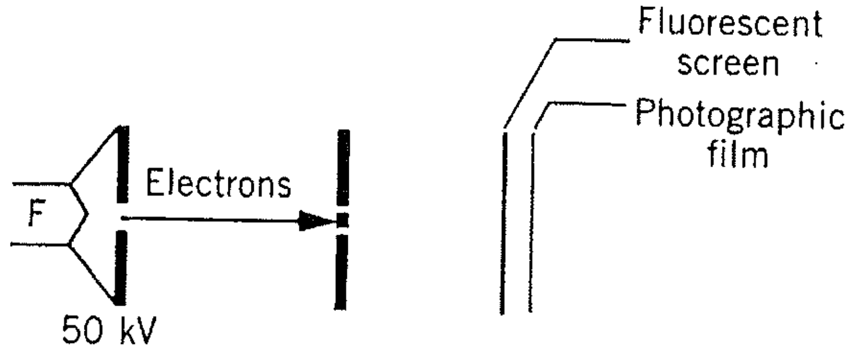
wave-particle duality

light: electromagnetic waves / photons

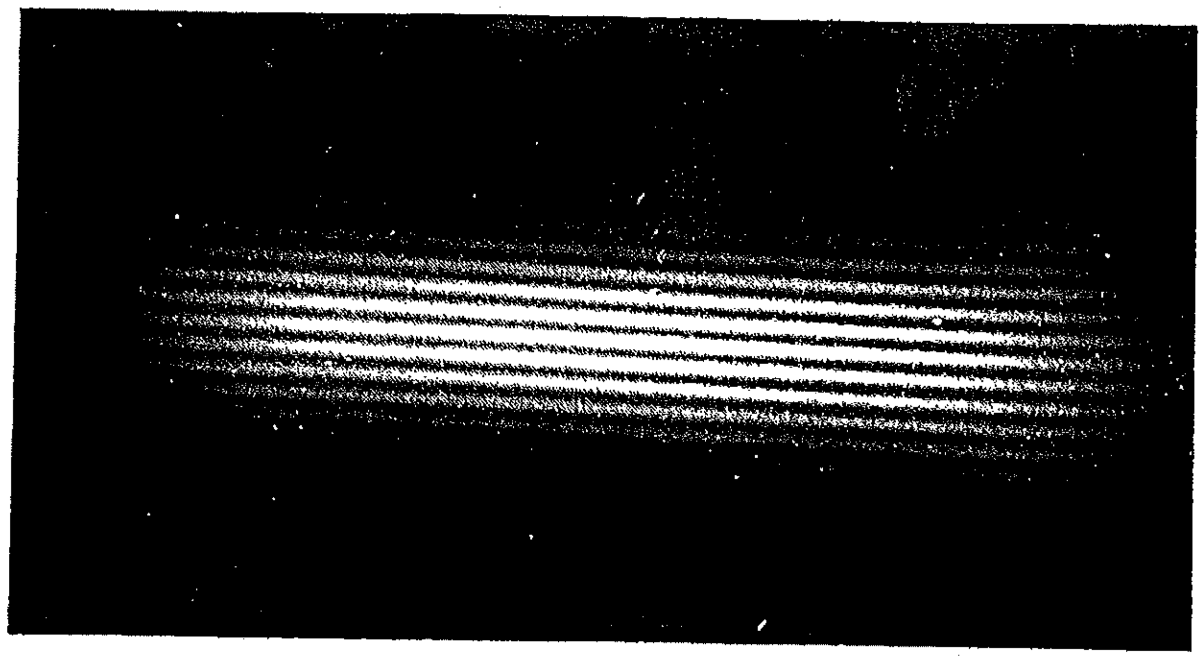
matter: matter waves / electrons, protons, etc.

In 1926 de Broglie proposed that electrons
can behave like waves

[evidence? interference. see 3 transparencies]



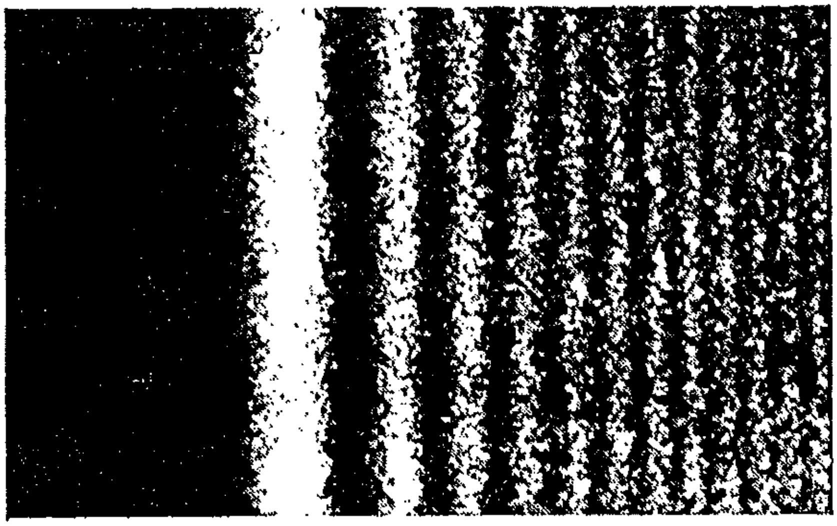
(*a*)



