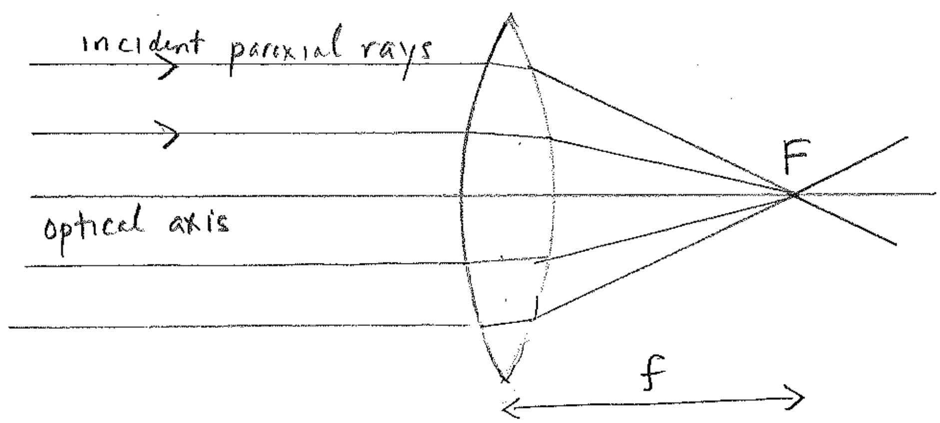
Lenses

mirrors use reflection to focus light;
lenses use refraction to focus light

Converging lens



Paraxial rays (= rays parallel and close to the optical axis) are refracted to converge to a focal point F

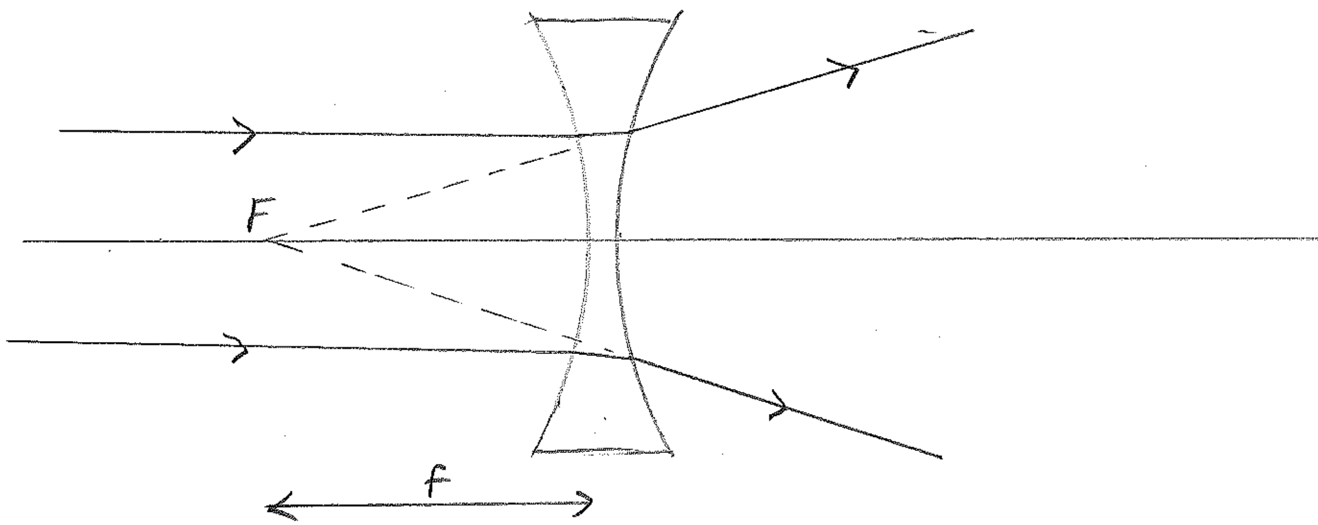
F is a <u>real</u> focus because rays actually pass through it

∴ F is on the R-side

The right hand side of a lens is the R-side (assuming rays incident from the left)

Focal length $f > 0$ for a converging lens

Diverging lens



For a diverging lens, F is a *virtual* focus because rays appear to come from it, but do not actually pass through it

$\therefore$ F is on V-side

The left hand side of a lens is the V-side

$f < 0$ for a diverging lens

Lensmaker's equation

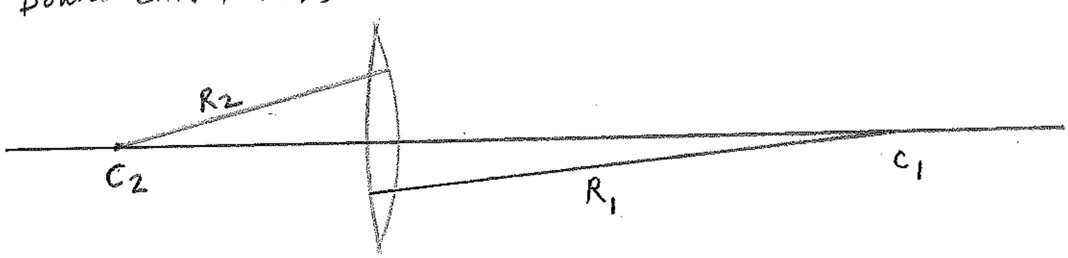
For a mirror, $f = \frac{1}{2}R$
but for a lens, f depends on both ① shape of lens, and ② refraction index

$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$ lensmaker's eqn

Since n depends on λ (dispersion), the focal lengths of different colors are slightly different.
This is called chromatic aberration

R_1 = radius of curvature of the surface first struck by incident light
R_2 = " " " " " struck second by " "

