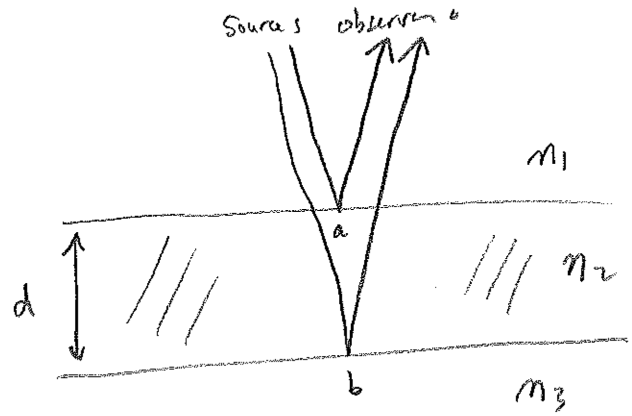
Thin film interference

→ e.g. oil on water
soap bubbles

Consider a thin layer of material w/ different index of refraction
(e.g. soap bubble, layer of oil)



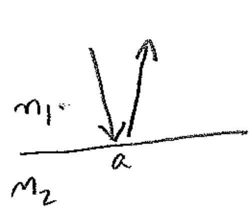
Light reflects off front + back surfaces, causing interference

Two paths from source to observer

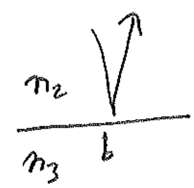
SaO w/ path length x_a
SbO w/ path length $x_b = x_a + \Delta x$ where $\Delta x = 2d$

$$E_{tot} = E_{saO} + E_{sbO} = A\sin(\omega t - kx_a + \delta_a) + A\sin(\omega t - kx_b + \delta_b)$$

δ_a, δ_b = possible phase shift upon reflection



$\delta_a = \pi$ if $n_1 < n_2$
$\delta_a = 0$ if $n_1 > n_2$



$\delta_b = \pi$ if $n_2 < n_3$
$\delta_b = 0$ if $n_2 > n_3$

Phase difference between 2 waves

$$\Delta\phi = (-kx_a + \delta_a) - (-kx_b + \delta_b) = k\,\Delta x + \delta_a - \delta_b$$
$$= \frac{2\pi}{\lambda_{n_2}}(2d) + \delta_a - \delta_b$$

where λ_{n_2} is the wavelength of light in medium w/ index of refraction n_2

Example: soap bubble

$n_1 = 1$ (air) $\Rightarrow \delta_a = \pi$

$n_2 = n = 1.33$ (water) $\Rightarrow \delta_b = 0$

$n_3 = 1$ (air)

$$\Rightarrow \Delta\phi = 2\pi\left(\frac{2d}{\lambda_n}\right) + \pi$$

If $2d = m\lambda_n$ for some integer m, then

$$\Delta\phi = 2\pi m + \pi = 2\pi(m + \tfrac{1}{2}) \quad \text{(destructive interf.)}$$

Therefore, reflection from front + back surfaces cancels out.

If $2d = (m + \frac{1}{2})\lambda_n$ for some integer m then

$$\Delta\phi = 2\pi(m + \tfrac{1}{2}) + \pi = 2\pi(m+1) \quad \text{(constructive interf.)}$$

Light w/ wavelength λ_n in medium is strongly reflected when film has thickness $d = (m + \frac{1}{2})\frac{\lambda_n}{2}$

ie $\frac{\lambda_n}{4}, \frac{3\lambda_n}{4}, \frac{5\lambda_n}{4}$, etc.

Because light travels through medium w/ speed $v = \frac{c}{n}$, the wavelength λ_n of a given color is different from its wavelength λ in vacuum. In vacuum $\lambda = \frac{c}{f}$

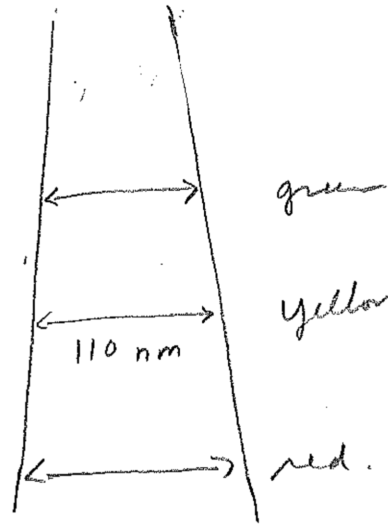
In medium $\lambda_n = \frac{v}{f} = \frac{c}{nf} = \frac{\lambda}{n}$.

