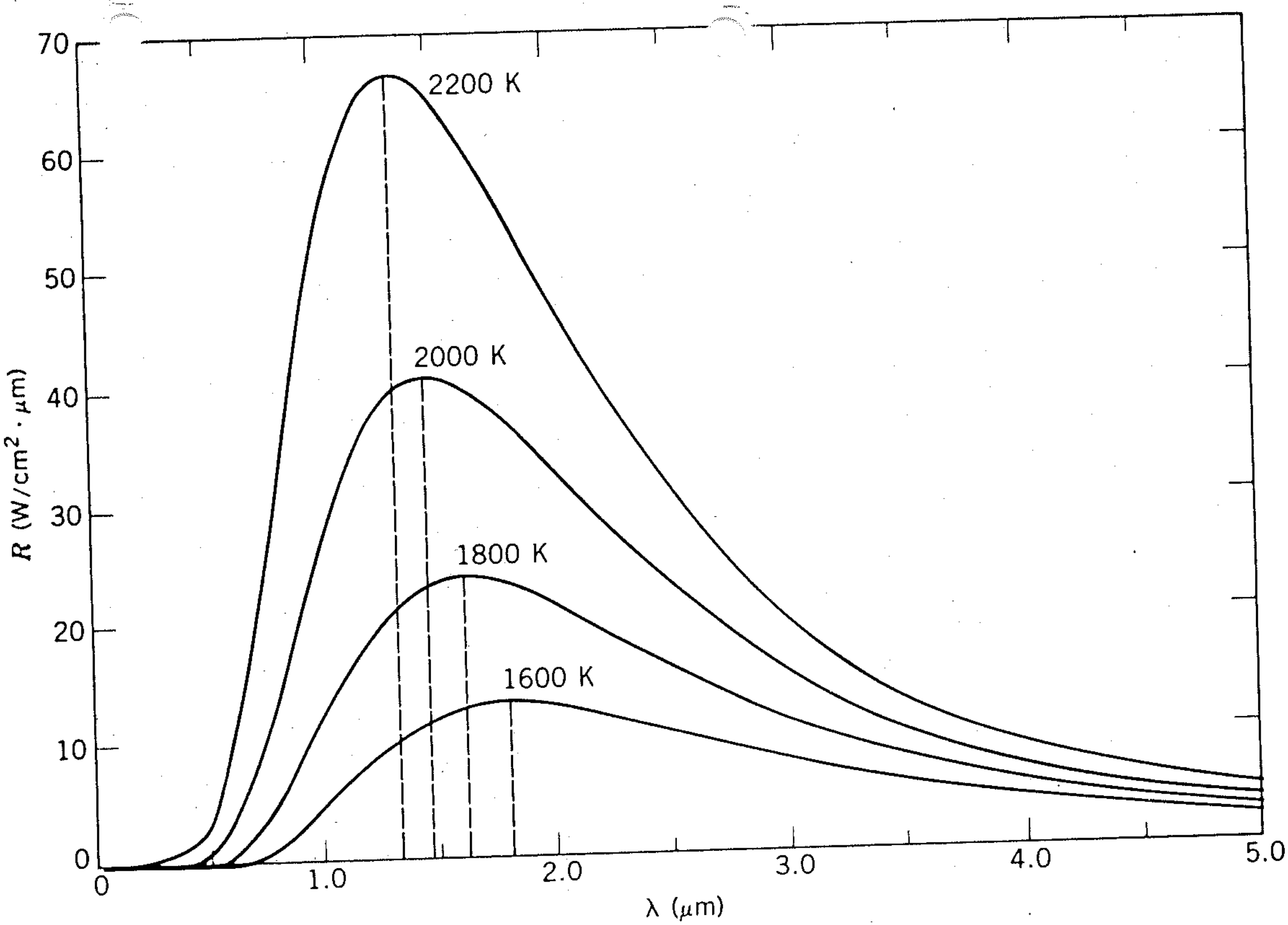

Figure 3 Spectral radiancy curves for cavity radiation at four selected temperatures. Note that as the temperature increases, the wavelength of the maximum spectral radiancy shifts to lower values.