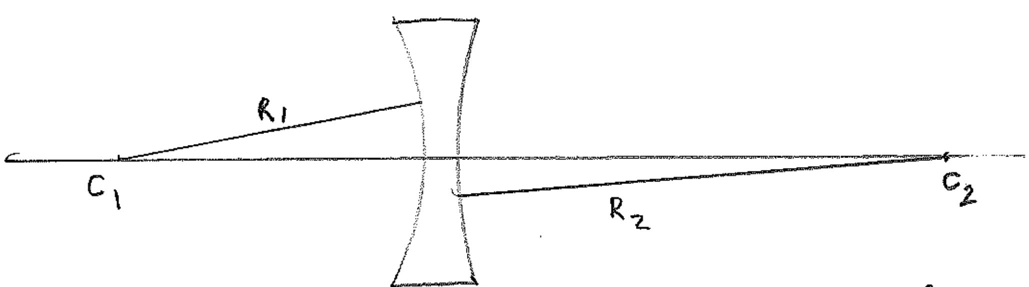
Double convex lens



$R_1 > 0$ because C_1 on R-side
$R_2 < 0$ because C_2 on V-side

$$\underbrace{\frac{1}{R_1}}_{+} - \underbrace{\frac{1}{R_2}}_{-} > 0 \Rightarrow f > 0$$

Biconvex is converging



$R_1 < 0$, $R_2 > 0$ $\qquad \frac{1}{R_1} - \frac{1}{R_2} < 0 \Rightarrow f < 0$

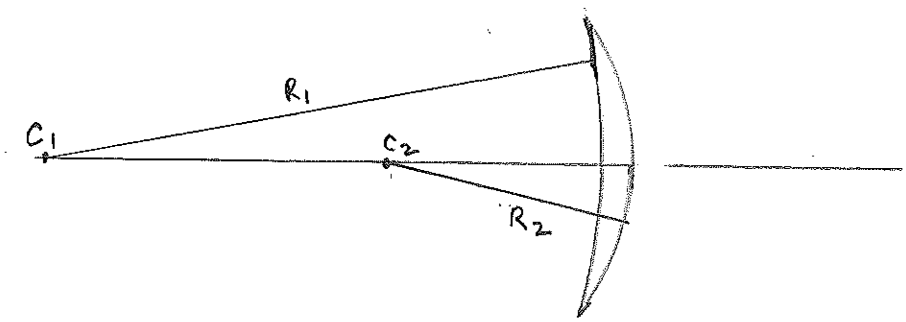
Biconcave is diverging

Meniscus:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

What is sign of f?

Meniscus



$R_1 < 0$

$R_2 < 0$

$|R_1| > |R_2|$

$\frac{1}{|R_1|} < \frac{1}{|R_2|}$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (n-1)\left(-\frac{1}{|R_1|} + \frac{1}{|R_2|}\right) > 0$$

$f > 0 \Rightarrow$ meniscus is converging

Rule of thumb: thicker in center $\Rightarrow$ converging

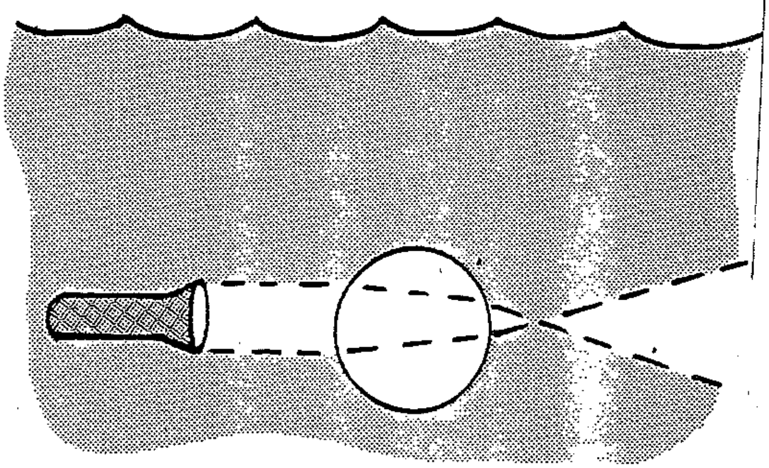
thinner in center $\Rightarrow$ diverging

BUBBLE LENS

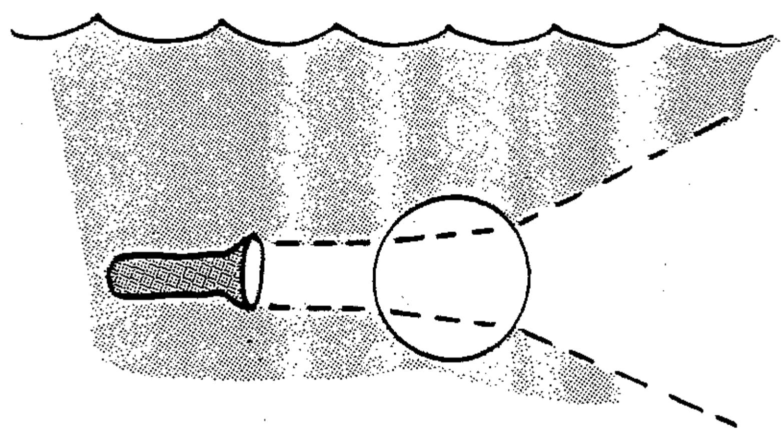
optional P6

Underwater is a bubble. A light beam shines through it. After passing through the bubble the light beam

