Electric field $\vec{E}(\vec{r})$

Key takeaways:

1) vector field produced by an electric charge in the space around it

2) points radially away from (or toward) the charge

3) obeys inverse square law

} Coulomb's law

4) superposition principle (field produced by several charges is the vector sum of the fields produced by each)

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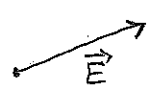
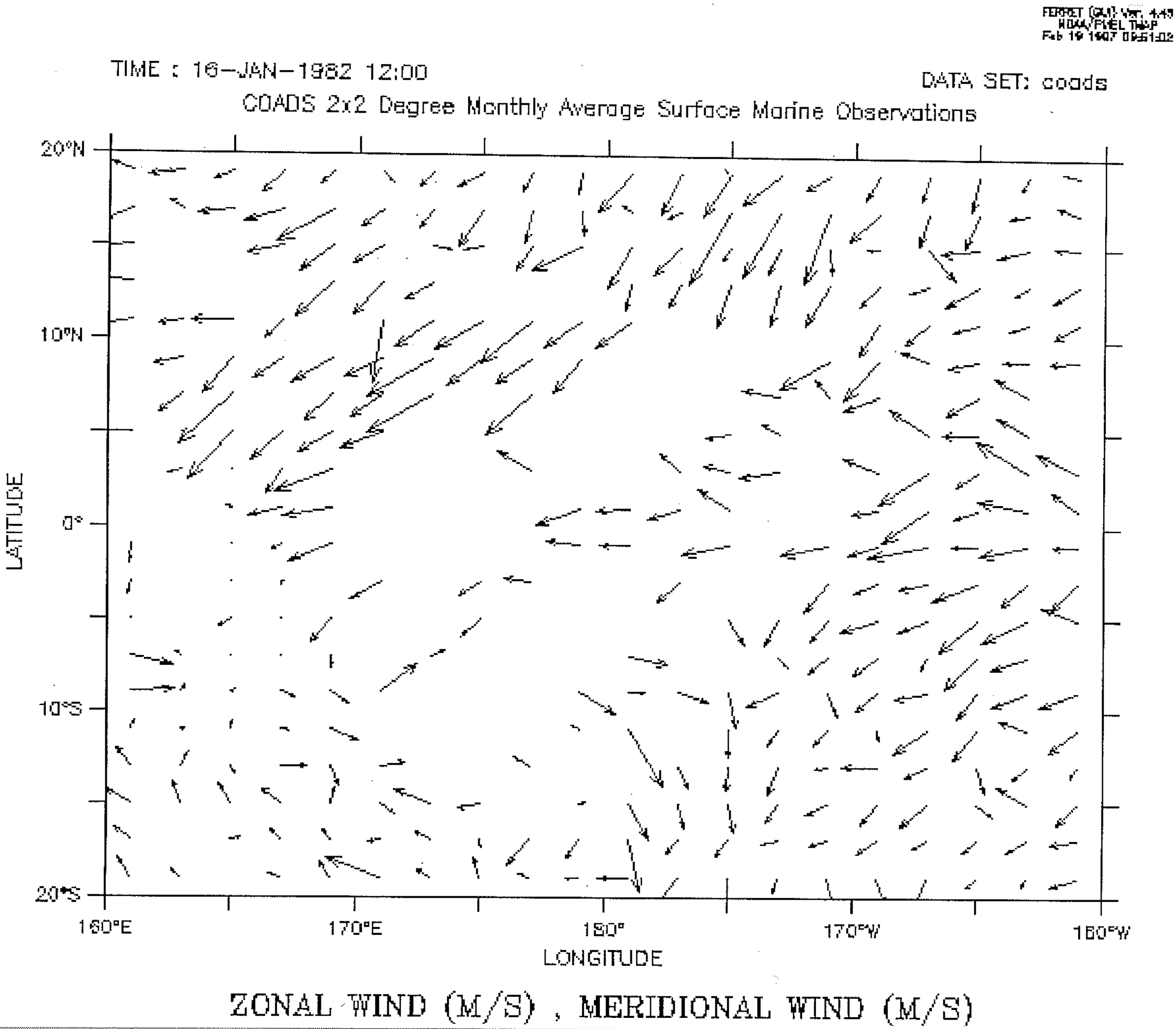
field: function of position [has a value at each point in space]
[e.g. temperature in room]

vector field: has a magnitude and a direction at each point



$|\vec{E}|$ = magnitude of $\vec{E}$ = "length" of vector (positive or zero)
= "strength" of field