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Rough explanation of Wien's law.

Typical $E_{atom} \sim k_B T$

If atom gives that energy to a photon

$$E_{photon} \sim E_{atom}$$

$$hf \sim k_B T$$

$$f \sim \frac{k_B T}{h}$$

$$\lambda = \frac{c}{f} \sim \frac{hc}{k_B T}$$

higher T $\Rightarrow$ higher E_{atom} $\Rightarrow$ more energetic photons $\Rightarrow$ shorter λ

$$\lambda_{peak} = \frac{2.9 \times 10^6 \text{ nm} \cdot \text{K}}{T}$$

candle
incandescent bulb

[DEMO: hand out diffraction gratings]

$T = 300K$ (room temp) $\Rightarrow \lambda_{peak} = 10{,}000$ nm $= 10\ \mu m$ (IR)

[heat scopes]

$T = 1000$ K (hot stove element) $\Rightarrow \lambda_{peak} = 3{,}000$ nm

[dull red glow]

$T = 1750K$ (candle) $\Rightarrow \lambda_{peak} = 1{,}700$ nm [yellow]

[carbon particles in the flame" not gas atoms]

[show transparency; light candle]

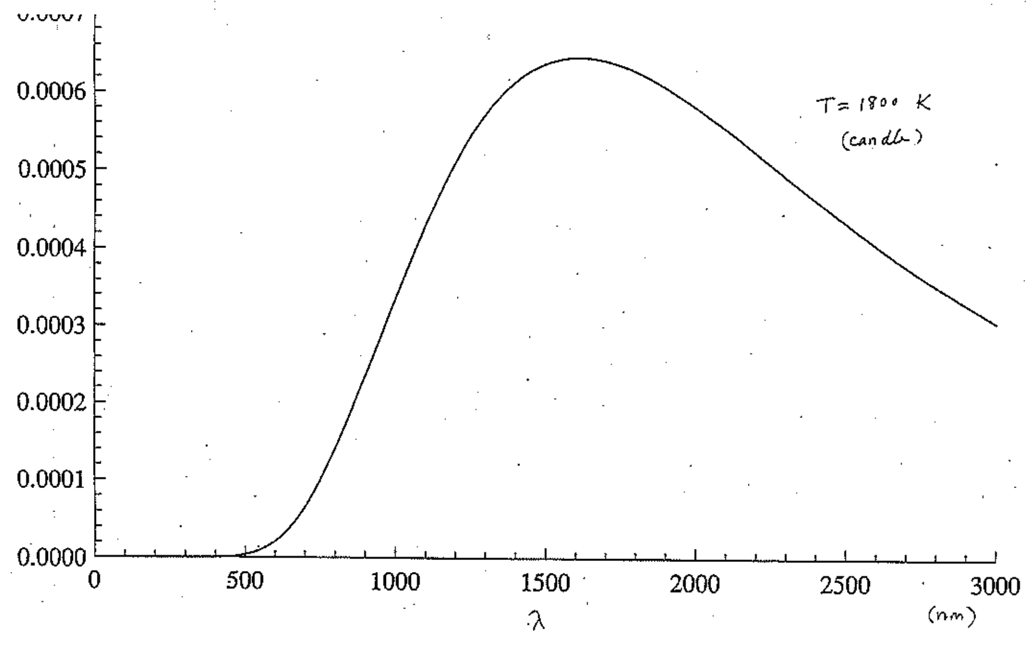
$T = 2800$ K (filament of incandescent bulb) $\lambda_{peak} = 1100$ nm

[show transparency; adjustable bulb]

