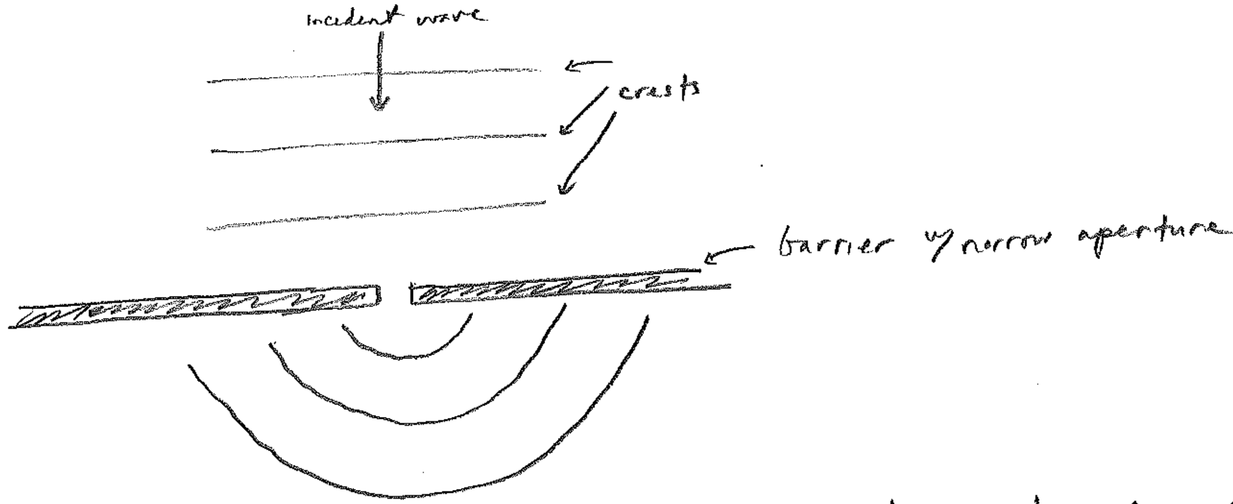
Diffraction through a narrow slit



When a wave passes through an aperture whose size $\ll \lambda$, the aperture acts as a source of radially spreading waves

[see OH, QB 2]

[Explain physical basis for this in terms of scattering.

An opaque screen "blocks" light by setting up counterbalancing waves that cancel incident wave through complete destructive interference.

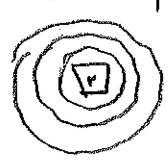
Consider barrier w/ a plug:

electrons in barrier + plug scatter waves.

