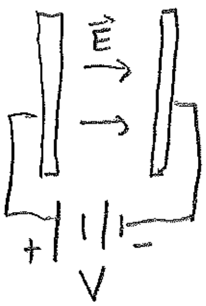
[HRK, 5e, ch 29-3, -4]

Electrical conductors

[Let's put these abstract concepts to use]

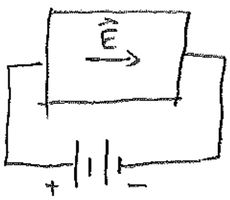


In vacuum, an electric field $\vec{E}$ acting on an electron results in acceleration

$$\vec{a} \propto -e\vec{E}$$

[see "Electron gun" problem]

because $\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$



In a conductor, an electric field $\vec{E}$ acting on conduction electrons results in constant drift velocity

$$\vec{v}_d \propto -e\vec{E}$$

[Discuss conductor: solid material w/ fixed lattice of positive nuclei, tightly bound core electrons + "sea" of conduction electrons]

e.g. copper: 1 conduction electron per atom

n_c = conduction electron number density $= 8.5 \times 10^{28} \frac{\text{electrons}}{m^3}$

[Conduction electrons move in response to electric field maintaining local neutrality, w/ others moving in to replace them]

[Collisions w/ impurities (similar to drag force) cause accelerating electrons to reach a terminal, or drift, velocity.
(Random velocities are much higher).]

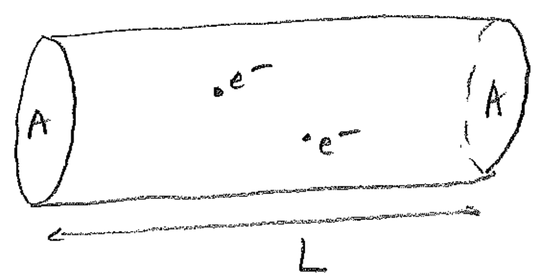
Key takeaways:

1) $\vec{J} = \frac{1}{\rho}\vec{E}$ where $\vec{J}$ = current density
ρ = resistivity of material

2) $V = iR$ (Ohm's law) where R = resistance

3) For a cylindrical resistor w/ length L and area A,

$$R = \frac{\rho L}{A}$$



How much charge due to conduction electrons in resistor?

$$\Delta q = (\text{charge per electron})\left(\frac{\#\text{ conduction electrons}}{\text{volume}}\right)(\text{volume})$$

$$= (-e)\, n_c\, (AL)$$

If conduction electrons move w/ constant speed v_d how long Δt until all pass through a plane bisecting the conductor?

$$v_d \Delta t = L \quad \Rightarrow \quad \Delta t = \frac{L}{v_d}$$

What is the current through the plane?

$$i = \frac{\Delta q}{\Delta t} = \frac{-n_c e A L}{\left(\frac{L}{v_d}\right)} = -n_c e A v_d$$

Define <u>current density</u> $j = \frac{\text{current}}{\text{cross-sectional area}} = \frac{i}{A} = -nev_d$

Current density is a vector pointing in direction of current flow

$$\vec{j} = -ne\vec{v}_d$$

[Why is $\vec{j}$ opposite $\vec{v}_d$? Because charge carriers are negative]

Earlier, we learned the $\vec{v}_d \propto -e\vec{E}$

therefore $\vec{j} \propto ne^2\vec{E}$ [$\vec{j}$ same direction as $\vec{E}$]

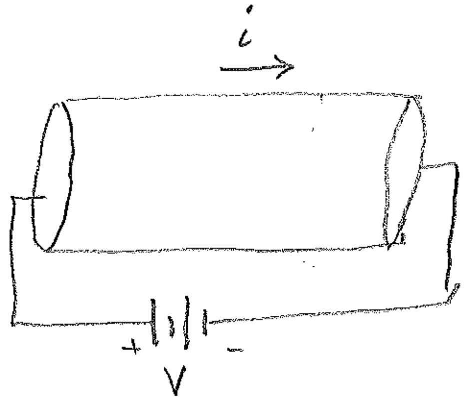
Call the constant of proportionality between $\vec{j}$ and $\vec{E}$

$\sigma \equiv$ <u>conductivity</u> of the material

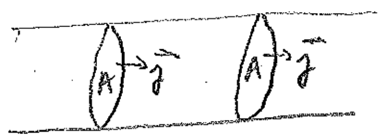
$\vec{j} = \sigma\vec{E}$

Alternatively, define <u>resistivity</u> $\rho \equiv \frac{1}{\sigma}$

$$\boxed{\vec{j} = \frac{\vec{E}}{\rho}} \Rightarrow \vec{E} = \rho\vec{j}$$



- In steady-state conductions current is same at all points in circuit (If not, charge will build up at certain points $\Rightarrow$ not steady state)



- If resistor has constant cross-section, then $\vec{j} = \text{constant}$

- Since $\vec{E} = \rho \vec{j} \Rightarrow \vec{E}$ is uniform through resistor

- Voltage difference across resistor

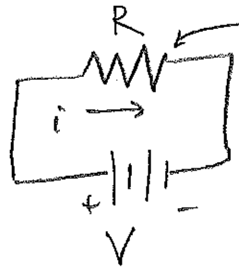
$$\Delta V = V_+ - V_- = -\int_-^+ \vec{E} \cdot d\vec{r} = EL$$