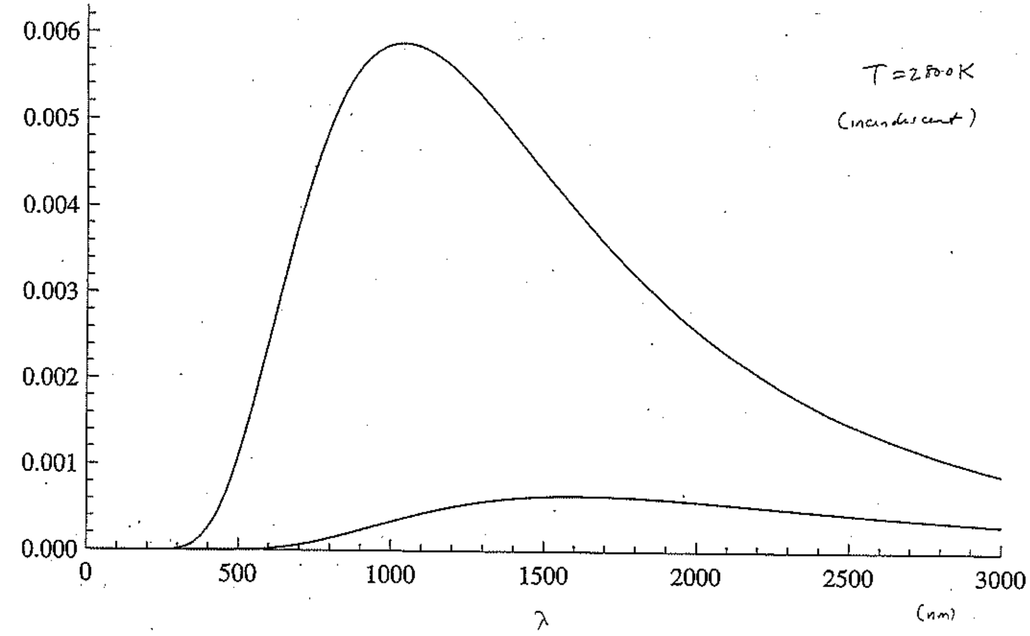
$T = 5800K$ (sun's surface) $\lambda_{peak} = 500$ nm [white]

$T = 12{,}000$ K (hot stars) $\lambda_{peak} = 250$ nm [UV peak] [blue]

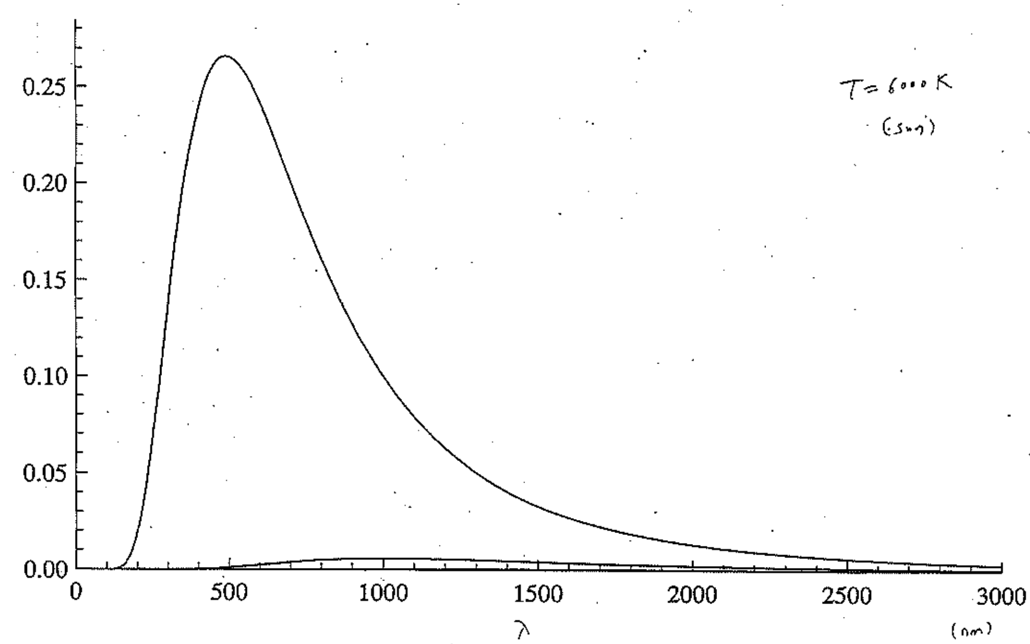
[20,000K: B-type stars]