$$E_{inc} + E_{barrier} + E_{plug} = 0.$$

If screen is not thick enough (thin gold film), cancellation is not complete

Scattered wave produced by plug



E_{plug}

$\Rightarrow$

Incident + scattered wave produced by barrier w/o plug is the same

$$E_{inc} + E_{barrier} = -E_{plug}$$

QB2

OH

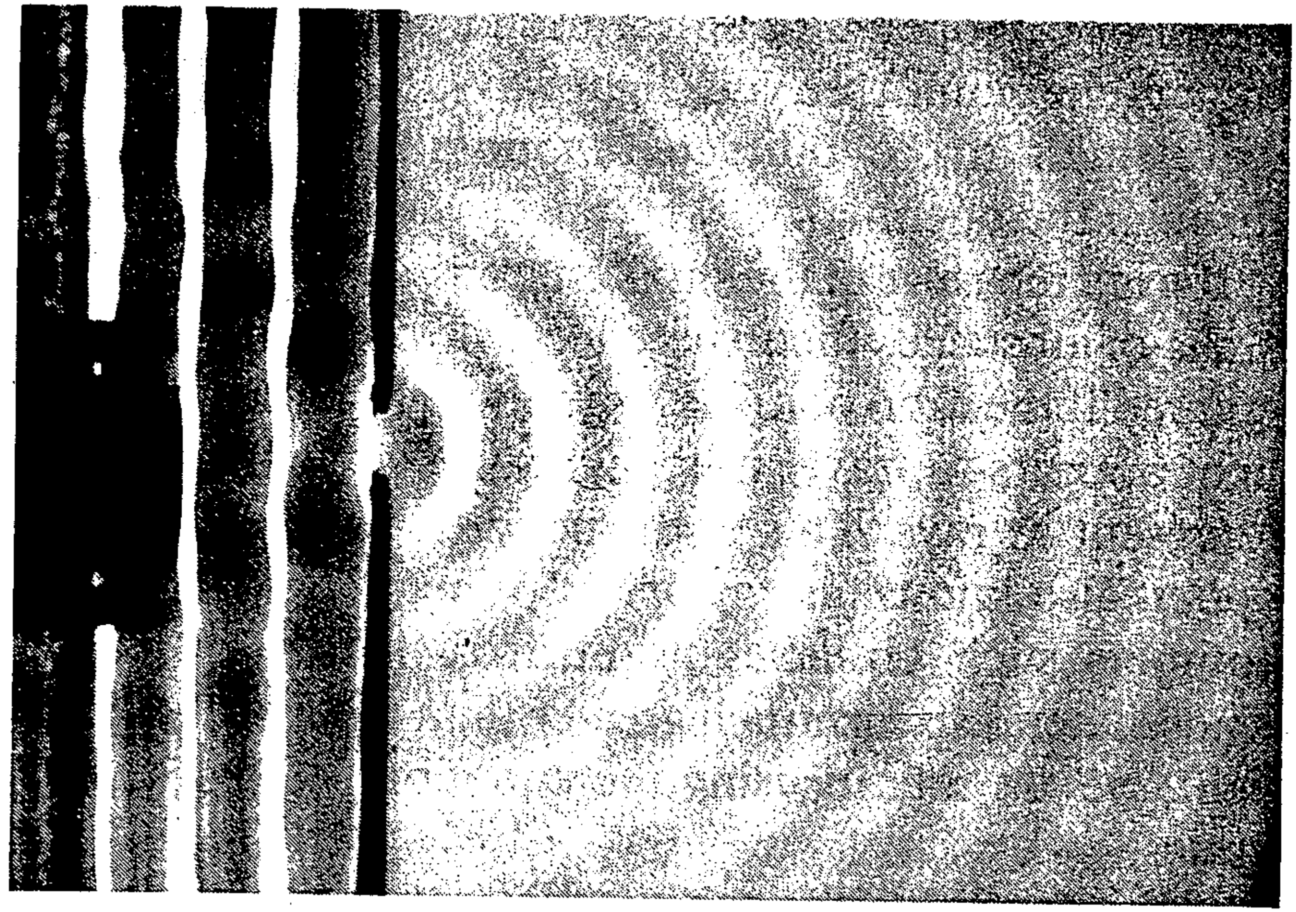
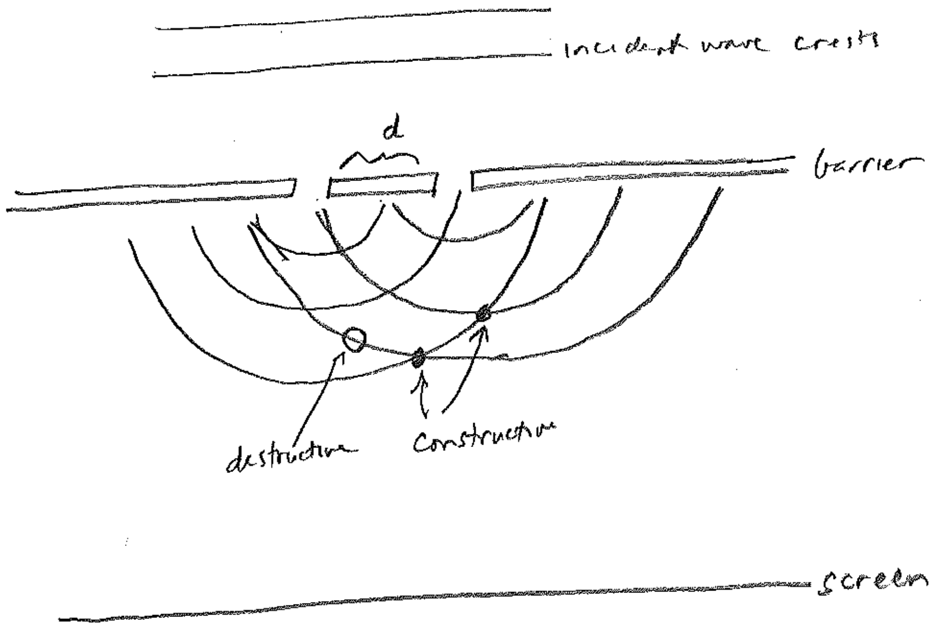