Therefore

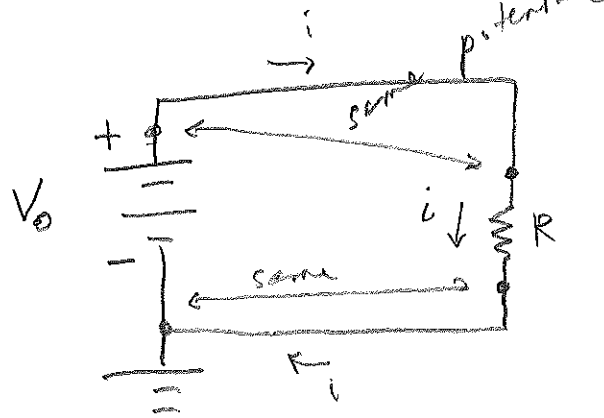
$$\Delta V = EL = \rho j L = \rho \frac{i}{A} L = \left(\frac{\rho L}{A}\right) i$$

"Voltage drop" across resistor is proportional to current (Ohm's law)

$$\boxed{V = iR}$$ where for cylindrical resistors $$\boxed{R = \frac{\rho L}{A}}$$



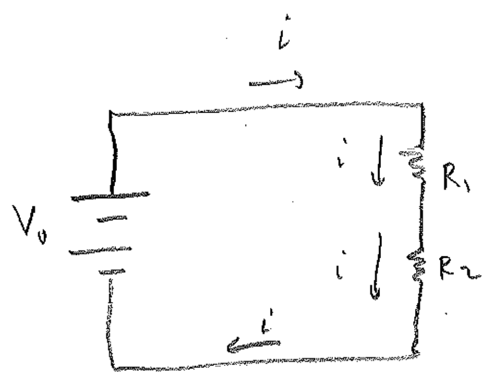
symbol for resistor



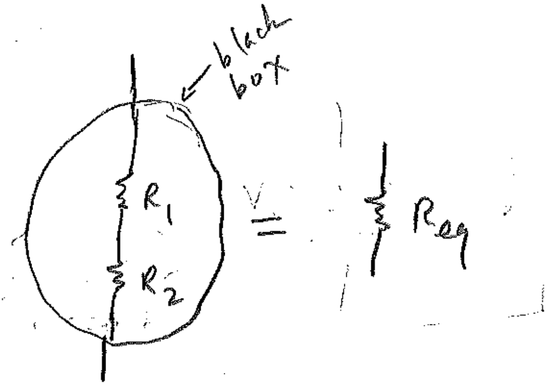
The potential difference or "voltage drop" across a resistor is $\Delta V = iR$

$$V_0 = \Delta V \Rightarrow i = \frac{V_0}{R}$$

[the wires in a circuit are conductors w/ very low resistance ($\rho \approx 0$) so voltage drop (for given i) is very small. Thus voltage at top of R is same as voltage at positive terminal of battery & the voltage at bottom is ≈ neg. terminal of battery]

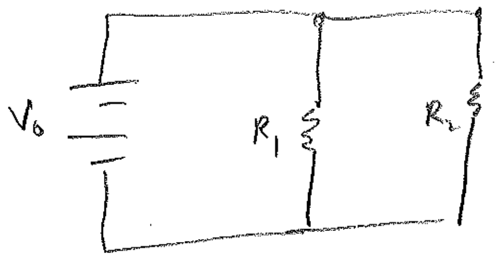


$$\Delta V_1 = iR_1$$
$$\Delta V_2 = iR_2$$
$$V_0 = \Delta V_1 + \Delta V_2 = iR_1 + iR_2 = i(R_1 + R_2)$$
$$i = \frac{V_0}{R_1 + R_2}$$



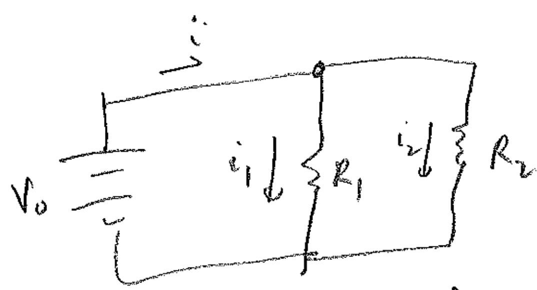
Two resistors connected <u>in series</u> are equivalent to a single resistor w/ resistance

$$R_{eq} = R_1 + R_2$$



Voltage drop across each resistor connected in parallel is the same

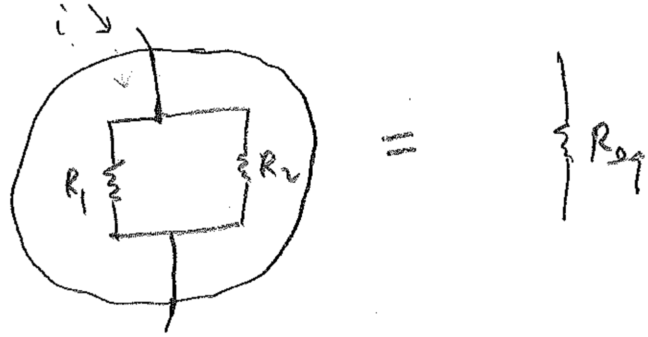
$$\Delta V_1 = \Delta V_2 = V_0$$



Kirchoff's current law $\Rightarrow$ $i = i_1 + i_2$

Ohm $\Rightarrow$ $i_1 = \dfrac{V_0}{R_1}$, $i_2 = \dfrac{V_0}{R_2}$

$$i = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_0 = \frac{1}{R_{eq}} V_0$$



Two resistors connected in parallel are equivalent to a single resistor R_{eq} where

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$