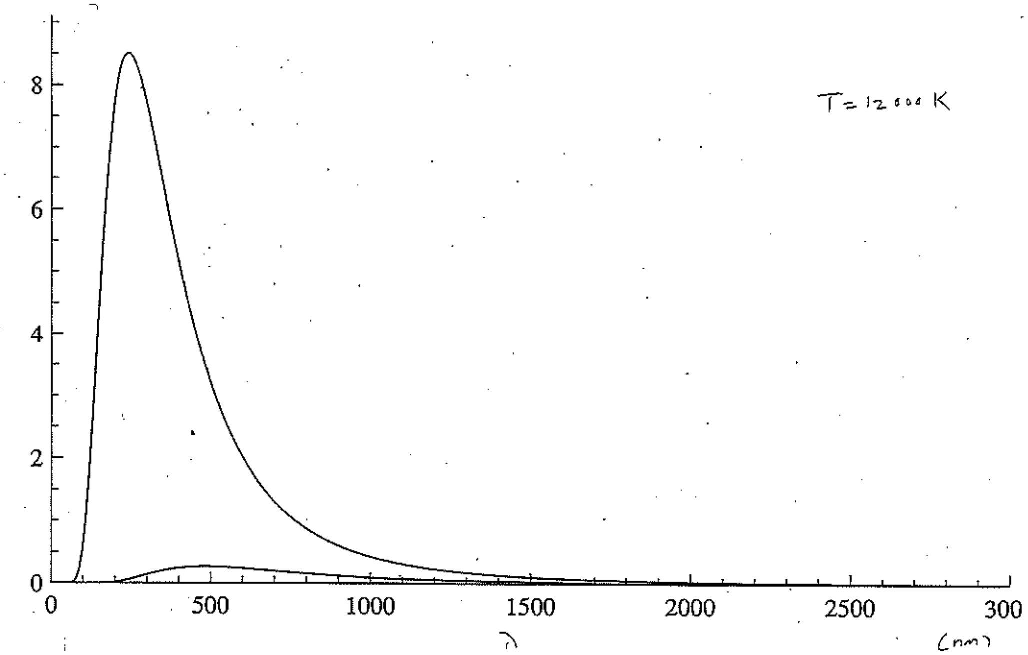
T = 1800 K
(candle)
0.0000
0.0001
0.0002
0.0003
0.0004
0.0005
0.0006
0 500 1000 1500 2000 2500 3000
λ
(nm)



T = 2800 K
(incandescent)
0.000
0.001
0.002
0.003
0.004
0.005
0.006
0 500 1000 1500 2000 2500 3000
λ
(nm)



T = 6000 K
(sun)
0.00
0.05
0.10
0.15
0.20
0.25
0 500 1000 1500 2000 2500 3000
λ
(nm)



T = 12000 K
0
2
4
6
8
0 500 1000 1500 2000 2500 300
λ
(nm)

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U10

[First derivatives, now integrals]

Total radiation emitted = area under curve

$$I = \int_0^\infty R(\lambda)\, d\lambda = 2\pi h c^2 \int_{\lambda=0}^{\lambda=\infty} \frac{d\lambda}{\lambda^5 \left(e^{\frac{hc}{\lambda k_B T}} - 1\right)}$$

[How to do an integral w/o doing an integral]