Figure 2 Diffraction of water waves at a slit in a ripple tank. Note that the slit width is about the same size as the wave-length. Compare with Fig. 1*c*.

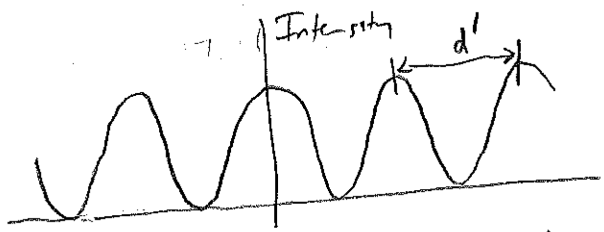
H+R, ch 43, fig 2

Double slit interference



[show OH, QB4]

Diffracted waves from two narrow slits, separated by d, interfere to produce an interference pattern of light on the screen behind the barrier



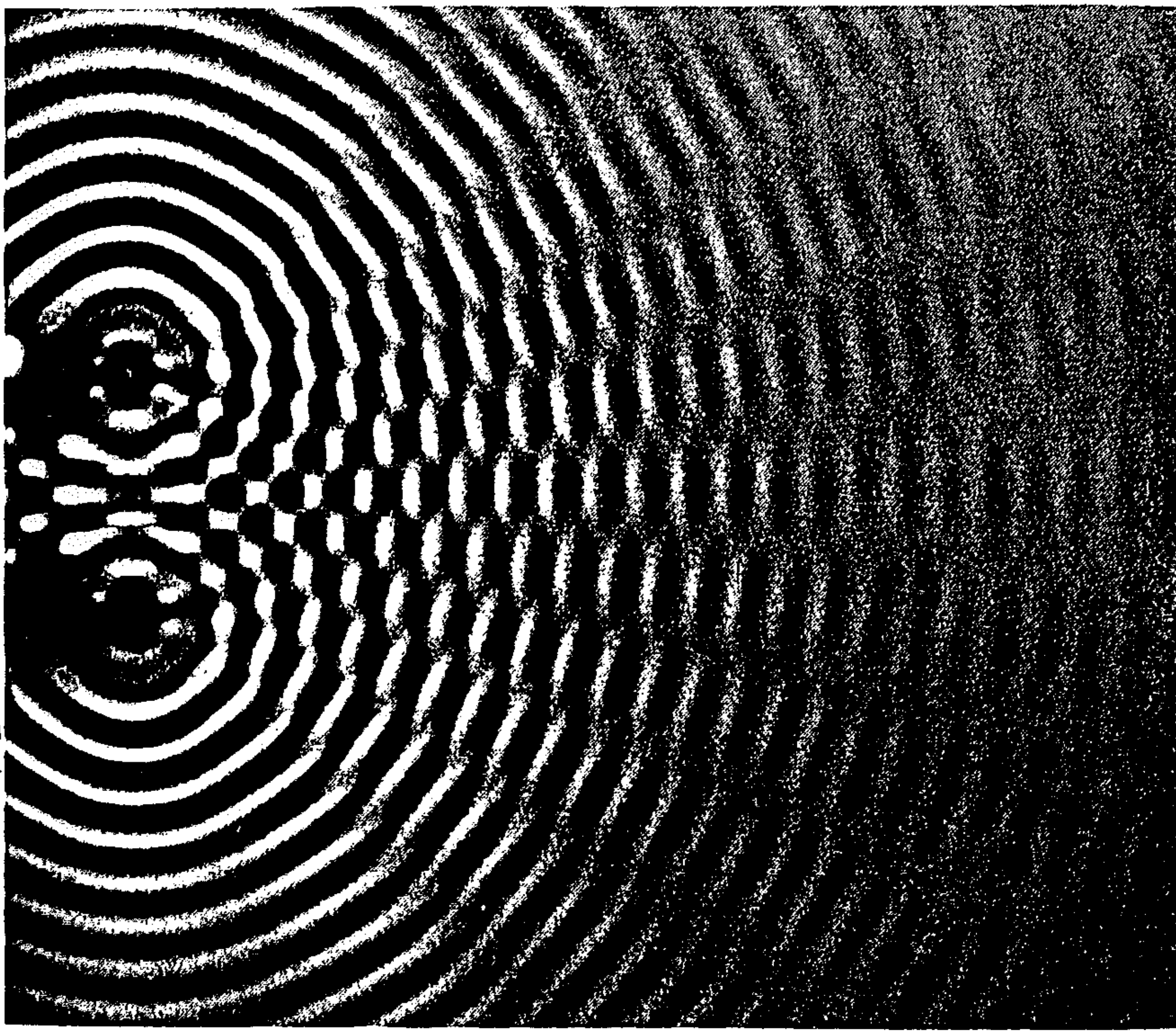
As d increases, the separation d' between the intensity peaks decreases and vice versa.

[Show, using QB4, QB5, QB6, how constructive & destructive interference occurs along "lines". Vary d to show change in intensity pattern.]