For example: yellow in vacuum has $\lambda = 580$ nm

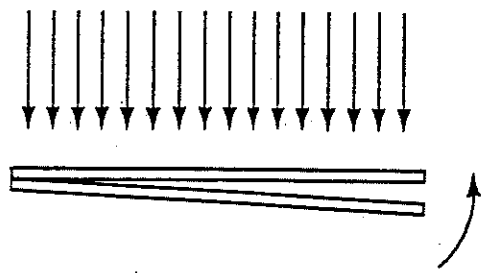
in water has $\lambda = 440$ nm

Q: Why different colors in soap films & oil slicks?

Because thickness varies across the film



Consider two identical microscope slides in air illuminated with monochromatic light. The bottom slide is rotated (counterclockwise about the point of contact in the side view) so that the wedge angle gets a bit smaller. What happens to the fringes?

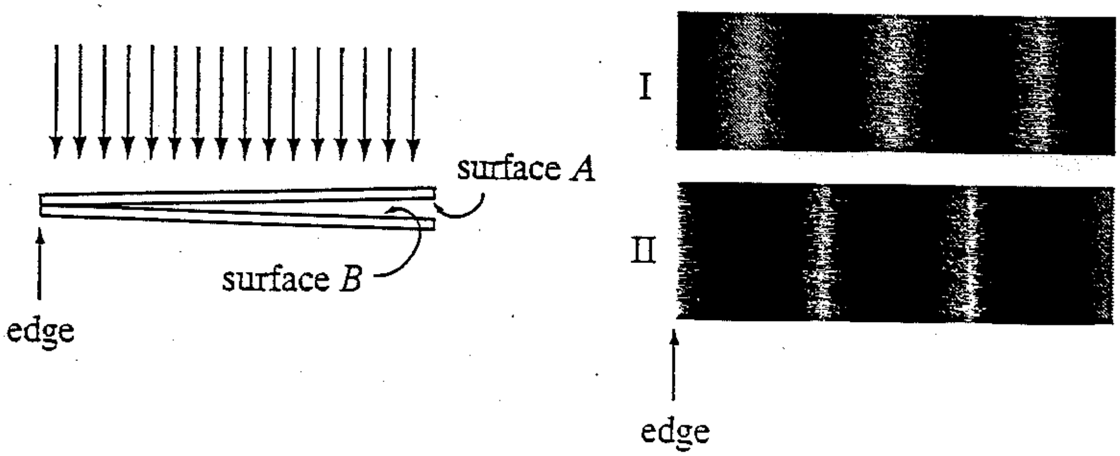


1. They are spaced farther apart.
2. They are spaced closer together.
3. They don't change.

QD5

0 M

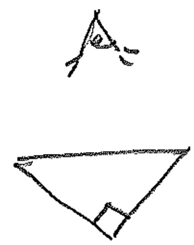
Monochromatic light shines on a pair of identical glass microscope slides that form a very narrow wedge. The top surface of the upper slide and the bottom surface of the lower slide have special coatings on them so that they reflect no light. The inner two surfaces (A and B) have nonzero reflectivities. A top view of the slides looks like

