Electrostatic potential, or voltage V

Associated with the vector field $\vec{E}$
produced by a collection of static charges
is a scalar field V called the electrostatic potential (or voltage)

Key takeaways:

1) Only the difference of V between two points
is physically meaningful, so we talk
about "potential difference" or "voltage drop"

2) The difference $V_B - V_A$ between two points A and B
is defined by the line integral of $\vec{E}$

$$\boxed{V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{r}}$$

[minus sign important]

3) Electrostatic field $\vec{E}$ is conservative
which means line integral does not depend on path from A to B
$\Rightarrow$ we can choose a convenient path

4) We can choose $V = 0$ at an arbitrary reference point,
called "ground"

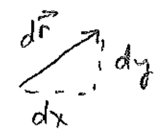
How to do a line integral in 3 easy steps

a) choose a path from A to B

b) do the dot product $\vec{E} \cdot d\vec{r}$

$\vec{E} = (E_x, E_y, E_z)$

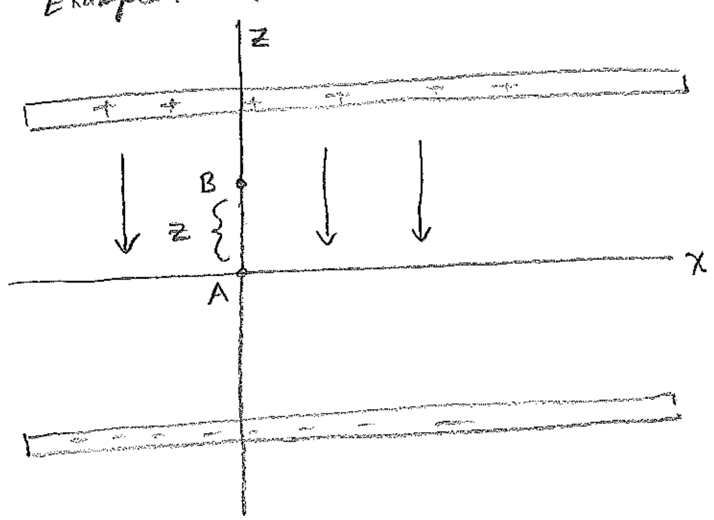
$d\vec{r} = (dx, dy, dz)$



$\vec{E} \cdot d\vec{r} = E_x\, dx + E_y\, dy + E_z\, dz$

c) do the integral

Example: uniform electric field (between parallel plates)



$\vec{E} = (0, 0, -E)$

E = magnitude of field $|\vec{E}|$

a) path: along z axis from A to B

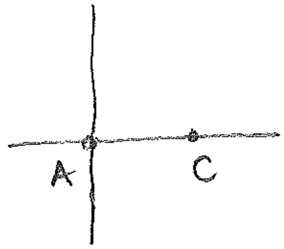
$\vec{r} = (0, 0, z)$

$d\vec{r} = (0, 0, dz)$

b) dot product: $\vec{E} \cdot d\vec{r} = E_z\, dz = -E\, dz$

c) integral: $V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{r} = -\int_0^z (-E\, dz) = Ez\Big|_0^z = Ez$

If A is ground ($V_A = 0$), then $V_B = Ez$



a) path along x-axis from A to C

$\vec{r} = (x, 0, 0)$

$d\vec{r} = (dx, 0, 0)$

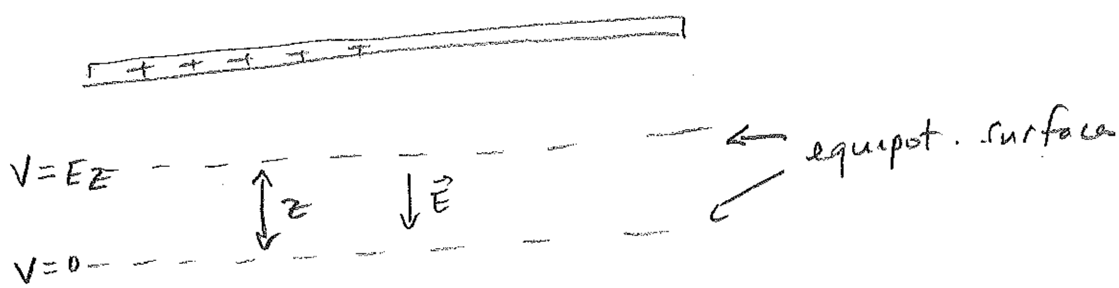
b) dot product $\vec{E} \cdot d\vec{r} = E_x \, dx = 0$

c) $V_C - V_A = -\int_A^C \vec{E} \cdot d\vec{r} = 0$

$\Rightarrow V_C = 0,$

A and C are equipotential points

All points w/ same voltage form an equipotential surface



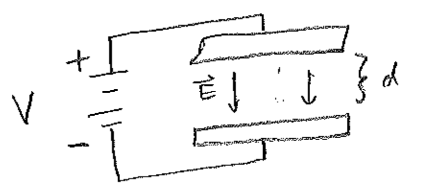
Two general points:

1) $\vec{E}$ is always perpendicular to equipotential surface

2) $\vec{E}$ points from higher to lower voltage

<u>Capacitor</u> : device for storing charge

Attach a V-volt battery between parallel plates of area A, separated by distance d