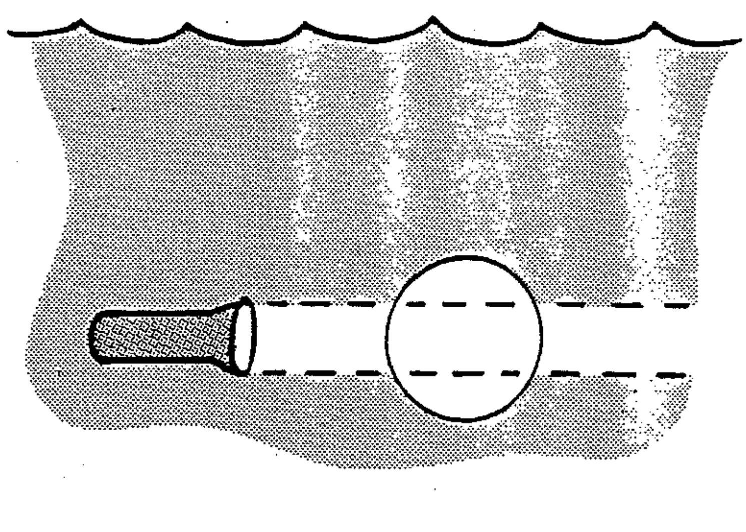
a) converges



b) diverges



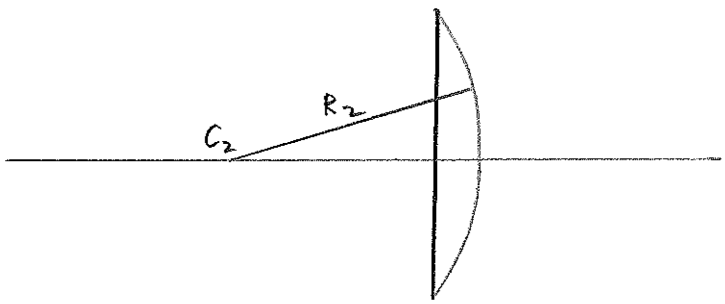
c) is unaffected



Let's derive lens maker's formula

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

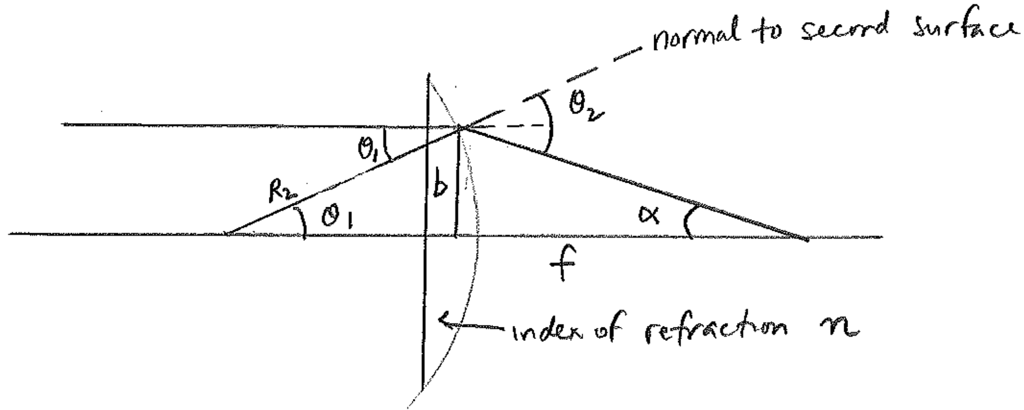
for the special case of a plano-convex lens



$R_1 = \infty$

$R_2 < 0$

$$\frac{1}{f} = (n-1)\left(\frac{1}{\infty} - \frac{1}{-|R_2|}\right) = \frac{n-1}{|R_1|}$$



Snell's law: $n \sin\theta_1 = \sin\theta_2$

geometry $\theta_1 + \alpha = \theta_2$

$$\sin\theta_1 = \frac{b}{|R_2|}$$

$$\tan\alpha = \frac{b}{f}$$

Paraxial rays are close to optical axis
so all angles are small (in radians)

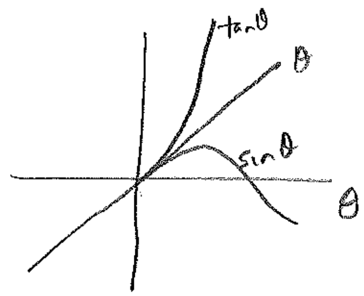
Small angle approximation: $\sin\theta \approx \theta \approx \tan\theta$

↑ radians

Observe: $\sin(30°) = \frac{1}{2}$

But $30° \cdot \left(\frac{\pi}{2}\right) = \frac{\pi}{6} = 0.52$ radians

$\sin(0.52\text{ rad}) = 0.50 \approx 0.52$ [4% error]



Snell: $n\theta_1 \approx \theta_2$

geometry: $\theta_1 + \alpha = \theta_2 \approx n\theta_1$

$$\Rightarrow \alpha = (n-1)\theta_1$$

$$\frac{b}{f} = (n-1)\frac{b}{|R_2|}$$

$$\frac{1}{f} = \frac{n-1}{|R_2|} \quad \checkmark$$

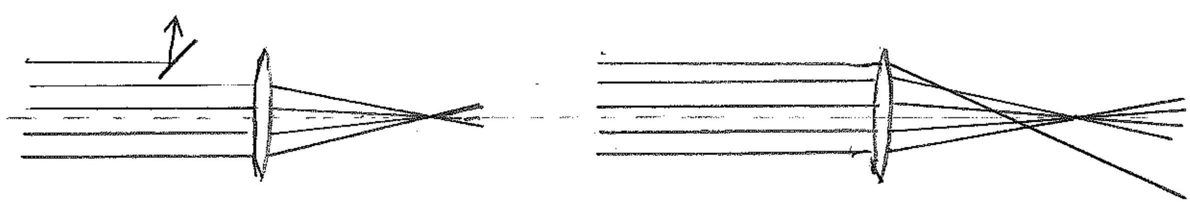
Spherical aberration [optional]

paraxial rays converge to the focus F
but nonparaxial rays converge to a point between the lens + F

