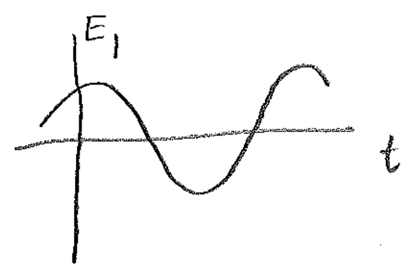
Wave optics

Key concept: interference is caused by the superposition of 2 or more waves

Consider an EM wave at a fixed point in space.

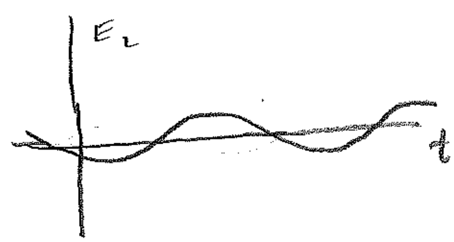
Suppose one of the components of the electric field (say E_y) varies in time as

$$E_1 = A_1 \sin(\omega t + \phi_1)$$



Consider another wave with same frequency but different amplitude and phase

$$E_2 = A_2 \sin(\omega t + \phi_2)$$

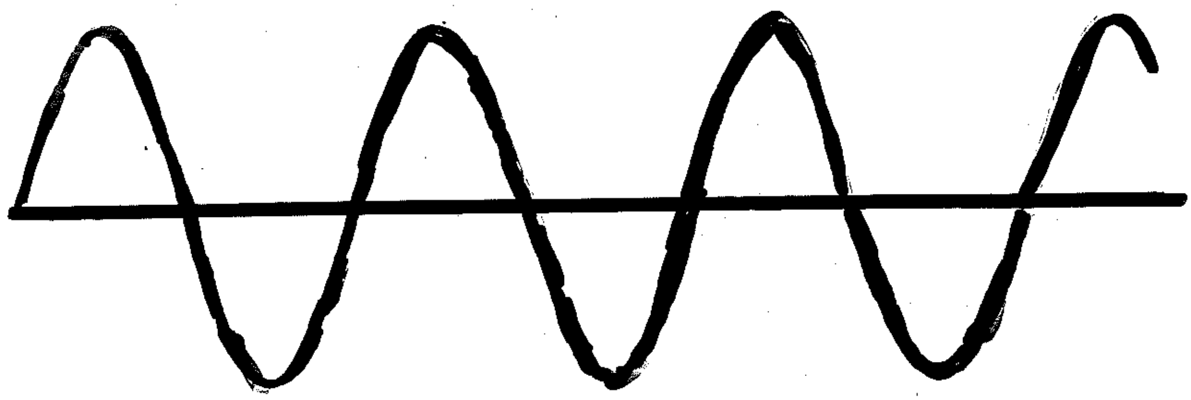


Superposition principle

$$E_{tot} = E_1 + E_2$$

[Show how to add these graphically on OH projector]

OH



Interference depends on the difference of the phases

Define phase difference $\Delta\phi = \phi_2 - \phi_1$

Takeaways:

1) If $\Delta\phi = 0$ (or a multiple of 2π), waves are "in phase"

$$E_{tot} = (A_1 + A_2) \sin(\omega t + \phi_1)$$

Amplitudes add $\Rightarrow$ constructive interference [see OH →]

2) If $\Delta\phi = \pi$ (or π + multiple of 2π), waves are "180° out of phase"

$$E_{tot} = A_1 \sin(\omega t + \phi_1) + A_2 \underbrace{\sin(\omega t + \phi_1 + \pi)}_{-\sin(\omega t + \phi_1)}$$

$$= (A_1 - A_2) \sin(\omega t + \phi_1)$$

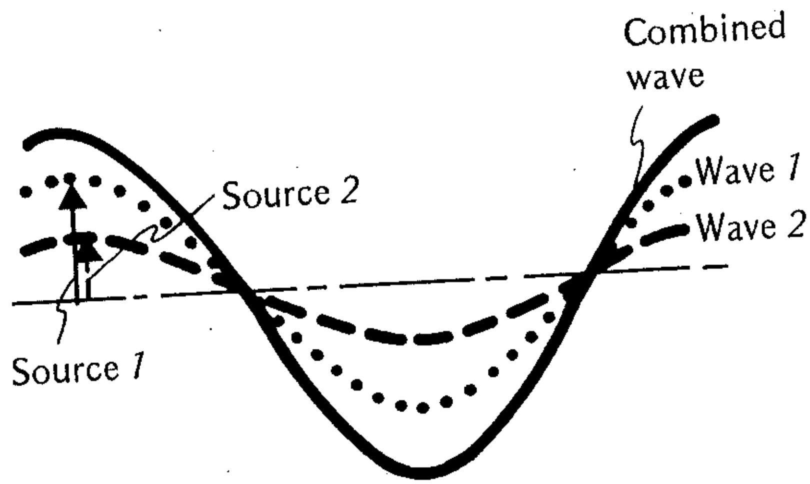
Amplitudes subtract $\Rightarrow$ destructive interference [see OH →]