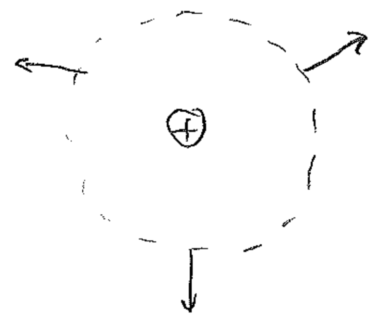
How to visualize? [wind velocity map]


FERRET (GUI) Ver. 4.45
NOAA/PMEL TMAP
Feb 19 1997 09:51:02
TIME : 16-JAN-1982 12:00
DATA SET: coads
COADS 2x2 Degree Monthly Average Surface Marine Observations
20°N
10°N
0°
10°S
20°S
LATITUDE
160°E
170°E
180°
170°W
160°W
LONGITUDE
ZONAL WIND (M/S) , MERIDIONAL WIND (M/S)

OH

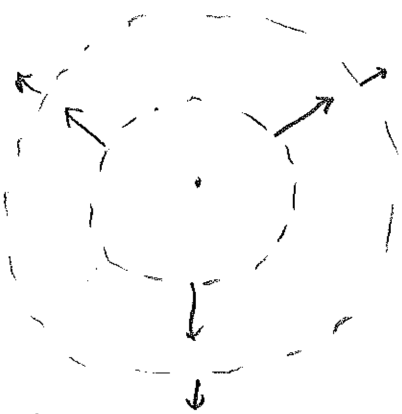
R2

Positive charge $\Rightarrow$ field radially outward

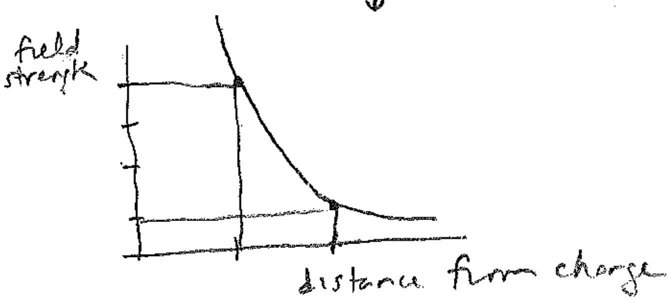


[actually in 3 dimensions]

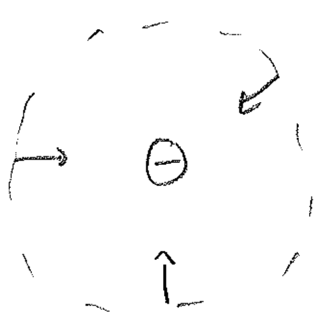
Inverse square law: $|\vec{E}| \sim \frac{1}{r^2}$



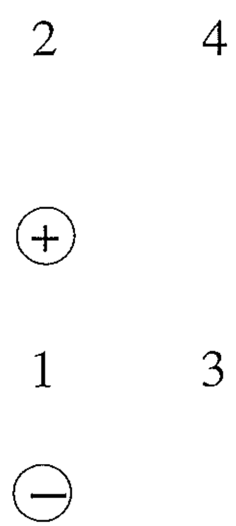
Twice as far, one quarter as strong



Negative charge $\Rightarrow$ field radially inward



What is the direction of the electric field produced by the two charges (⊕ and ⊖) at each of the indicated locations (1, 2, 3, 4)? The possible answers are shown at right.



A →

B ←

C ↑

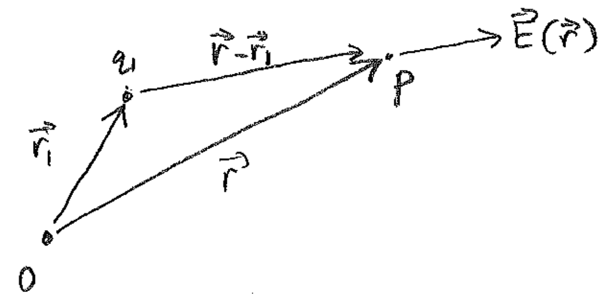
D ↓

E no direction

[review head-to-tail rule]

How represent electric field mathematically?