DEMO: Blackboard optics
Multiple ray source w/ double convex lens

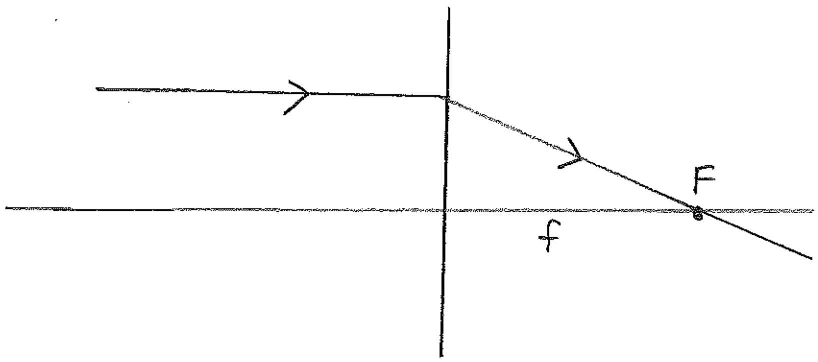
Use 4 parallel rays to see focus (block the 5th)

Then show that the fifth ray does not go to focus



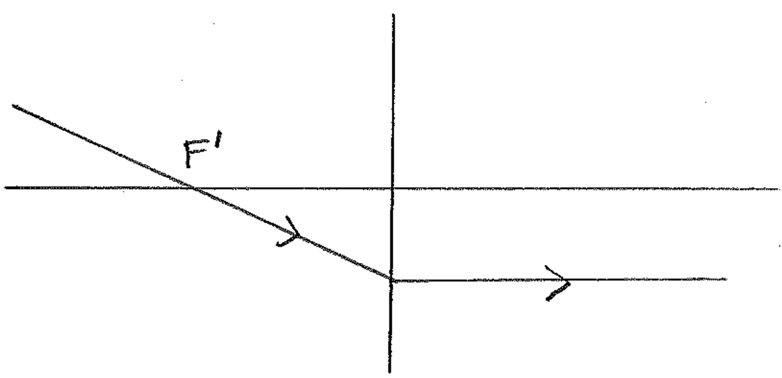
[can use stops to block outer rays,
or multiple lens systems to correct.]

Three ray tracing rules for converging lenses



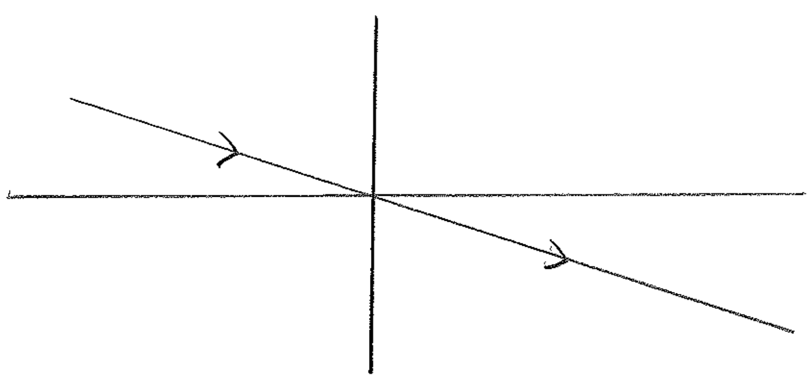
① a paraxial ray is refracted through F

F = true focus since $f > 0$ for a converging lens



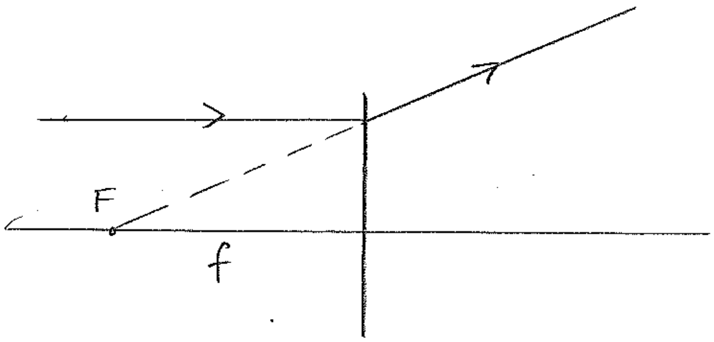
② a ray passing through F' is refracted parallel to axis

F' is on opposite side of lens from F and the same distance from lens



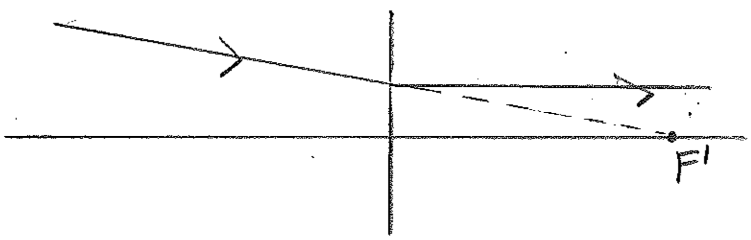
③ a ray passing through the center of the lens is undeviated (parallel surfaces)

