Electric field produced by a continuous charge distribution

A) infinite line of uniformly distributed charge

Key takeaways:

1) Field points radially away from (or toward) the line of charge [see figure →]

2) field strength inversely proportional to distance ρ from line ["rho"]

3) proportional to linear charge density $\lambda \equiv \frac{\text{charge}}{\text{length}}$ ["lambda"]

Thus, $$\boxed{|\vec{E}| = \frac{2K\lambda}{\rho} = \frac{\lambda}{2\pi\epsilon_0 \rho}}$$

[Unlike $\vec{E}$ due to point charge,
we will not take that as a given
but will derive it from Coulomb's law + superposition.
Useful if we want to consider other charge distributions.]

C2

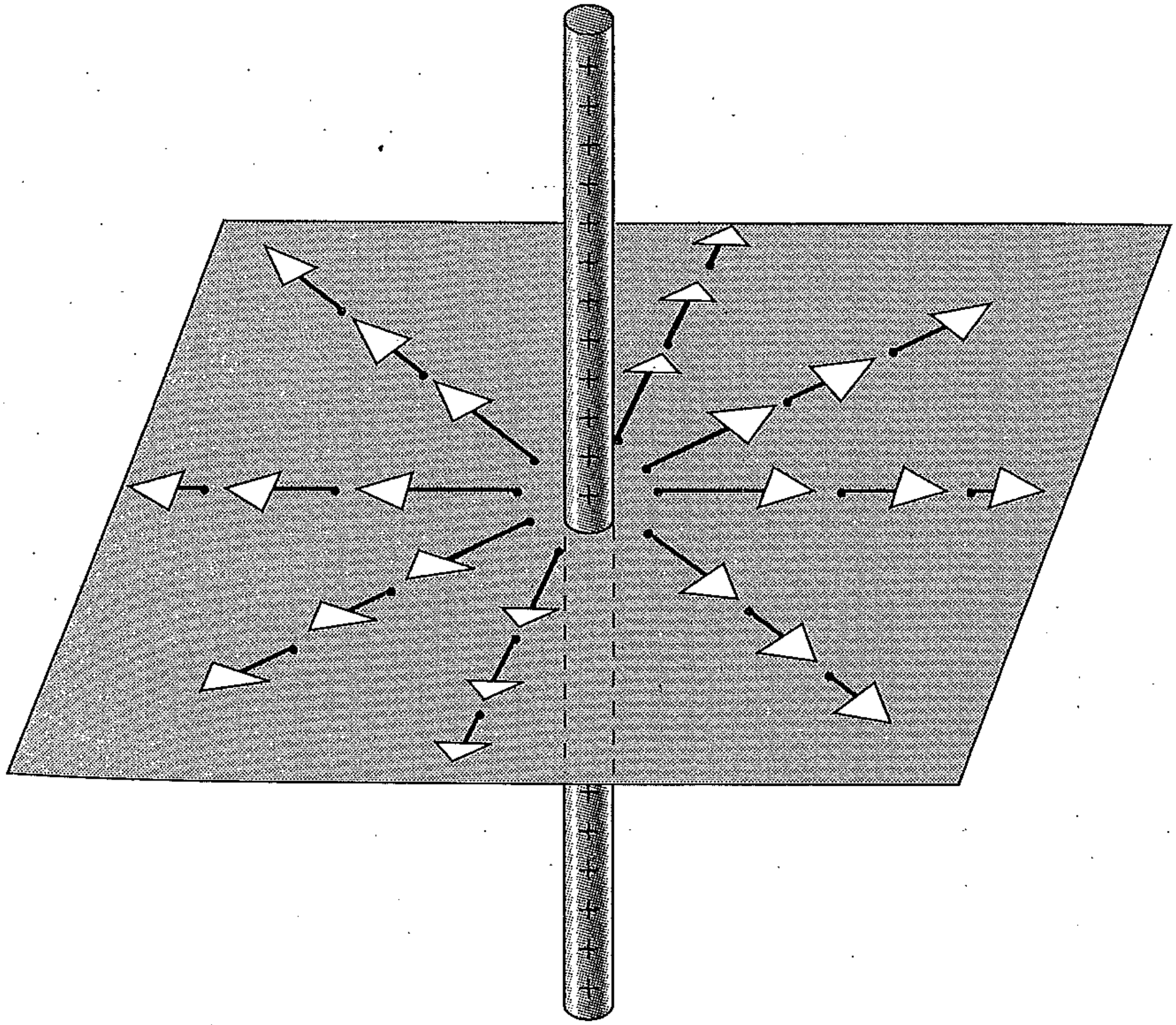
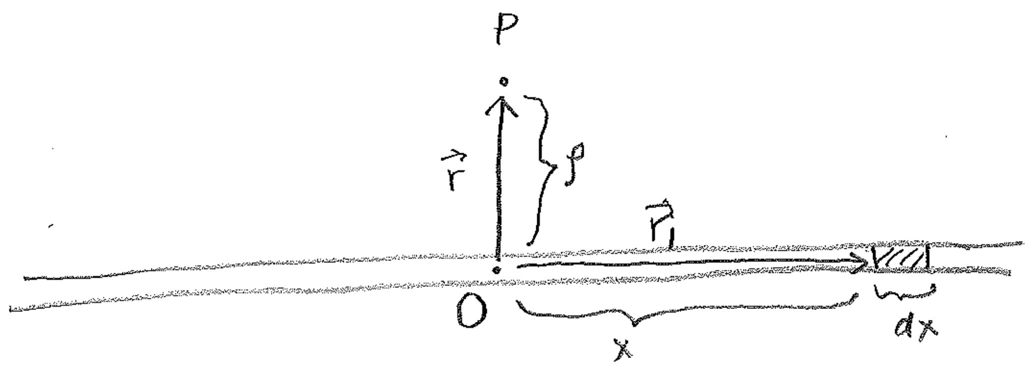