Change to dimensionless variable $x = \frac{hc}{kT}\frac{1}{\lambda}$

$$\lambda = \frac{hc}{k_B T}\frac{1}{x}$$

$$\frac{d\lambda}{dx} = -\frac{hc}{k_B T}\frac{1}{x^2}$$

$$d\lambda = -\frac{hc}{k_B T}\frac{dx}{x^2}$$

$$I = 2\pi h c^2 \int_{x=\infty}^{x=0} \frac{\left(-\frac{hc}{k_B T}\frac{dx}{x^2}\right)}{\left(\frac{hc}{k_B T}\frac{1}{x}\right)^5 (e^x - 1)}$$

$$= 2\pi h c^2 \left(\frac{k_B T}{hc}\right)^4 \int_\infty^0 \frac{(-x^3\, dx)}{e^x - 1}$$

$$= \frac{k_B^4 T^4}{h^3 c^2}\, 2\pi \underbrace{\int_0^\infty \frac{x^3\, dx}{e^x - 1}}_{\text{Tables} \Rightarrow \frac{\pi^4}{15}}$$

$$= \underbrace{\left(\frac{2\pi^5}{15}\frac{k_B^4}{h^3 c^2}\right)}_{\sigma_B \,=\, 5.67 \times 10^{-8} \frac{J}{s \cdot m^2 \cdot K^4}} T^4$$

= Stefan Boltzmann constant

$$\boxed{I = \sigma_B T^4}$$ Stefan Boltzmann law

Thermal radiation emitted by sun

$I = \sigma_B T_s^4$

T_s = temp of sun's surface $\approx 5800\,K$

$I = \dfrac{\text{power}}{\text{area}}$

P_s = (intensity at sun's surface)(surface area of sun)

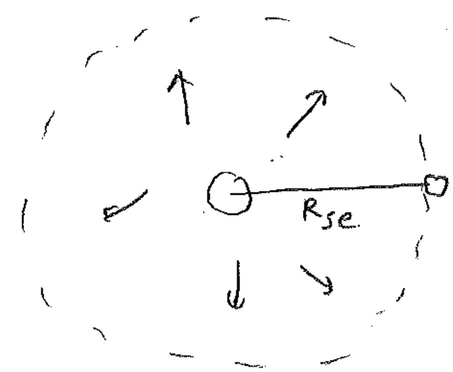
$= (\sigma_B T_s^4)(4\pi R_s^2)$

$R_s \approx 6.95 \times 10^5$ km

$\Rightarrow P_s = 3.9 \times 10^{26}$ W

[where does it come from?
chemical rxns? $< 10^5$ yrs
gravitational collapse?
nuclear fusion $\Rightarrow 10^{10}$ yrs]

Intensity of sunlight at earth's surface



$I = \dfrac{P_s}{4\pi R_{se}^2}$

R_{se} = distance between earth + sun

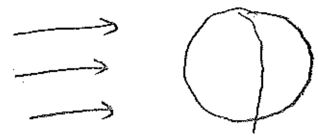
$= 1.5 \times 10^8$ km

$I = 1380\ \dfrac{W}{m^2}$

[In a previous problem, you used I to find P_s]

Solar energy intercepted by earth?

Treat earth as a flat disk
w/all radiation perpendicular



$$P_{in} = I \cdot (\pi R_e^2)$$

R_e = radius of earth = 6400 km

$$P_{in} = 1.8 \times 10^{17}\ W$$

Thermal power emitted by earth (mostly infrared)

$$P_{out} = (\sigma_B T_e^4)(4\pi R_e^2)$$

Assume thermal equilibrium

$$P_{out} = P_{in}$$

$$4\pi R_e^2 \sigma_B T_e^4 = I \pi R_e^2$$

$$T_e^4 = \frac{I}{4\sigma_B}$$

$$T_e = \left(\frac{I}{4\sigma_B}\right)^{\frac{1}{4}} \approx 280\ K$$

[do this, or do U15]