Let $\vec{r}$ = position of point P relative to origin O



Let $\vec{r}_1$ = position of charge q_1

$\vec{r}-\vec{r}_1$ = position of P relative to q_1 [head-to-tail]

$\vec{E}(\vec{r})$ is parallel to $\vec{r}-\vec{r}_1$ [place tail of $\vec{E}$ at P]

<u>Coulomb's law:</u> electric field produced by q_1 is

$$\boxed{\vec{E}(\vec{r}) = \frac{K q_1}{|\vec{r}-\vec{r}_1|^3}(\vec{r}-\vec{r}_1)}$$

K = Coulomb's constant $= 8.99 \times 10^9 \ \frac{N \cdot m^2}{C^2}$

Also $K = \frac{1}{4\pi\epsilon_0}$ where ϵ_0 = permittivity of vacuum

Coulomb's law satisfies inverse square law

$$|\vec{E}| = \frac{K|q_1|}{|\vec{r}-\vec{r}_1|^3}\,|\vec{r}-\vec{r}_1| = \frac{K|q_1|}{|\vec{r}-\vec{r}_1|^2}$$

where $|\vec{r}-\vec{r}_1|$ = distance from q_1 to P

<u>Superposition principle</u>: electric field produced at P by a collection of charges q_i is the vector sum of fields produced by each

$$\boxed{\vec{E}(\vec{r}) = \sum_{i=1}^{N} \frac{K q_i (\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}}$$

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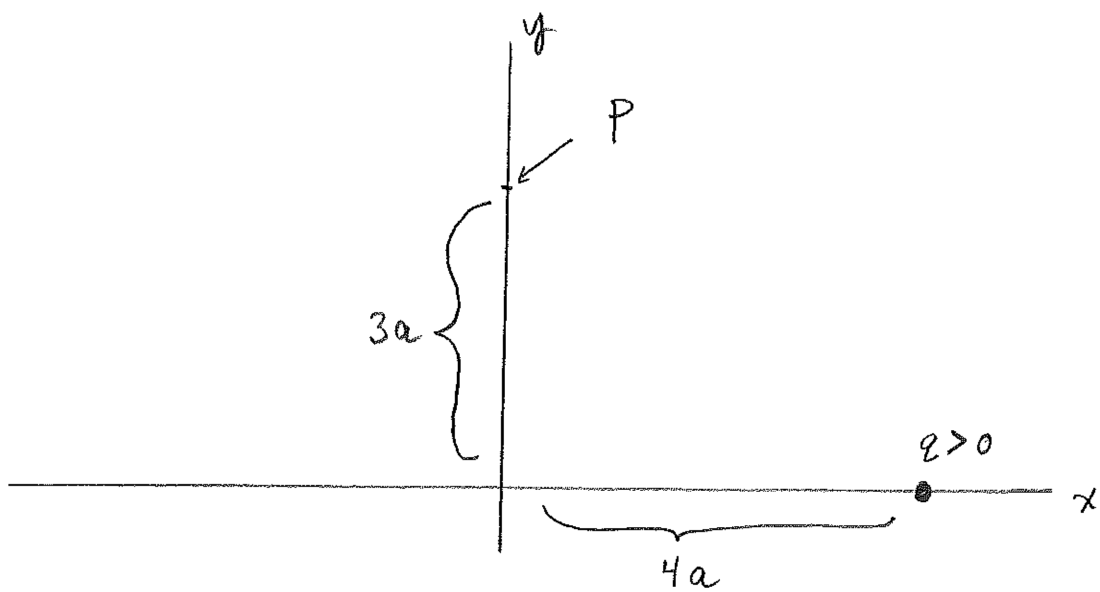


What is the magnitude $|\vec{E}|$ of the electric field at point P?

(A) $\dfrac{Kq}{a^2}$ (B) $\dfrac{Kq}{5a^2}$ (C) $\dfrac{Kq}{25a^2}$ (D) $\dfrac{Kq}{125a^2}$ (E) None of these

What is the y-component E_y of the electric field at point P?

(A) $\dfrac{4Kq}{5a^2}$ (B) $\dfrac{3Kq}{25a^2}$ (C) $\dfrac{4Kq}{125a^2}$ (D) $\dfrac{3Kq}{125a^2}$ (E) None of these

What is the x-component E_x of the electric field at point P?