Three ray tracing rules for diverging lenses

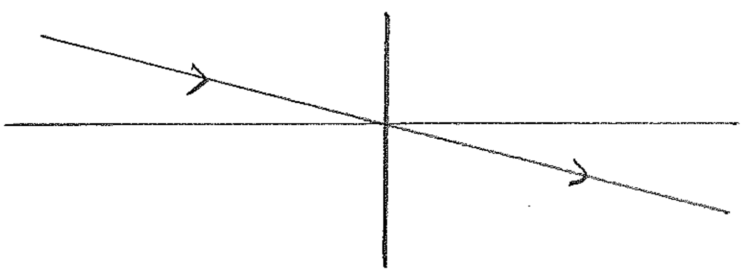


① a paraxial ray is refracted so as to appear to come from F

F = true focus since $f<0$ for a diverging lens



② a ray that would have passed thru F' is refracted parallel to axis



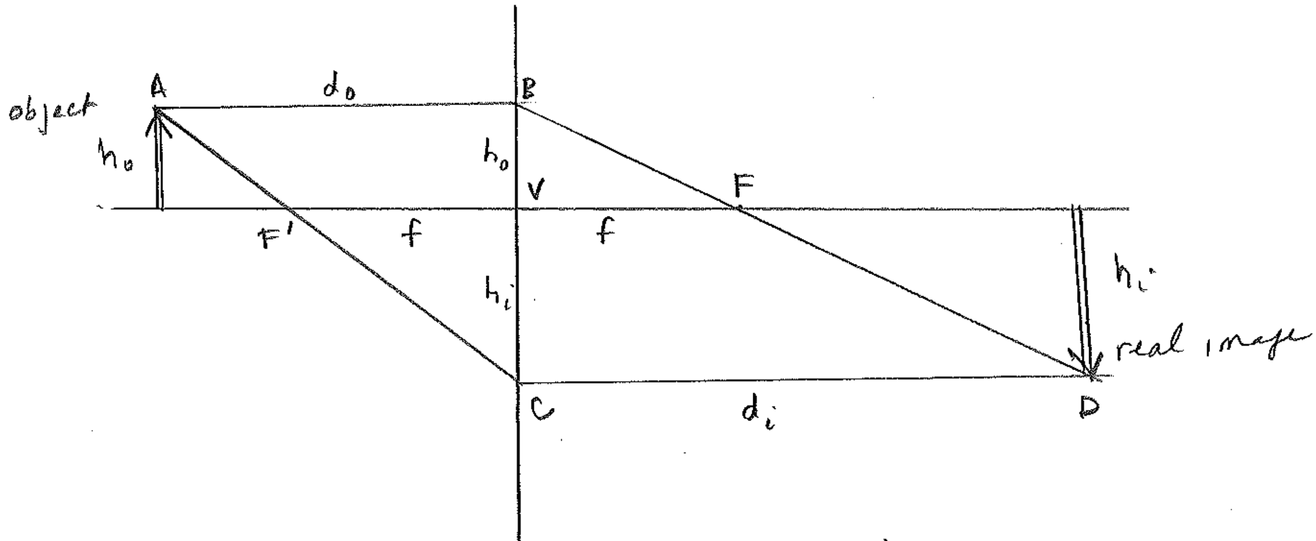
③ a ray passing through the center of the lens is undeviated

Important exception to the sign convention!

An object on the V-side of a lens is nevertheless a real object, and $d_o > 0$

An object on the R-side of a lens is a virtual object, and $d_o < 0$

Formation of real image by a converging lens



Similar triangles: ABC = F'VC $\qquad \dfrac{h_o + h_i}{d_o} = \dfrac{h_i}{f}$

BCD = BFV $\qquad \dfrac{h_o + h_i}{d_i} = \dfrac{h_o}{f}$

Add equations: $\dfrac{h_o+h_i}{d_o} + \dfrac{h_o+h_i}{d_i} = \dfrac{h_i}{f} + \dfrac{h_o}{f} = \dfrac{h_o+h_i}{f}$

$$\boxed{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}}$$ (thin lens equation)

Divide equations

$$\frac{\left(\frac{h_o+h_i}{d_o}\right)}{\left(\frac{h_o+h_i}{d_i}\right)} = \frac{\left(\frac{h_i}{f}\right)}{\left(\frac{h_o}{f}\right)}$$

$$\frac{d_i}{d_o} = \frac{h_i}{h_o}$$

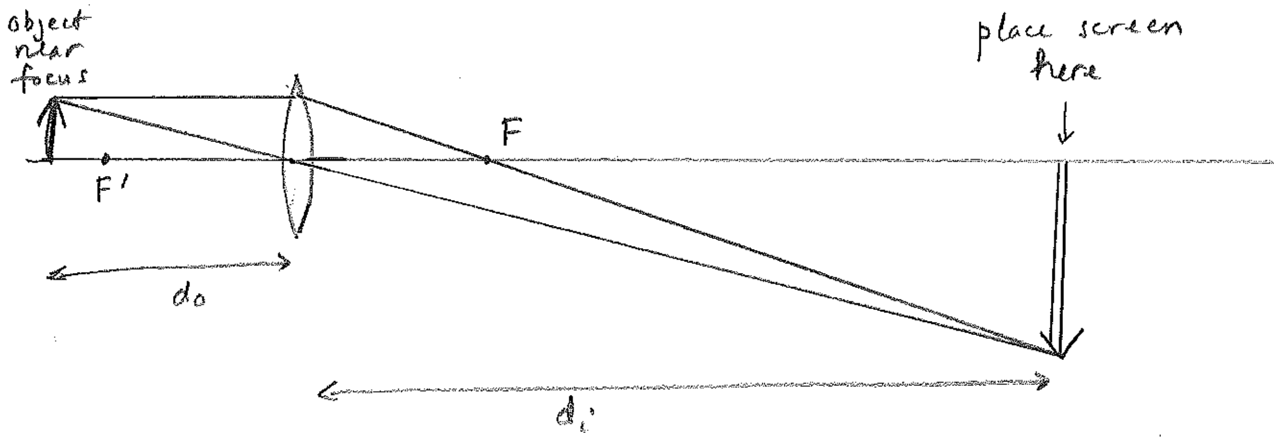
Magnification $m = -\dfrac{h_i}{h_o} = -\dfrac{d_i}{d_o}$ $\qquad \boxed{m = -\dfrac{d_i}{d_o}}$

(magnification equation)

Image inverted

Applications: slide (or overhead) projector

Lens makes a large real image of a small object placed just beyond F'



DEMO: old brown overhead projector (single lens!)

transparency w/ arrow.

focus the arrow, then move lens up ($d_o \uparrow$)

where is the image? find it w/ piece of paper (or ground glass)

($d_o \uparrow \Rightarrow d_i \downarrow$)