QB4

011

HRK (Se) 41-2



HRK (Se) 41-4

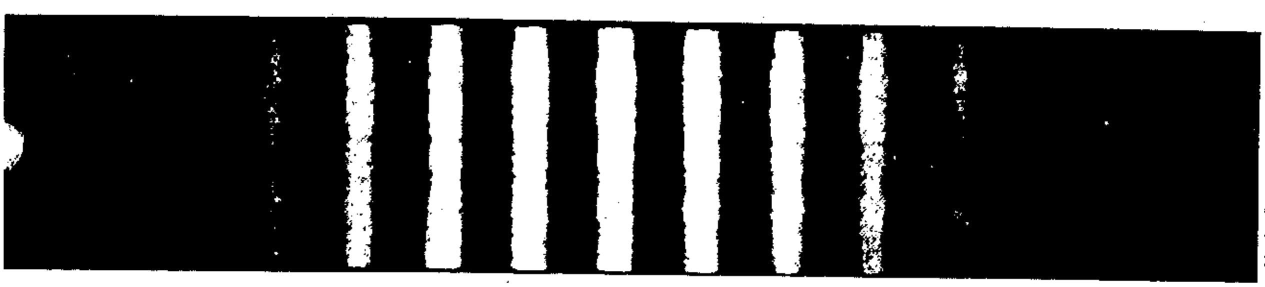
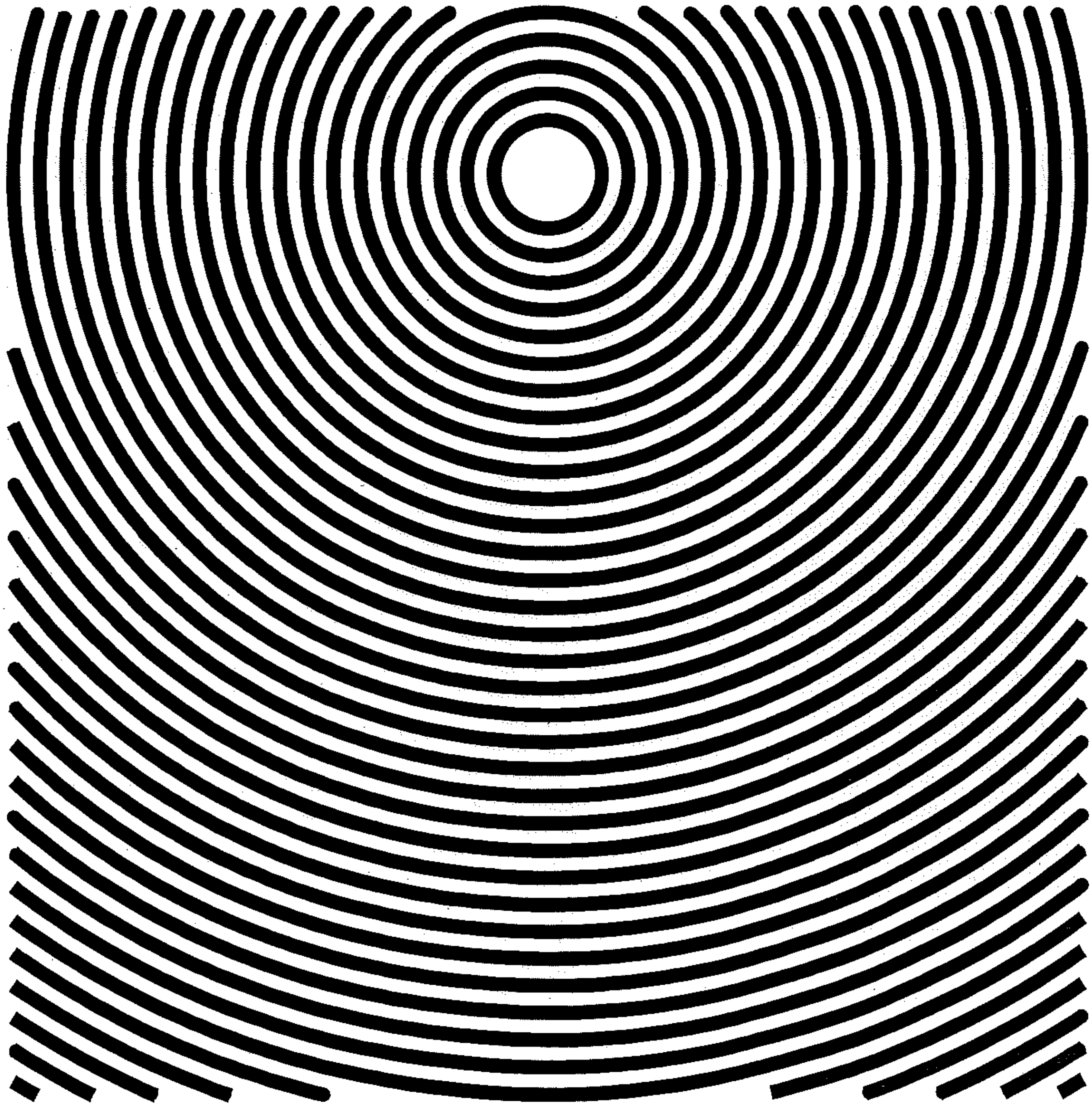