Does T_e ↑ or ↓ if

① R_s increases? [P_s ↑, so T_e ↑]

② R_{se} increases? [I ↓ so T ↓]

③ R_e increases? [nothing]

④ T_s increase? [P_s ↑ so T_e ↑]

$$\left[T_e = T_s \left(\frac{R_s}{2R_{se}} \right)^{\frac{1}{2}} \right]$$

4/5

OH

What would happen to the temperature of the Earth if:

1. R_{sun} doubled
2. R_{earth} doubled
3. $D_{earth\text{-}sun}$ doubled

Choose from:

A. $T_{new} = \frac{1}{4} T_{old}$
B. $T_{new} = \frac{1}{2} T_{old}$
C. $T_{new} = 1/\sqrt{2}\, T_{old}$
D. $T_{new} = T_{old}$
E. $T_{new} = \sqrt{2}\, T_{old}$
F. $T_{new} = 2\, T_{old}$
G. $T_{new} = 4\, T_{old}$

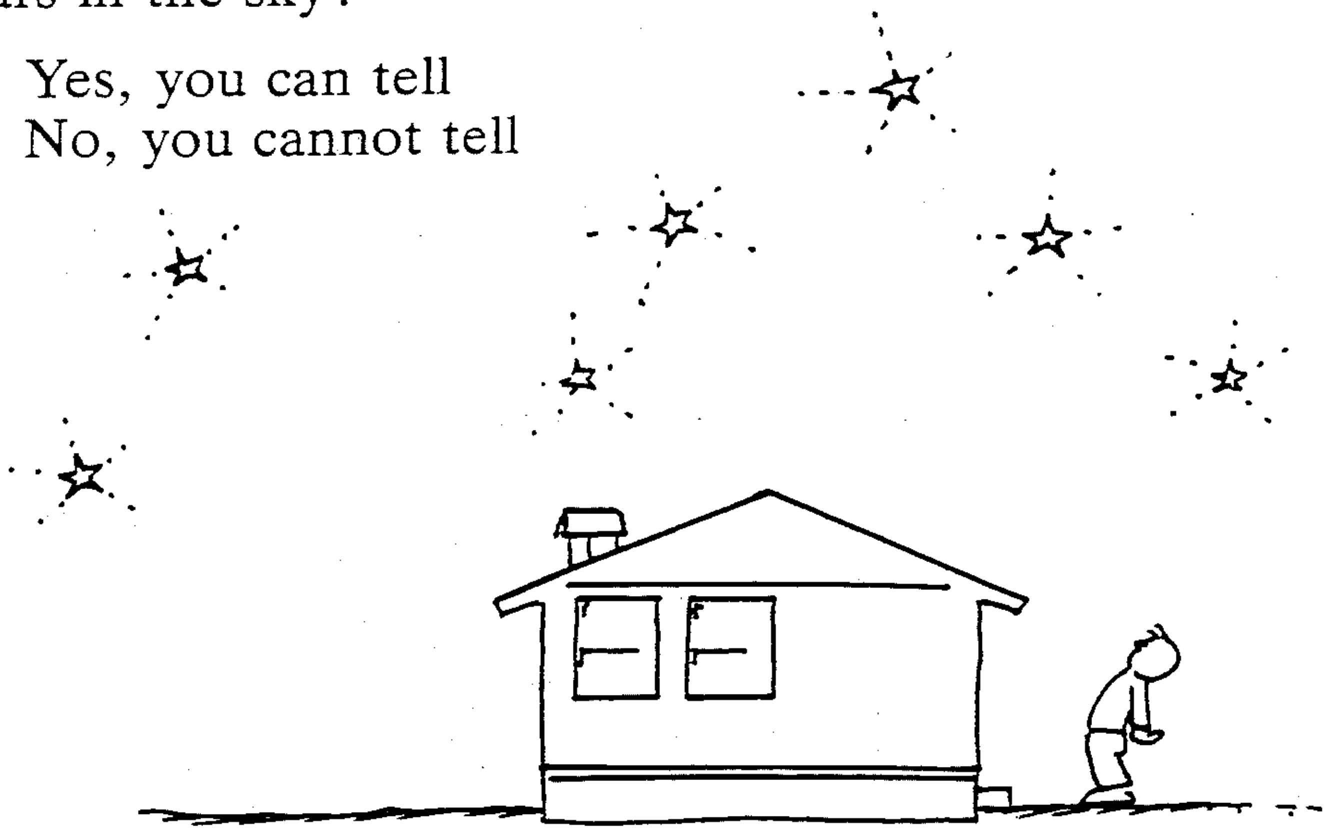
u16

0M

HOT STAR

Can you tell, just by looking, which are the hottest stars in the sky?

a) Yes, you can tell