FIGURE 26-7. Electric field due to a positively charged rod. The field has cylindrical symmetry about the axis of the rod.

[orient line along x-axis]



Let $d\vec{E}(\vec{r})$ = field at P produced by infinitesimal charge dq at $\vec{r}_1$

$$= \frac{K\, dq\, (\vec{r}-\vec{r}_1)}{|\vec{r}-\vec{r}_1|^3}$$

$$dq = \lambda\, dx$$
$$\vec{r} = (0, \rho, 0)$$
$$\vec{r}_1 = (x, 0, 0)$$
$$\vec{r}-\vec{r}_1 = (-x, \rho, 0)$$
$$|\vec{r}-\vec{r}_1| = \sqrt{\rho^2 + x^2}$$

$\vec{E}(\vec{r})$ = field at P produced by entire line of charge

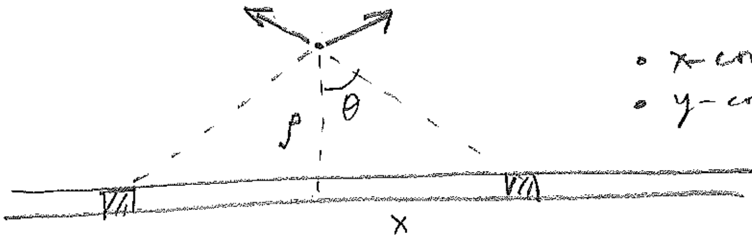
= vector sum of fields produced by each element

$$= \int d\vec{E} = \int_{x=-\infty}^{x=\infty} \frac{K\lambda\, dx \cdot (-x, \rho, 0)}{(\rho^2 + x^2)^{3/2}}$$

$$E_x = -K\lambda \int_{-\infty}^{\infty} \frac{x\, dx}{(\rho^2+x^2)^{3/2}}$$

$$E_y = K\lambda\rho \int_{-\infty}^{\infty} \frac{dx}{(\rho^2+x^2)^{3/2}}$$

Can do integral to show $E_x = 0$, but unnecessary



- x-components cancel
- y-components add

To calculate E_y, try trig substitution

$$x = \rho \tan\theta \qquad \text{Then } x = \infty \Rightarrow \theta = \frac{\pi}{2}$$
$$x = -\infty \Rightarrow \theta = -\frac{\pi}{2}$$

$$dx = \rho \sec^2\theta \, d\theta$$

$$\rho^2 + x^2 = \rho^2(1+\tan^2\theta) = \rho^2 \sec^2\theta$$

$$E_y = K\lambda\rho \int_{\theta=-\frac{\pi}{2}}^{\theta=\frac{\pi}{2}} \frac{\rho \sec^2\theta \, d\theta}{(\rho\sec\theta)^3}$$

$$= \frac{K\lambda}{\rho} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta \, d\theta$$

$$= \frac{K\lambda}{\rho} \sin\theta \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{K\lambda}{\rho}[1-(-1)]$$

$$= \frac{2K\lambda}{\rho}$$

B) infinite sheet of uniformly distributed charge

Key takeaways:

1) Field points away from (or toward) the sheet of charge [see figure]