(A) $\dfrac{4Kq}{5a^2}$ (B) $\dfrac{3Kq}{25a^2}$ (C) $\dfrac{4Kq}{125a^2}$ (D) $\dfrac{3Kq}{125a^2}$ (E) None of these

[review vector components]

$\vec{r} = (0, 3a, 0)$

$\vec{r}_1 = (4a, 0, 0)$

$\vec{r}-\vec{r}_1 = (-4a, 3a, 0)$

$|\vec{r}-\vec{r}_1| = 5a$

ans: C, D, E

Electrostatic force $\vec{F}$ exerted on charge q by field $\vec{E}$

Key takeaway:

1) force proportional to electric field strength

2) force proportional to charge

3) force parallel to field if charge positive (antiparallel if negative)

$$\vec{F} = q\vec{E}$$

where $\vec{E}$ = field at the location of the charge
that is produced by all <u>other</u> charges

Consider two charges q_1 and q_2

Let $\vec{F}_{12}$ = force exerted on q_1 by (field produced by) q_2

$$= q_1 \vec{E}_2(\vec{r}_1)$$

Field produced by q_2 at $\vec{r}$ is

$$\vec{E}_2(\vec{r}) = \frac{K q_2}{|\vec{r}-\vec{r}_2|^3}(\vec{r}-\vec{r}_2)$$

Thus

$$\vec{F}_{12} = \frac{K q_1 q_2}{|\vec{r}_1-\vec{r}_2|^3}(\vec{r}_1-\vec{r}_2)$$

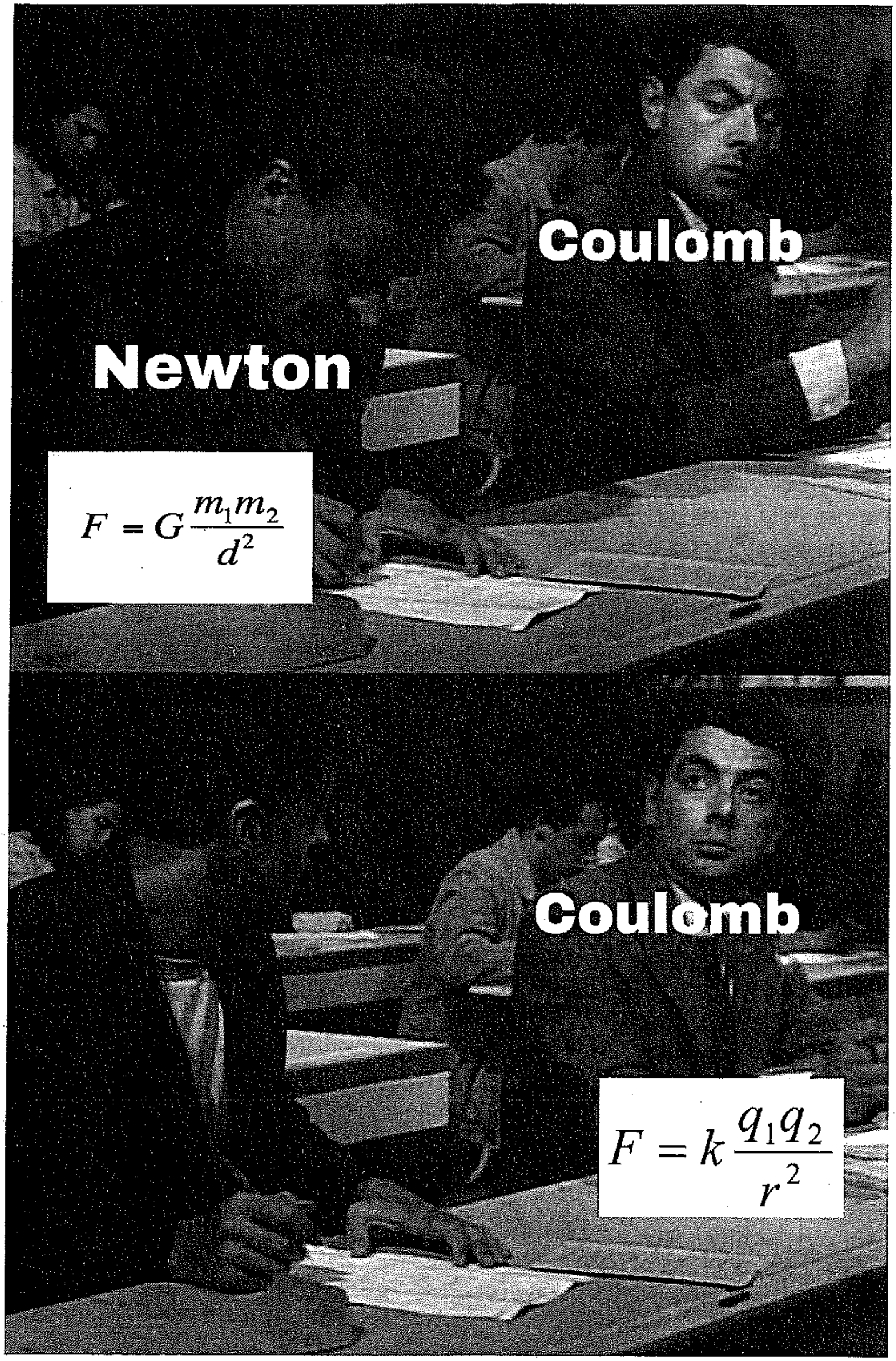
Define $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$ = position of q_1 relative to q_2
(points from $\vec{r}_2$ to $\vec{r}_1$)

$$\boxed{\vec{F}_{12} = \frac{K q_1 q_2 \vec{r}_{12}}{|\vec{r}_{12}|^3}}$$ Coulomb's law of force