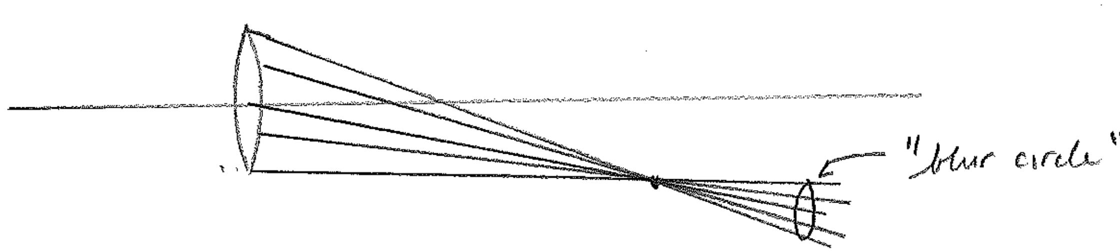
optional

use small hole cut in paper to form point-like image

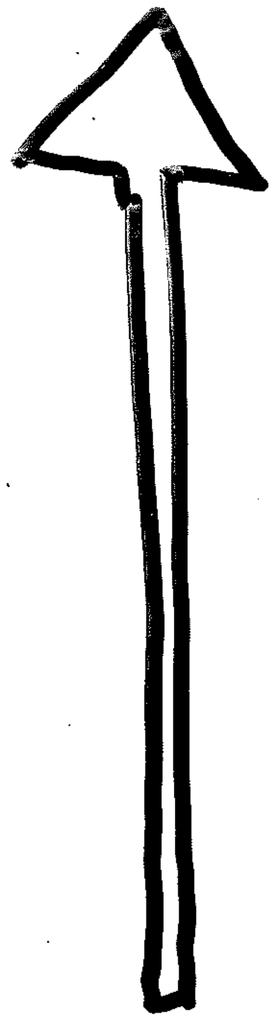
then move lens to see the blur circle

caused by the outer rays.

use stops to reduce size of blur circle



-OH



Use with overhead projector demo

small hole = object

Focus the "object"

Defocus it to show the "blur circle"

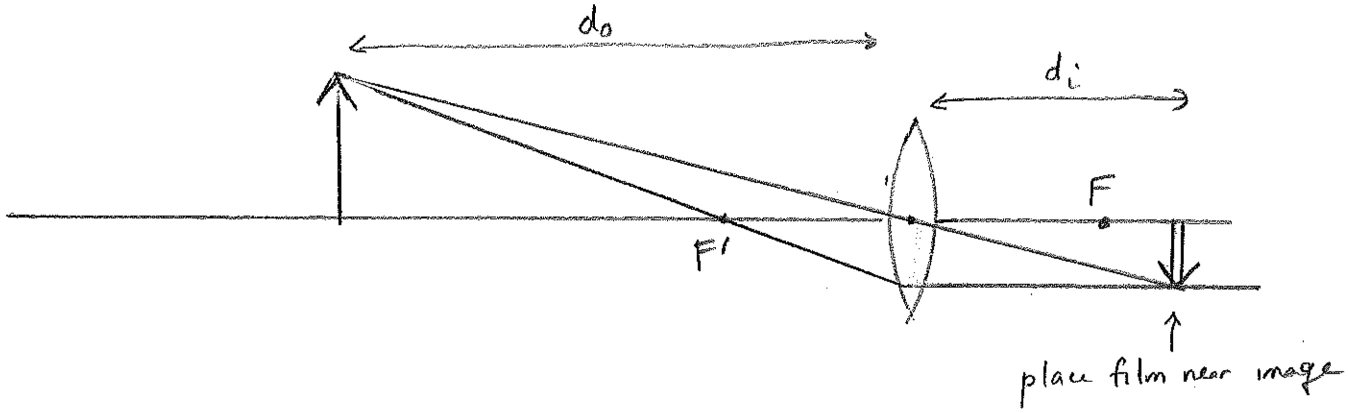
DEMO

Use with overhead projector demo

"Stop" placed over lens
to increase depth of field

Application: camera

Lens makes a small image just beyond F of a large object



"Focusing a camera": adjusting d_i (distance from lens to film) so object at d_o will form a sharp image on the film.

Objects at other distances will be "out of focus" (blur circle)

Can reduce blurriness by blocking outer rays, ie narrowing the aperture

"Depth of field"