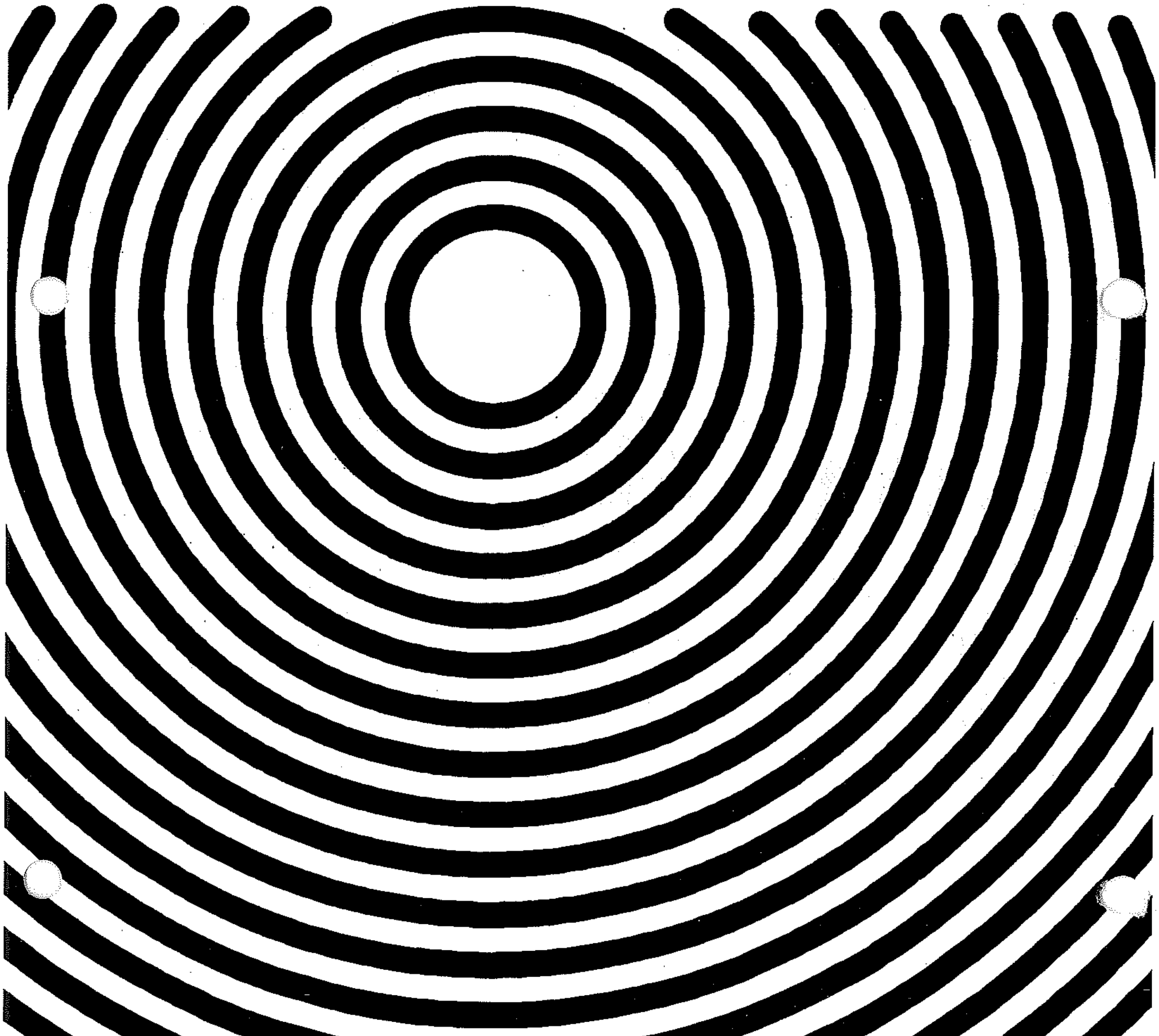
Figure 2 The interference pattern, consisting of bright and dark bands or fringes, that would appear on the screen of Fig. 1.