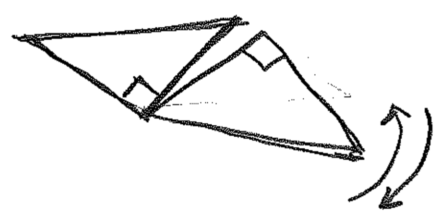


1. I.
2. II.

[DEMO] Hand out Porro prisms

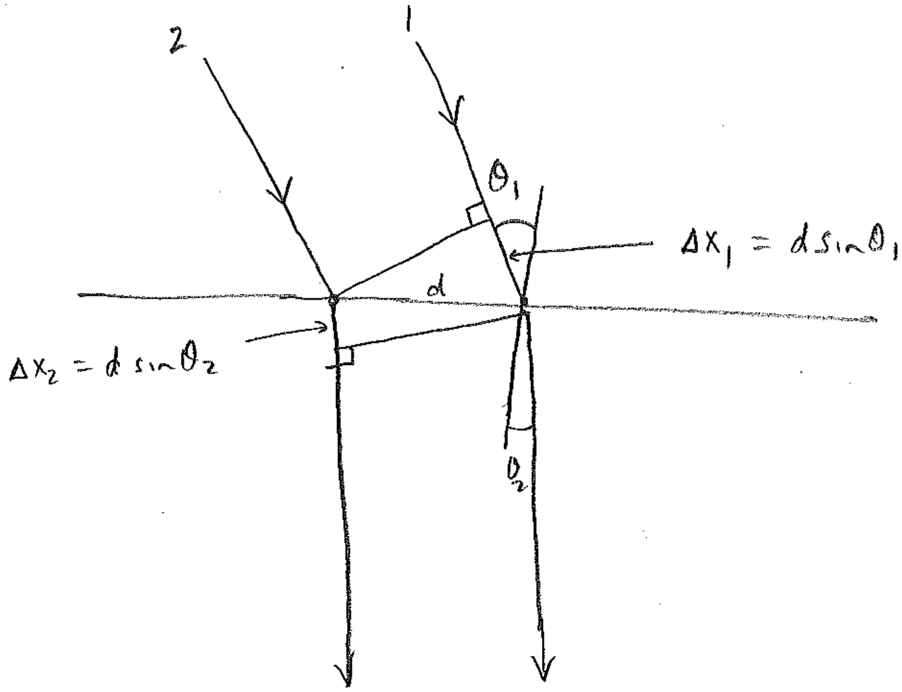
Put screen down in front of blackboard

- Polish prisms
- Look  until you see your eye
- Rotate slightly to see line
- Place 2nd prism behind to see interference fringes



rotate prism to see fringes move further apart or closer together

Snell's law of refraction can be understood via constructive interference and the difference of wavelengths in different media



$$\Delta\phi = k_1 \, \Delta x_1 - k_2 \, \Delta x_2$$

$$= \frac{2\pi}{\lambda_1}(d \sin\theta_1) - \frac{2\pi}{\lambda_2}(d \sin\theta_2)$$

Constructive interference occurs when $\Delta\phi = 0$

$$\frac{\sin\theta_1}{\lambda_1} = \frac{\sin\theta_2}{\lambda_2}$$

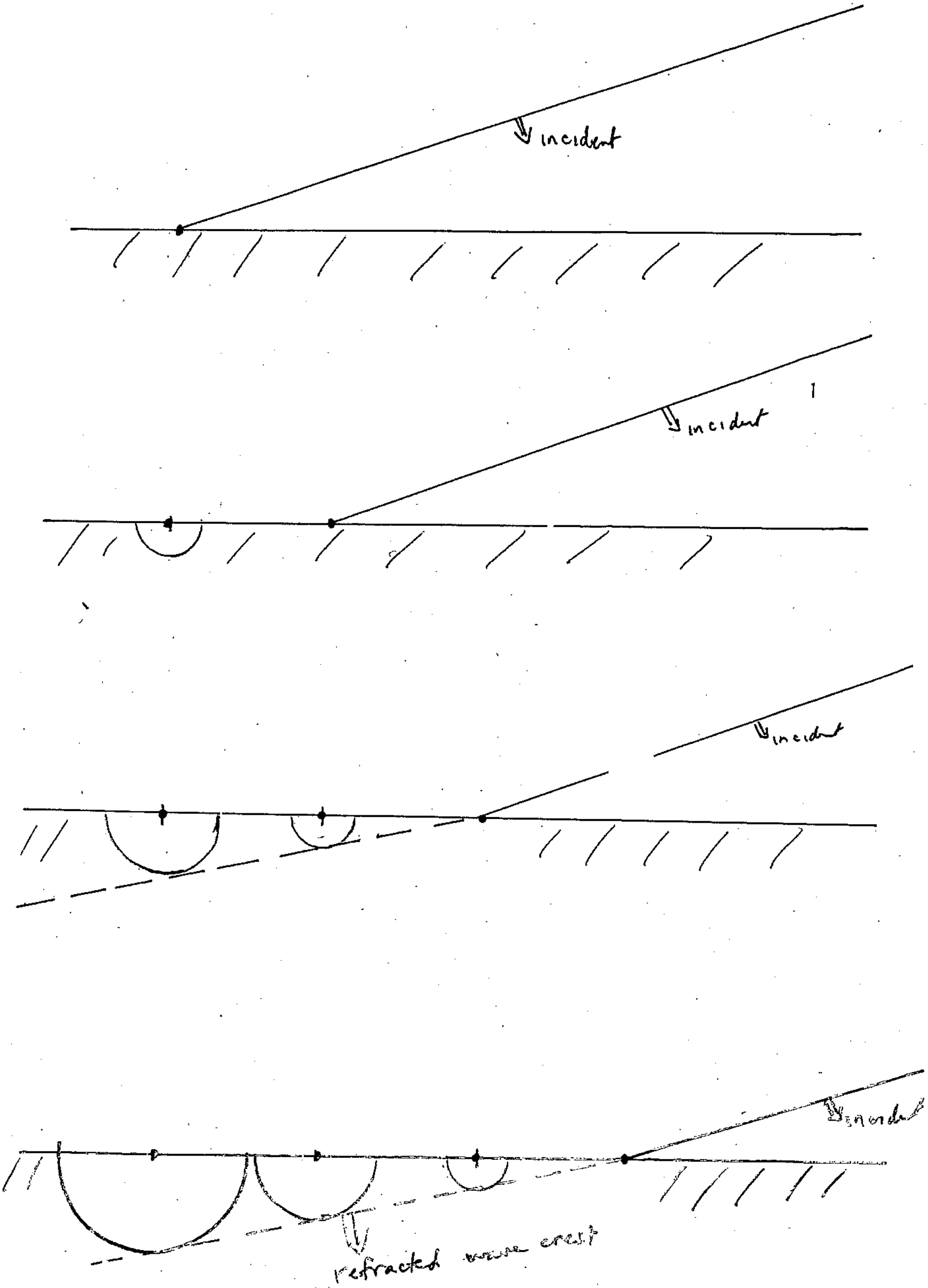
But $\lambda_1 = \frac{\lambda}{n_1}$ and $\lambda_2 = \frac{\lambda}{n_2}$