$|\vec{F}_{12}| = \frac{K q_1 q_2}{|\vec{r}_{12}|^2}$: inverse square law (like gravity)

units of K make sense: $\frac{N \cdot m^2}{C^2}$

oH



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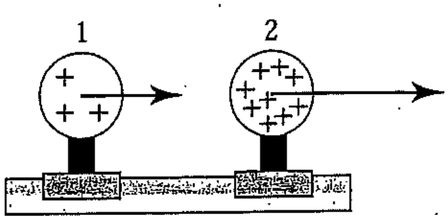
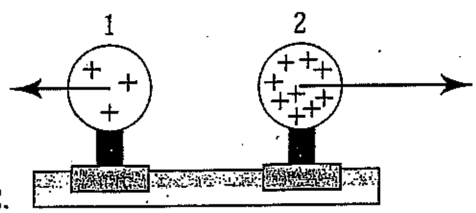
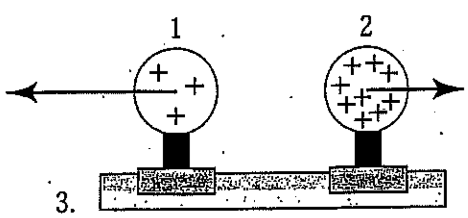
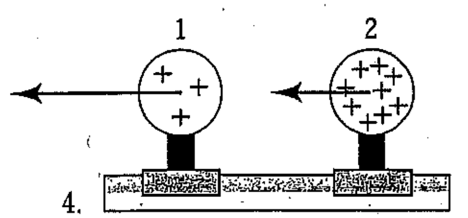
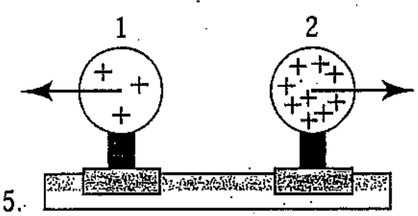
Two uniformly charged spheres are firmly fastened to and electrically insulated from frictionless pucks on an air table. The charge on sphere 2 is three times the charge on sphere 1. Which force diagram correctly shows the magnitude and direction of the electrostatic forces:

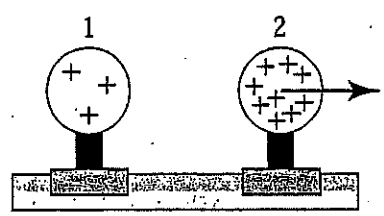
1. 

2. 

3. 

4. 

5. 

6. 

7. none of the above

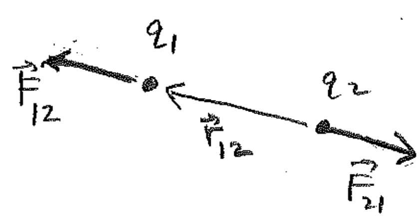
Let $\vec{F}_{21}$ = force exerted q_2 by (field produced by) q_1

$$= \frac{K q_2 q_1 \vec{r}_{21}}{|\vec{r}_{21}|^3}$$

Note: $\vec{r}_{21} = \vec{r}_2 - \vec{r}_1 = -(\vec{r}_1 - \vec{r}_2) = -\vec{r}_{12}$

$$\vec{F}_{21} = -\frac{K q_1 q_2 \vec{r}_{12}}{|\vec{r}_{12}|^3} = -\vec{F}_{12}$$

Newton's 3rd law is satisfied!



(assume $q_1 q_2 > 0$)

like charges repel, opposites attract.

To recap: charges both <u>produce</u> and <u>respond</u> to $\vec{E}$ fields

[Why introduce $\vec{E}$ field at all?
Why not just consider Coulomb force $\vec{F}_{12}$?
$\vec{F}_{12}$ implies instantaneous action-at-a-distance,
in violation of special relativity. (Explain).
OK if all charges are static, but need $\vec{E}$ to carry information if not.]