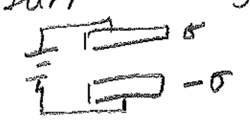


Let $\vec{E}$ be uniform field between plates (high to low voltage)

Potential difference between plates is $Ed = V$

$$E = \frac{V}{d}$$

Field is caused by surface charge density σ on the plates



$$E = \frac{\sigma}{\epsilon_0}$$

Then

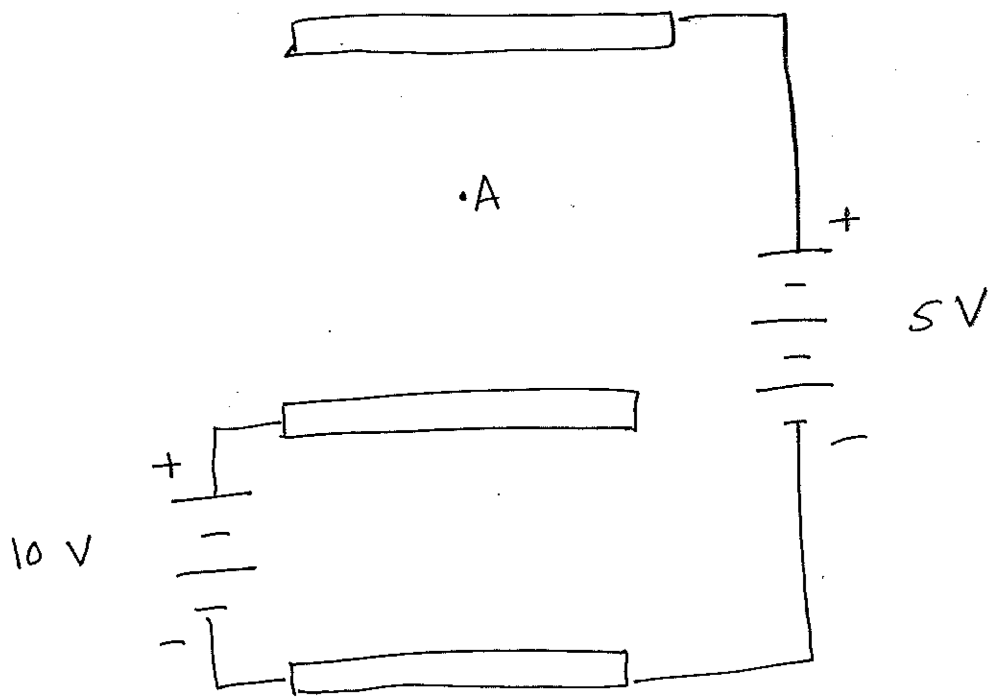
$$\sigma = \epsilon_0 E = \frac{\epsilon_0 V}{d}$$

Total charge on one of the plates is $q = \sigma A$

Thus $q = \underbrace{\left(\frac{\epsilon_0 A}{d}\right)}_{C} V$

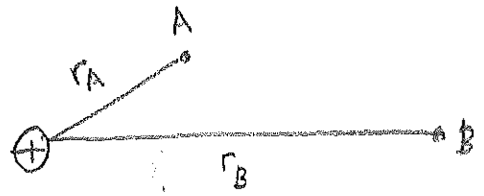
C = capacitance of parallel plates

In general $\boxed{q = CV}$ charge is proportional to voltage



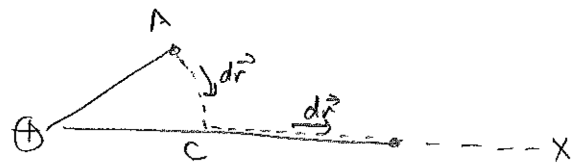
Which way does $\vec{E}$ point at A?

Potential due to a point charge q at origin



$$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{r} \qquad \text{where} \quad |\vec{E}| = \frac{Kq}{r^2}$$

a) choose path : $A \rightarrow C \rightarrow B$



b) $$V_B - V_A = -\int_A^C \underbrace{\vec{E}\cdot d\vec{r}}_{0 \text{ because } \vec{E}\perp d\vec{r}} - \int_C^B \underbrace{\vec{E}\cdot d\vec{r}}_{E_x\,dx} = -\int_{r_A}^{r_B} \frac{Kq}{x^2}\,dx$$

c) $$= \left.\frac{Kq}{x}\right|_{r_A}^{r_B}$$

$$= \frac{Kq}{r_B} - \frac{Kq}{r_A}$$

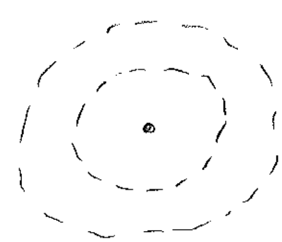
choose $V_A = 0$ at $r_A = 0$? No!

choose $V_A = 0$ at $r_A = \infty \Rightarrow V_B = \frac{Kq}{r_B}$

only depends on distance from charge, not direction.

The potential at $\vec{r}$, due to a point charge q_1 at $\vec{r}_1$ is

$$\boxed{V = \frac{Kq_1}{|\vec{r} - \vec{r}_1|}}$$



Equipotential surfaces are spheres