b) No, you cannot tell



The hottest stars in the sky are the brightest stars in the sky.

a) True
b) False

Notes: P_{in} reduced by albedo

(earth not a perfect absorber)

P_{out} reduced by greenhouse effect

(IR radiation trapped + reflected)

U18

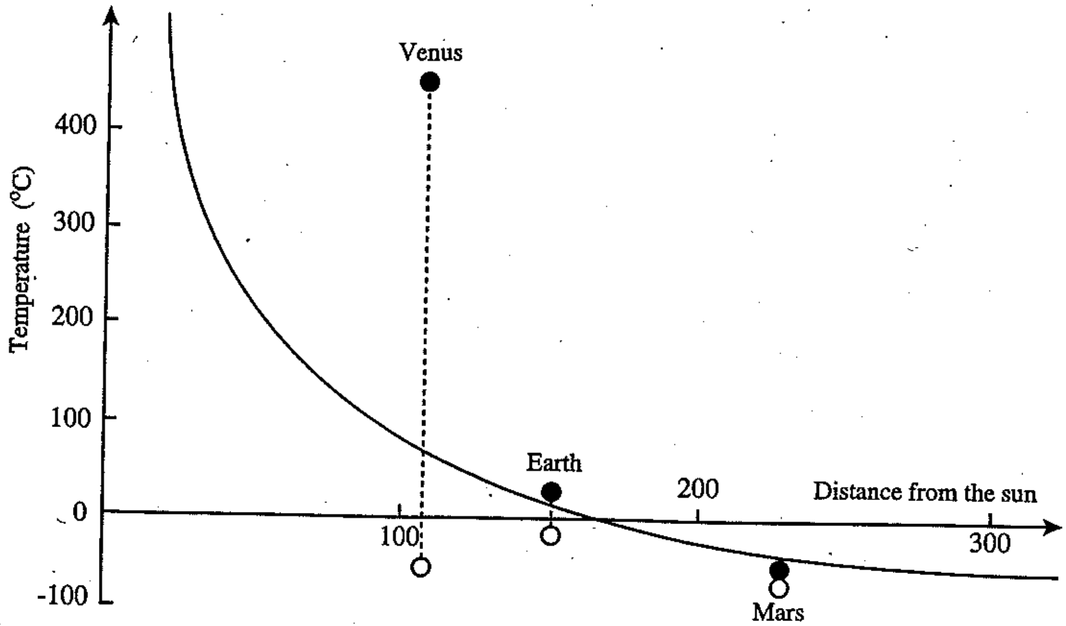


Figure 3.1 The curve shows the decrease in temperature, with increasing distance from the Sun (in units of 10^6 km), for planets that absorb all the incident sunlight and that have neither internal sources of energy nor atmospheres. The open circles take into account that each planet reflects some sunlight. The solid circles correspond to the actual temperatures at the surfaces of the planets. The length of each dashed line is a measure of the greenhouse effect.