Electrostatic potential energy U

Charge q in electric field $\vec{E}$

experiences a force $\vec{F} = q\vec{E}$

causing it to accelerate, gaining speed & $\therefore$ kinetic energy K.

Increase in K $\Rightarrow$ decrease in potential energy U

because total energy is conserved [as in gravity]

If charge moves from A to B then

$U_B + K_B = U_A + K_A$ (conservation of mechanical energy)

$\Rightarrow U_B - U_A = -(K_B - K_A)$

Work-energy theorem says change in kinetic energy is given by work done $\int \vec{F} \cdot d\vec{r}$

$$\Rightarrow \quad U_B - U_A = -\int_A^B \vec{F} \cdot d\vec{r}$$

$$= -q \int_A^B \vec{E} \cdot d\vec{r}$$

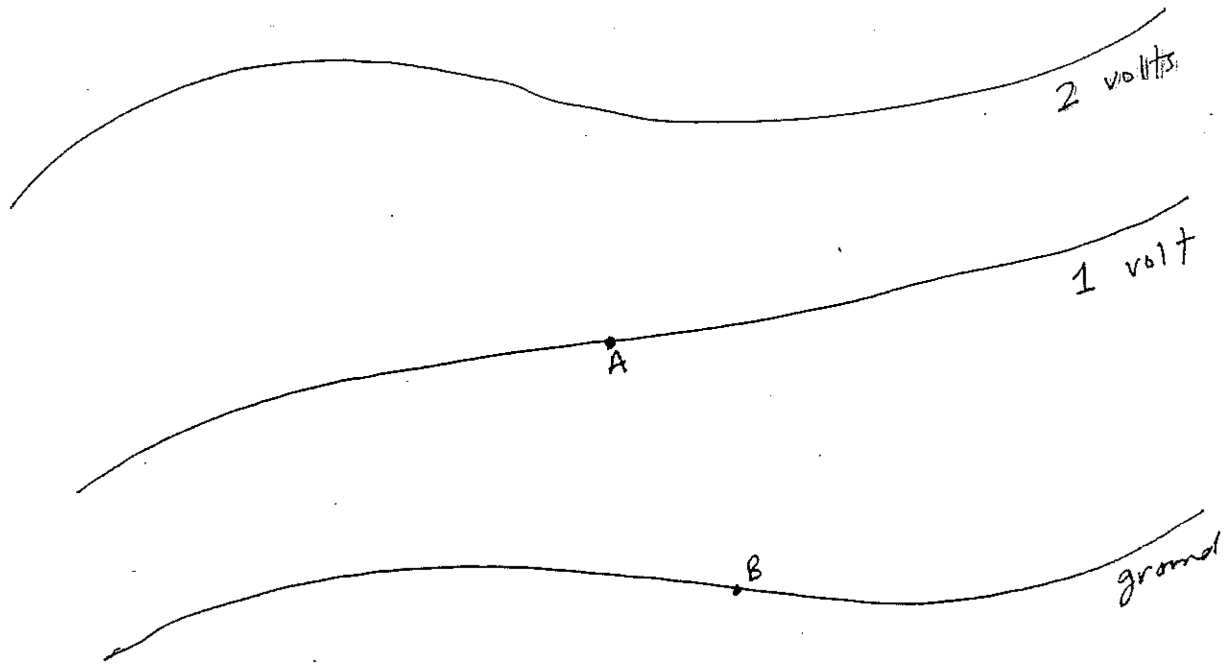
$$= q(V_B - V_A)$$

If chose $U_A = 0$ when $V_A = 0$, then $U_B = qV_B$

$\boxed{U = qV}$ electrostatic potential energy proportional to voltage

V in volts, q in Coulombs $\Rightarrow$ U in Joules

$1\,J = 1\,C \cdot V$



DOES AN ELECTRON HAVE MORE ELECTROSTATIC POTENTIAL ENERGY AT "A" OR "B"?

WHAT IS THE DIFFERENCE IN POTENTIAL ENERGY (IN JOULES) BETWEEN THESE TWO POINTS?

[$\Delta U = 1.6 \times 10^{-19}$ J

Use this to motivate/explain eV.

$$1 \text{ eV} = (e)(1\text{ V})$$
$$= (1.6 \times 10^{-19}\text{ C})(1\text{ V})$$
$$= 1.6 \times 10^{-19}\text{ J}$$]

Energy approach

A charge with initial speed v_i travels from $\vec{r}_i$ to $\vec{r}_f$

Use mechanical energy conservation to find final speed v_f.