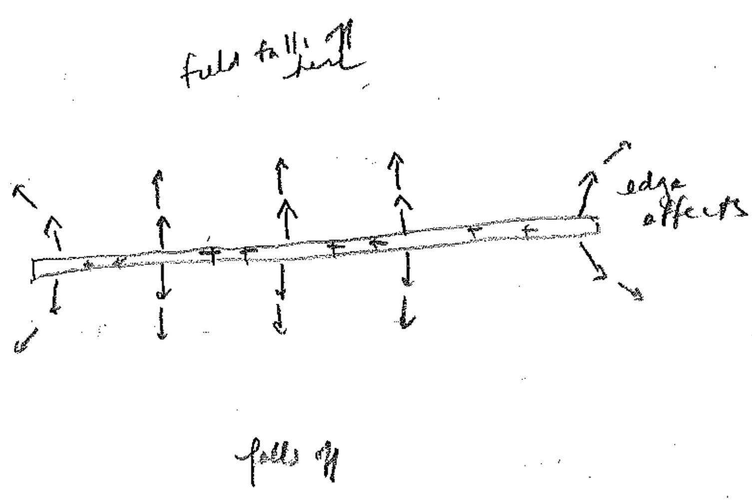
2) Field strength independent of distance from sheet

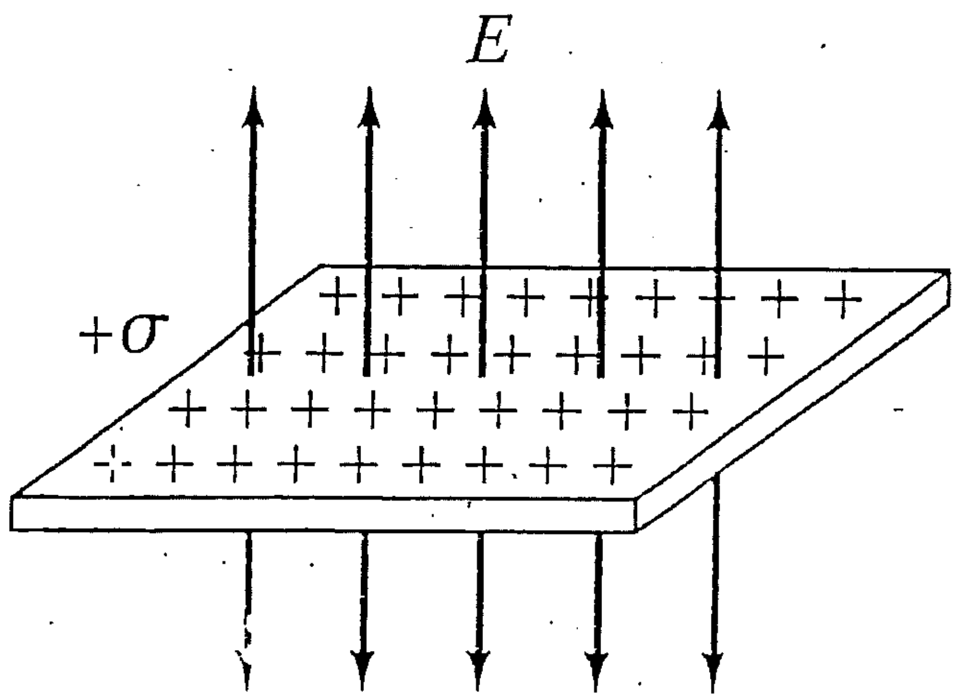
3) proportional to surface charge density $\sigma = \frac{\text{charge}}{\text{area}}$ ["sigma"]

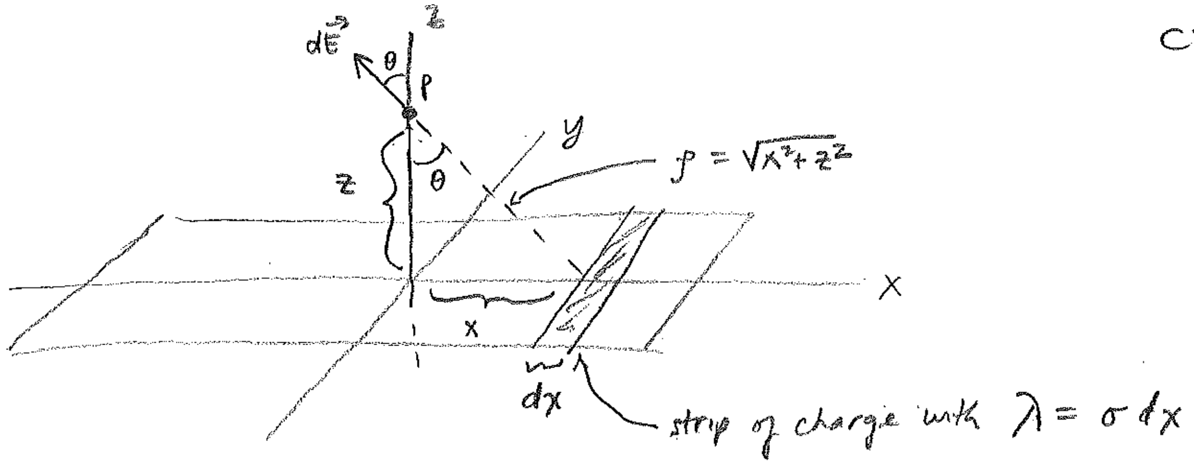
$$\boxed{|\vec{E}| = \frac{\sigma}{2\epsilon_0}}$$

We'll prove this using result for line of charge + superposition.