$$\Rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2$$

[see also OH → QD8, QD9]

QD8

OH


incident
incident
incident
incident
refracted wave crest

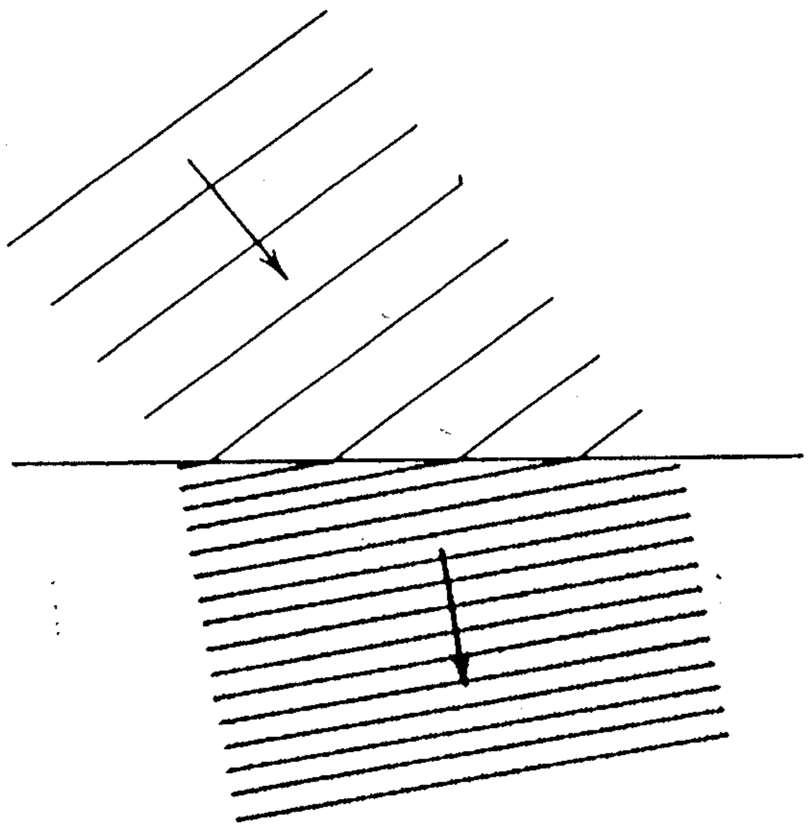


Fig. 5-3. Effect on the orientation of wavecrests and corresponding direction of travel when light passes into a medium in which it travels more slowly.

QD9

OH