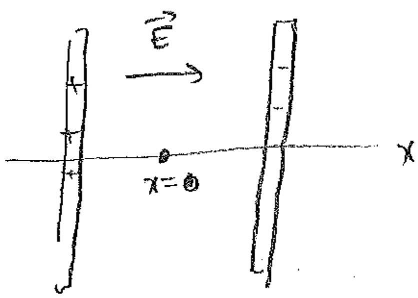
$$K_f + U_f = K_i + U_i$$

$$K_f = K_i + U_i - U_f$$

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + U(\vec{r}_i) - U(\vec{r}_f)$$

$$v_f = \sqrt{v_i^2 + \frac{2}{m}\left(U(\vec{r}_i) - U(\vec{r}_f)\right)}$$

Recall the potential for a uniform $\vec{E}$ field (magnitude E_0)



$$V(x) = \underbrace{V(0)}_{\text{set}=0} - \int_0^x \underbrace{E_x}_{E_0}\,dx = -E_0 x$$

A charge q has pot. energy $U(x) = qV(x) = -qE_0 x$

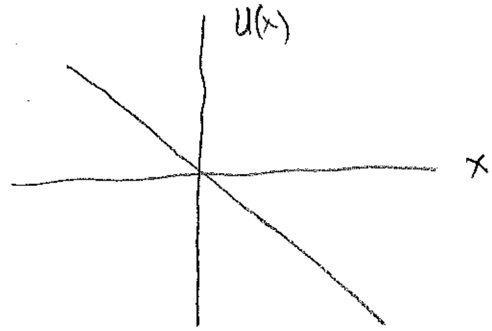
omit this as it is done in the electron gun problem

$$U_i - U_f = -qE_0(x_i - x_f)$$

$$= qE_0(x_f - x_i)$$

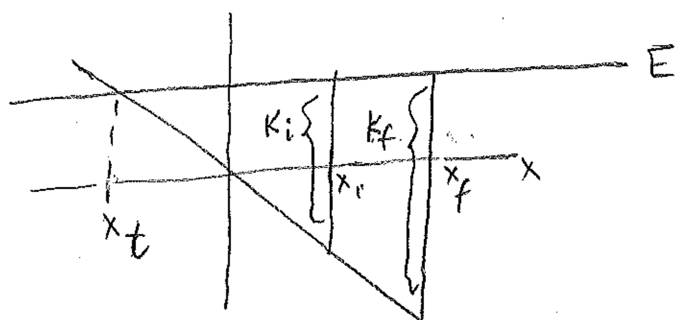
$$\Rightarrow v_f = \sqrt{v_i^2 + \frac{2qE_0}{m}(x_f - x_i)}$$

Potential energy diagrams



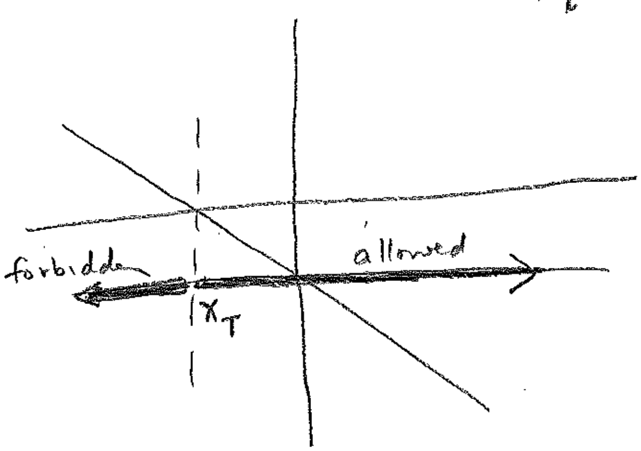
Initial speed v_i $\Rightarrow$ $K_i = \frac{1}{2} m v_i^2$

$\Rightarrow$ total mechanical energy $E = K_i + U_i = \frac{1}{2} m v_i^2 - q E_0 x_i$



If charge initially moving to the left, will travel to the turning point x_t, where $K_t = 0$

$$K_t + U_t = E$$
$$0 - qE_0 x_t = \frac{1}{2} m v_i^2 - qE_0 X_i$$
$$x_t = x_i - \frac{m v_i^2}{2qE_0}$$



$X > X_T$ is a classically allowed region because $U(x) \leq E$

$X < X_T$ is a classically forbidden region because $U(x) > E$