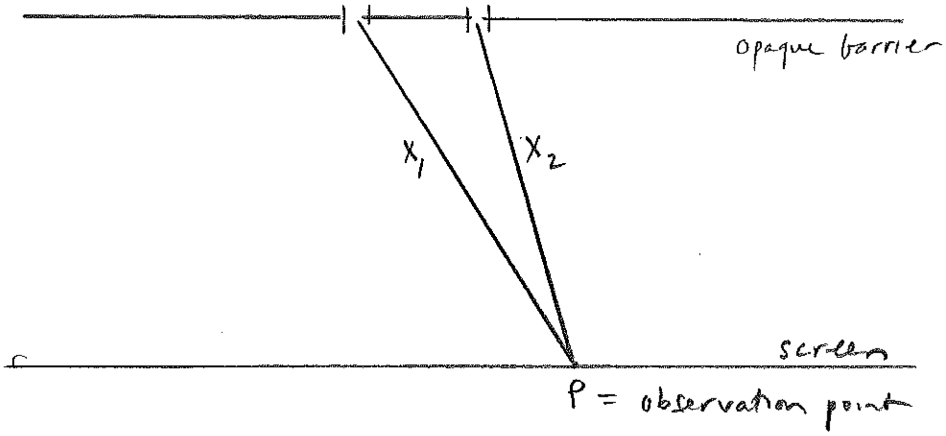
QB5



QB5

QB6





Electric field at P is

$$E_{tot} = E_1 + E_2 = A \sin(\omega t - kx_1) + A \sin(\omega t - kx_2)$$

The difference in path lengths $\Delta X = X_1 - X_2$ from the two slits causes a phase difference between the fields

$$E_{tot} = A \sin(\omega t - kx_1) + A \sin(\omega t - kx_1 + \underbrace{k\Delta X}_{\Delta\phi})$$

$$\Delta\phi = \phi_2 - \phi_1 = k\Delta X = \frac{2\pi}{\lambda}\Delta X$$

Constructive interference: $\Delta\phi = 2\pi m$ (m = integer)

$\Rightarrow \Delta X = m\lambda$ path lengths differ by whole # of wavelengths

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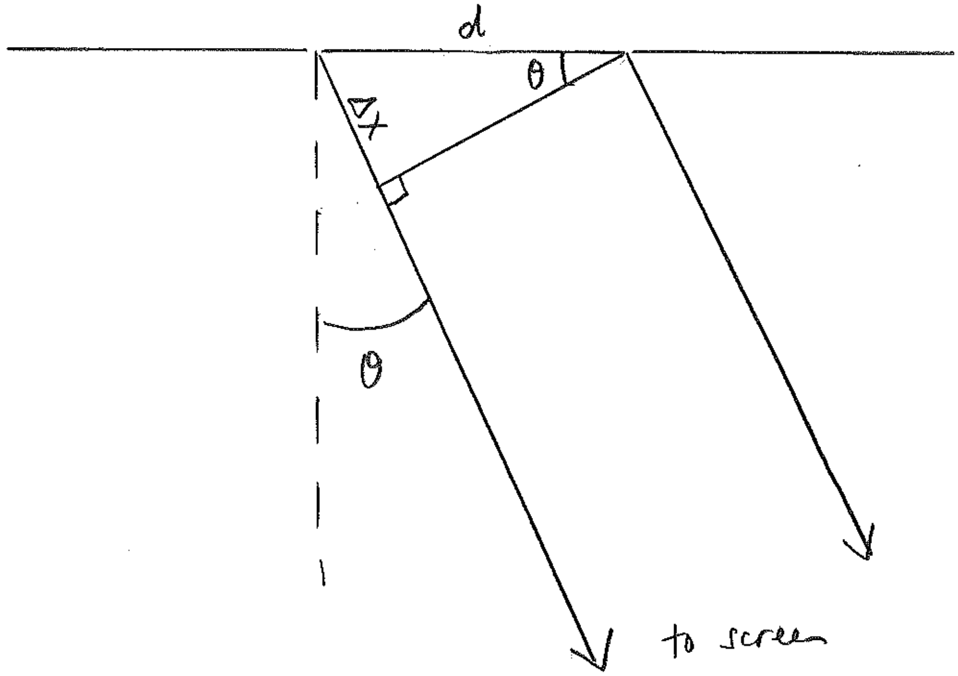
Destructive interference: $\Delta\phi = 2\pi\left(m + \frac{1}{2}\right)$