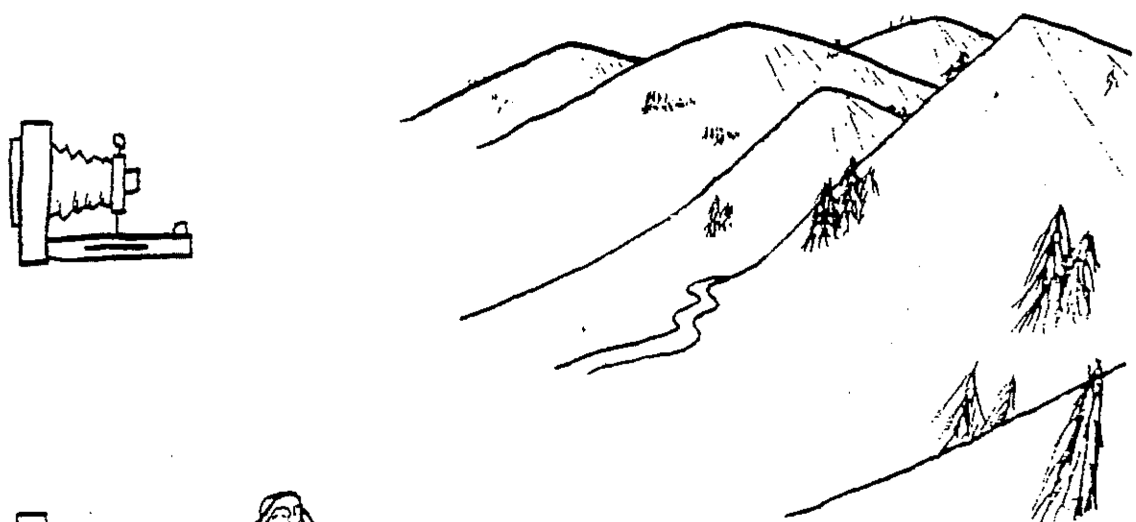
P15

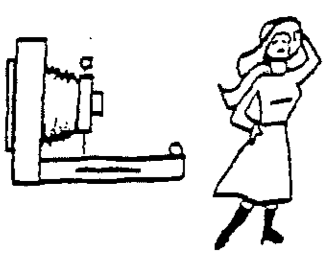
CLOSE-UP

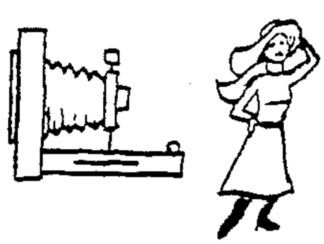
014

The first sketch shows a camera properly focused on the DISTANT MOUNTAINS. When the camera is re-focused on a very NEAR subject it will be set as shown in

a) the second sketch
b) the third sketch
c) the fourth sketch

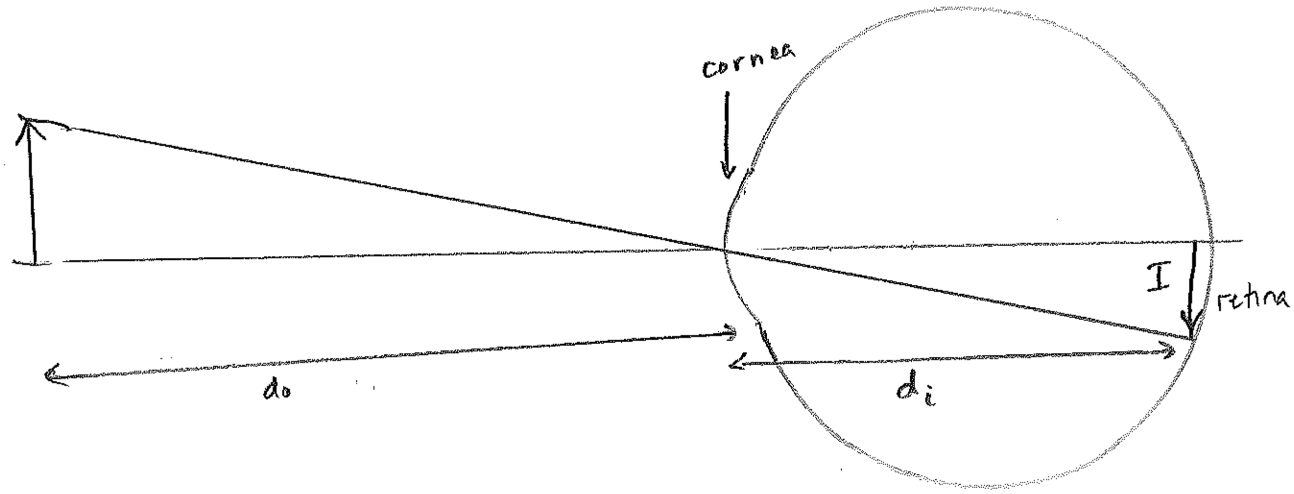


a) 

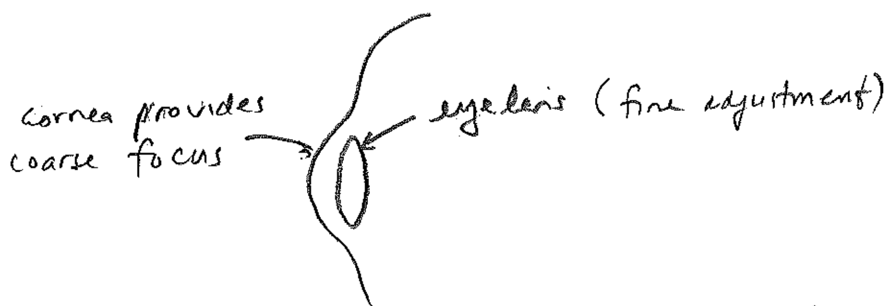
b) 

c) 

Applications: eye



d_i is fixed by size of eye,
so object at d_o is focused by changing f



"accommodation" = change in <u>shape</u> of eyelens to change f

DEMO: Hold finger in front of one eye (other closed) Notice that if finger in focus, background is blurry + vice versa. Find near point.

"near point", $d_o \approx 25$ cm (10 inches)

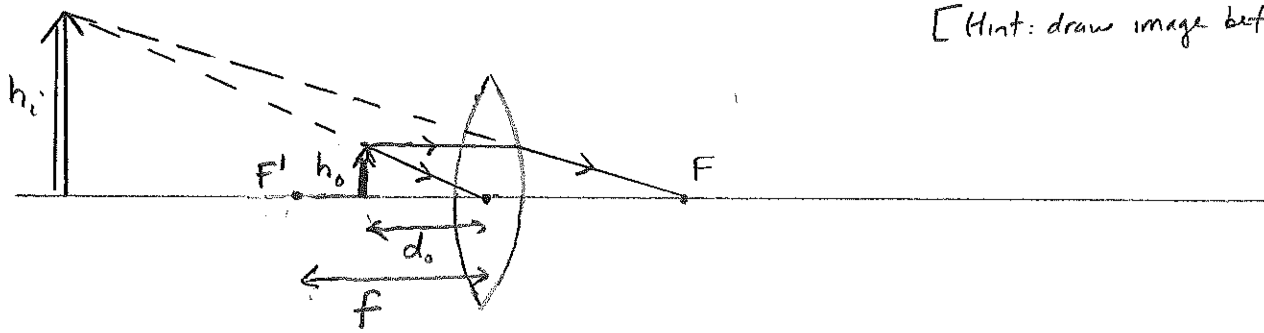
recedes w/ age (flexibility of eyelens decreases)

[have to hold newspaper further back]

Application: magnifier

Lens makes a large virtual image of a small object placed between F' and the lens.