If also $A_1 = A_2$ then $E_{tot} = 0$
total destructive interference

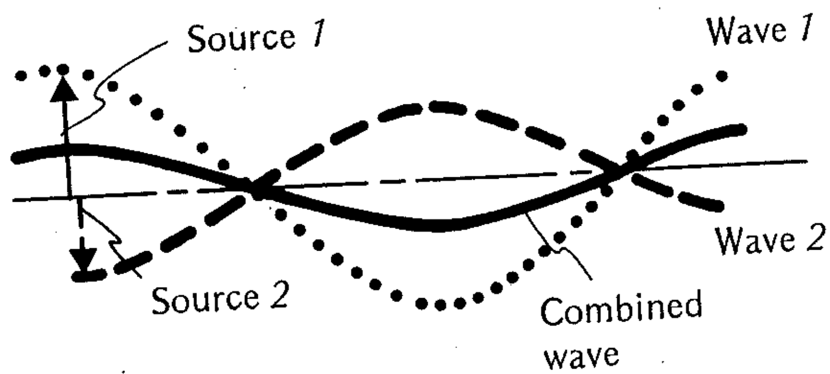
3) If $\Delta\phi \neq 0$ or π, then partial interference

QA 4

OH


Combined wave
Source 2
Wave 1
Wave 2
Source 1

(a)

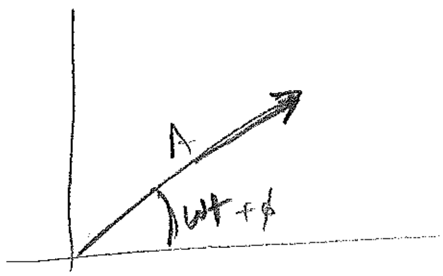

Source 1
Wave 1
Wave 2
Source 2
Combined wave

(b)