For finite sheets, result is valid close to sheet and away from edges




E
+σ



Let $d\vec{E}$ = field produced at P by infinitesimal strip of charge

$$|d\vec{E}| = \frac{2K\lambda}{\rho} = \frac{2K\sigma\, dx}{\sqrt{z^2+x^2}}$$

As before, the x-components cancel out

$$dE_z = |d\vec{E}|\cos\theta \qquad \text{where} \qquad \cos\theta = \frac{z}{\sqrt{z^2+x^2}}$$

E_z = field produced at P by entire sheet of charge

$$= \int dE_z = \int_{x=-\infty}^{x=\infty} \frac{2K\sigma z\, dx}{z^2+x^2}$$

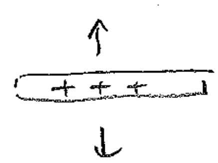
As before $x = z\tan\theta$

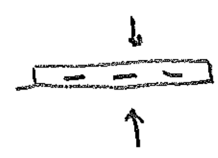
$$dx = z\sec^2\theta\, d\theta$$

$$z^2+x^2 = z^2(1+\tan^2\theta) = z^2\sec^2\theta$$

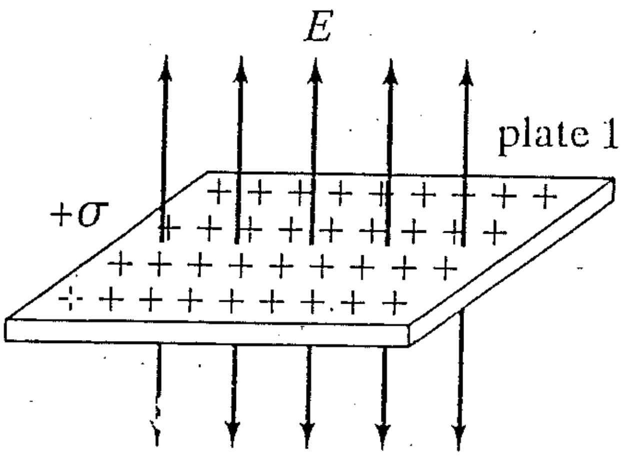
$$E_z = \int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2K\sigma z(z\sec^2\theta)\,d\theta}{z^2\sec^2\theta} = 2K\sigma\,\theta\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2\pi K\sigma = \frac{\sigma}{2\epsilon_0}$$

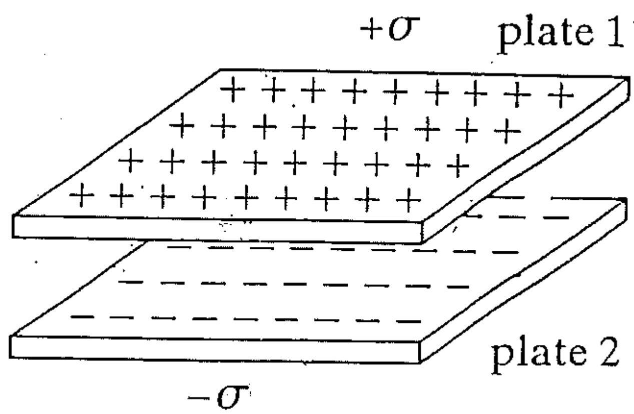
$$E_z = \begin{cases} \frac{\sigma}{2\epsilon_0} & \text{above sheet} \\ -\frac{\sigma}{2\epsilon_0} & \text{below sheet} \end{cases}$$





The charge per unit area is $+\sigma$ on plate 1 and $-\sigma$ on plate 2.
The magnitude of the electric field associated with plate 1 is $\sigma/2\epsilon_0$.





When the two plates are placed parallel to one another, the magnitude of the electric field is

(a) σ/ϵ_0 between, zero outside

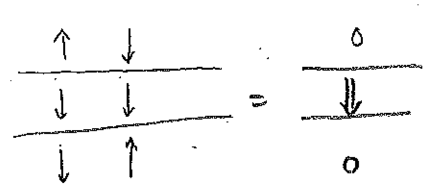
(b) σ/ϵ_0 between, $\pm\sigma/2\epsilon_0$ outside

(c) zero both between and outside

(d) $\pm\sigma/2\epsilon_0$ both between and outside

(e) none of the above

[After answers are in:



do this explicitly]