[Hint: draw image before object]



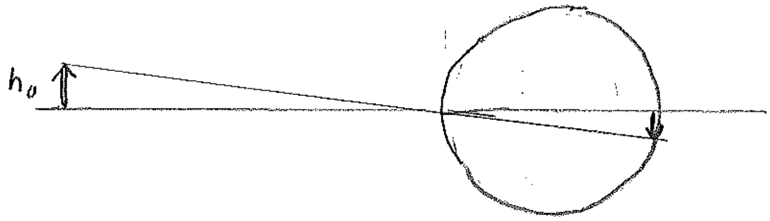
$0 < d_o < f \Rightarrow \frac{1}{d_o} > \frac{1}{f}$

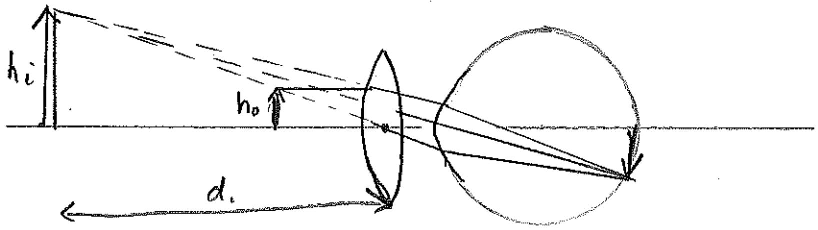
$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} < 0 \quad \Rightarrow \quad d_i < 0 \quad$ (virtual)

$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i}$

magnification

$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = -\frac{d_i}{f} + 1$





Optimally, $d_i \approx -25\text{ cm}$ (image placed at eye's near point)

$m \approx \frac{25\text{ cm}}{f} + 1$

usually $f \ll 25\text{ cm}$

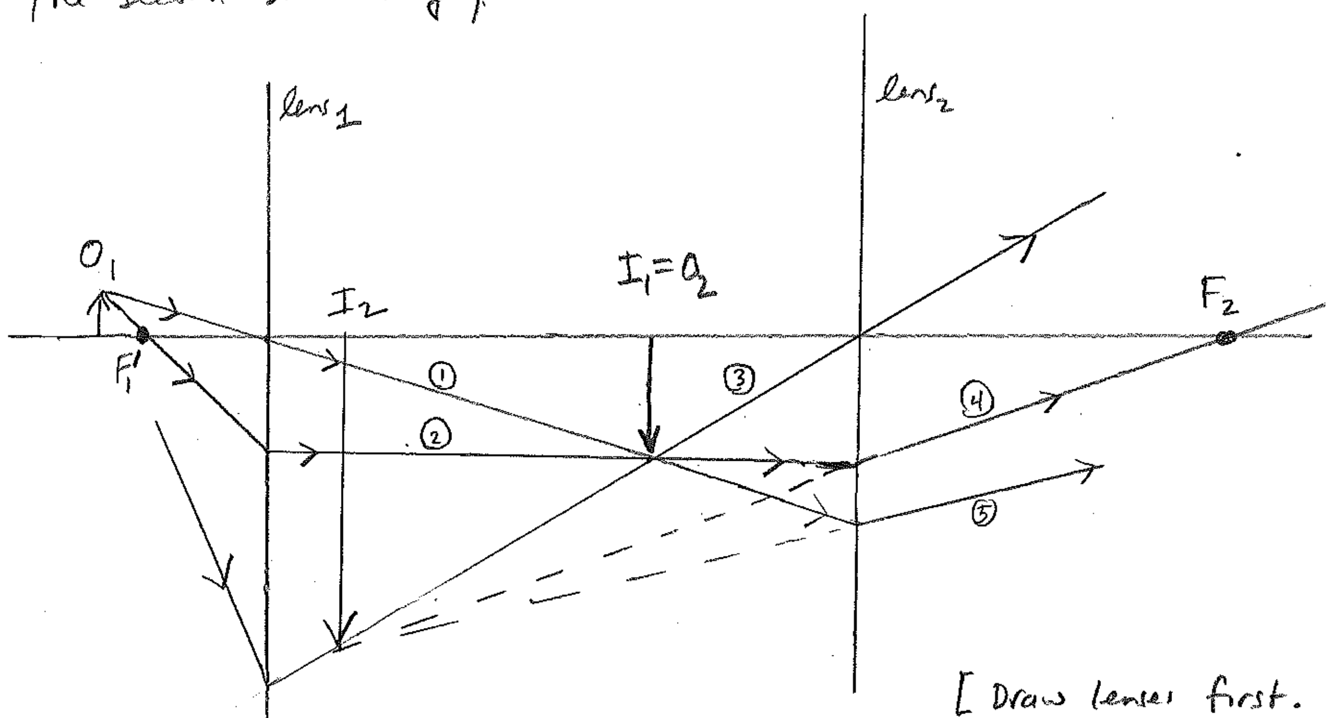
$\Rightarrow \boxed{m \approx \frac{25\text{ cm}}{f}}$

Application: compound microscope

The first lens (objective) forms a real image (I_1) of object O_1

I_1 acts as the object (O_2) of the second lens.

The second lens (eyepiece) forms a virtual image (I_2)



[Draw lenses first.
Then F_1' and I_1.
Then 1 and 2
Then O_1.
Then 3 and I_2
Then 4 and F_2
Finally 5.]

Total magnification m = product of magnification of each lens $= m_1 m_2$

Telescope is similar.