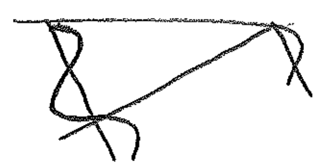
$\Rightarrow \Delta X = \left(m + \frac{1}{2}\right)\lambda$ path lengths differ by half a wavelength

Screen is usually distant relative to distance between slits so rays are nearly parallel

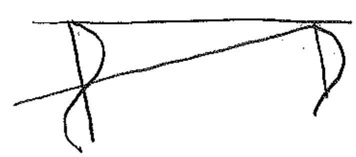


$$\sin\theta = \frac{\Delta x}{d} \Rightarrow \Delta x = d\sin\theta$$

Constructive interference: $d\sin\theta = m\lambda$, m = integer



Destructive interference: $d\sin\theta = (m + \frac{1}{2})\lambda$



Possible demos: laser + grating (middle double slit)

Mark's chains

Double slit intensity pattern

Using phasor methods, we found two interfering waves give

$E_{tot} = A_{tot} \sin(\omega t + \phi_{tot})$

$A_{tot} = 2A \cos\left(\frac{\Delta\phi}{2}\right)$

The intensity (time average of Poynting) is given by

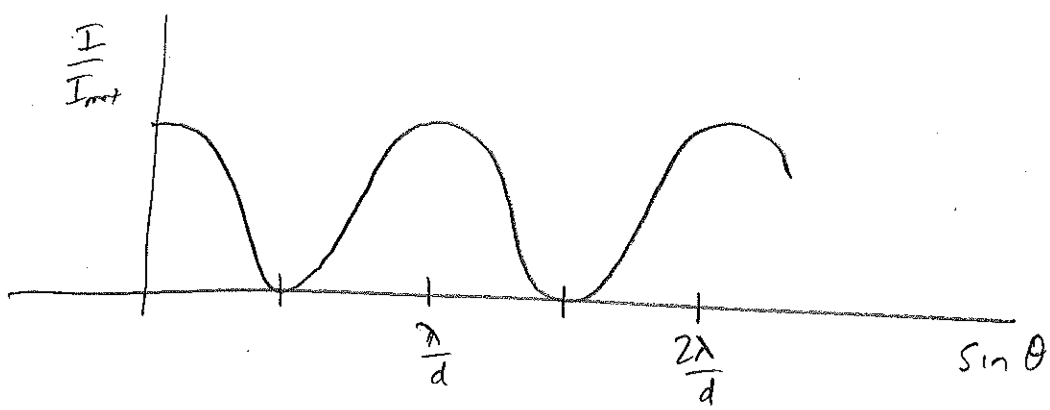
$$I = \frac{1}{2}\epsilon_0 c A_{tot}^2 = \frac{1}{2}\epsilon_0 c (2A)^2 \cos^2\left(\frac{\Delta\phi}{2}\right)$$

A_{tot} is maximized when $\Delta\phi = 0$, so $I_{max} = \frac{1}{2}\epsilon_0 c (2A)^2$

Relative intensity is thus $\frac{I}{I_{max}} = \cos^2\left(\frac{\Delta\phi}{2}\right)$

Recall $\Delta\phi = k\,\Delta x = \left(\frac{2\pi}{\lambda}\right)(d\sin\theta) =$

Thus $\boxed{\frac{I}{I_{max}} = \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)}$



Distance between intensity maxima is inversely proportional to d, the distance between the slits