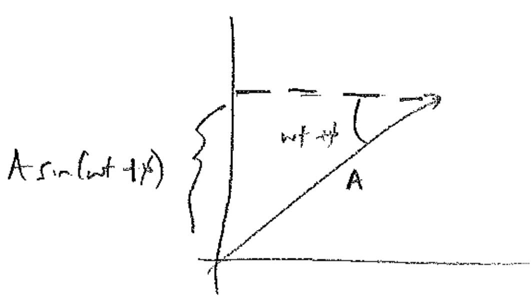
Phasor methods

Phasor: a graphical representation of an oscillating field

= a vector in a plane, w/ magnitude A & angle $(\omega t + \phi)$



The projection of phasor onto the vertical axis gives the field

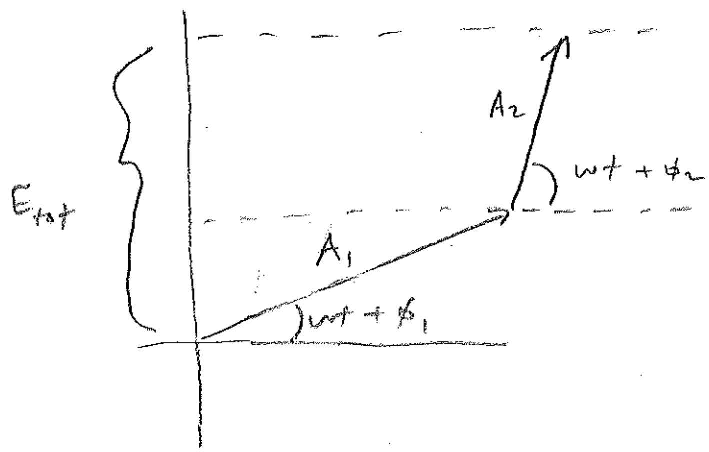


Phasor is rotating w/ const angular velocity ω

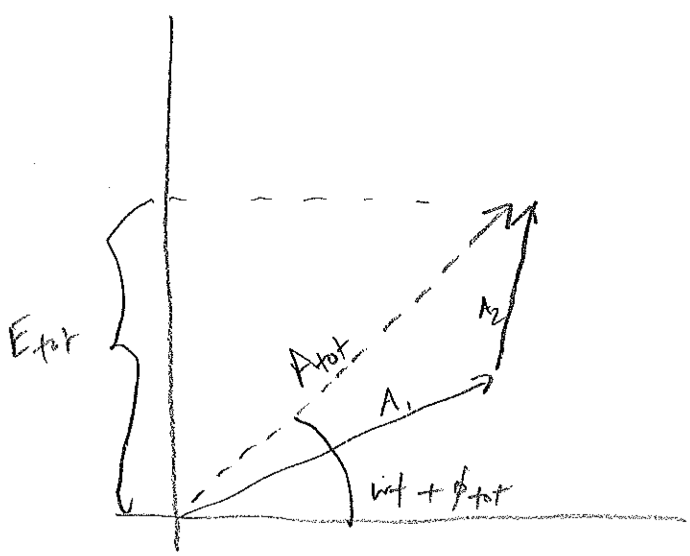
The superposition of two fields

$$E_{tot} = A_1 \sin(\omega t + \phi_1) + A_2 \sin(\omega t + \phi_2)$$

is represented by placing phasors head-to-tail



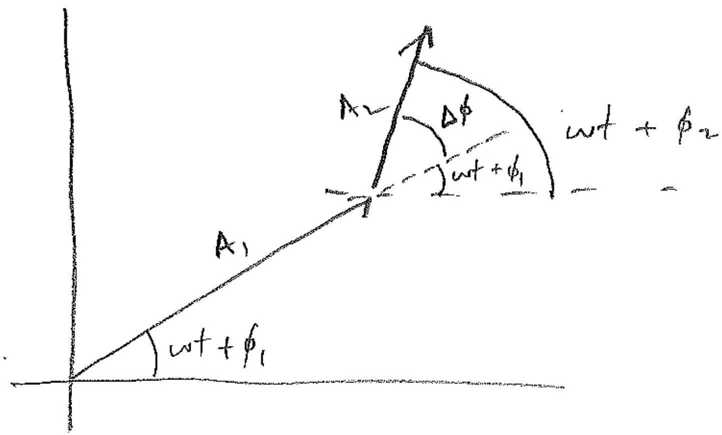
But this is the same as the field represented by the vector sum of the phasors



We need to find A_{tot} in terms of A_1, A_2, and $\Delta\phi$

Recall $\Delta\phi = \phi_2 - \phi_1$.

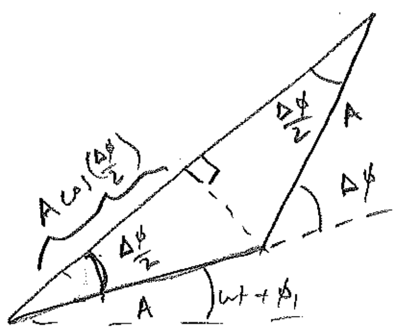
where is $\Delta\phi$ on the phasor diagram?



$\Delta\phi$ is the difference in angles between the phasors.

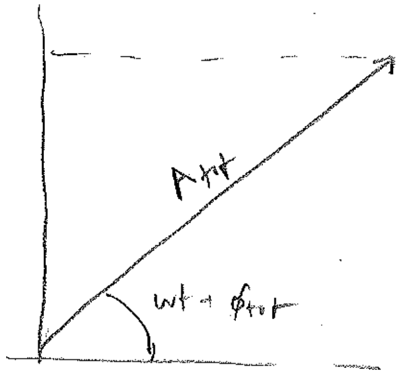
What are A_{tot} and ϕ_{tot}?

For simplicity, let $A_2 = A_1$ ⇒ phasors form an isosceles triangle



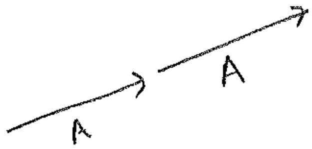
$$A_{tot} = 2A\cos\left(\frac{\Delta\phi}{2}\right)$$

Also $\phi_{tot} = \phi_1 + \frac{\Delta\phi}{2}$



$$E_{tot} = A_{tot}\sin(\omega t + \phi_{tot})$$

Constructive interference occurs when phasors are parallel

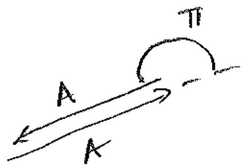


$\Delta\phi = 0$ (or a multiple of 2π)

$\Delta\phi = 2\pi m$ where m = integer

$|A_{tot}| = 2A$

Complete destructive interference occurs when phasors add up to zero



$\Delta\phi = \pi$ (or $\pi + 2\pi m$)

$\Delta\phi = 2\pi(m + \frac{1}{2})$

$A_{tot} = 2A\cos(\frac{\pi}{2}) = 0$

Partial interference occurs between $\Delta\phi = 0$ and π