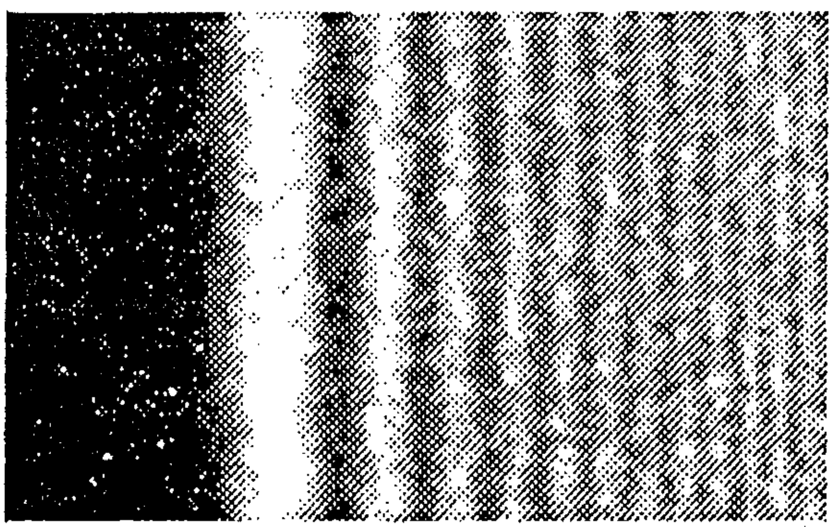
(*b*)

Figure 1 (*a*) Apparatus for producing double-slit interference with electrons. A filament F produces a spray of electrons, which are accelerated through 50 kV, pass through the single slit, and strike the double slit. They produce a visible pattern when they strike a fluorescent screen, which can be photographed. (*b*) The resulting electron double-slit interference pattern, showing the interference fringes.

Figure 2 An electron diffraction pattern using a circular aperture of diameter 30 μm and 100-keV electrons. Compare with the optical pattern (Fig. 2 of Chapter 46).



(*a*)



(*b*)

Figure 3 Diffraction of (*a*) light and (*b*) electrons from a straight edge.

If an electron is described by a wave,

what is its wavelength λ?

Depends on its momentum p.

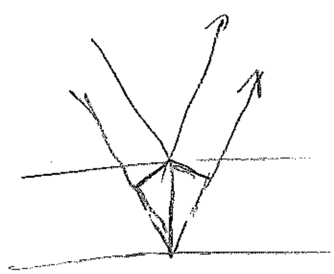
[analogous to light, where E depends on f]

Louis de Broglie (1926) proposed

$$\lambda = \frac{h}{p} \qquad \text{(de Broglie wavelength)}$$

Davisson & Germer (1926) confirmed this experimentally

using Bragg scattering off a nickel crystal.



wave number $k = \frac{2\pi}{\lambda} = \frac{2\pi}{h} p \implies p = \frac{h}{2\pi} k$

For convenience, define $\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}\ \mathrm{J \cdot s}$

Then $\boxed{p = \hbar k}$ de Broglie relation

Calculate λ for an electron w/ kinetic energy 1.0 eV.

$$E_{kin} = \frac{1}{2} m v^2 \quad \Rightarrow \quad v = \sqrt{\frac{2E_{kin}}{m}}$$

$$p = mv = \sqrt{2mE_{kin}}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_{kin}}} = \frac{hc}{\sqrt{2mc^2 E_{kin}}}$$

$hc = 1240 \text{ nm} \cdot \text{eV}$

$mc^2 = 0.511 \text{ MeV}$ for an electron

$E_{kin} = 1.0 \text{ eV}$

$\Rightarrow \lambda = 1.2 \text{ nm}$ (atomic dimensions)

Beware: for light $\lambda = \frac{c}{f} = \frac{hc}{E_{photon}}$

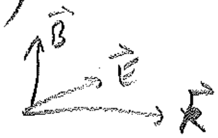
but for (non relativistic) particles $\lambda = \frac{h}{\sqrt{2mE_{kin}}}$

$v \ll c$ Not the same!

Light is described by an electromagnetic wave consisting of two real vector fields $\vec{E}$ + $\vec{B}$

A travelling EM wave w/ wave number $k = \frac{2\pi}{\lambda}$ takes the form

$$\vec{E} = \begin{pmatrix} 0 \\ A\sin(kx-\omega t) \\ 0 \end{pmatrix} \qquad \vec{B} = \begin{pmatrix} 0 \\ 0 \\ \frac{A}{c}\sin(kx-\omega t) \end{pmatrix}$$



Intensity $I = \epsilon_0 c \langle |\vec{E}|^2 \rangle = \frac{\epsilon_0 c}{2} A^2$

An electron is described by a matter wave which is a complex scalar field

$\psi = \psi_R + i\,\psi_I$ where $i = \sqrt{-1}$

complex number
$z = x + iy$
where x = real part
y = imag part

A travelling matter wave w/ wave number $k = \frac{2\pi}{\lambda}$ takes the form

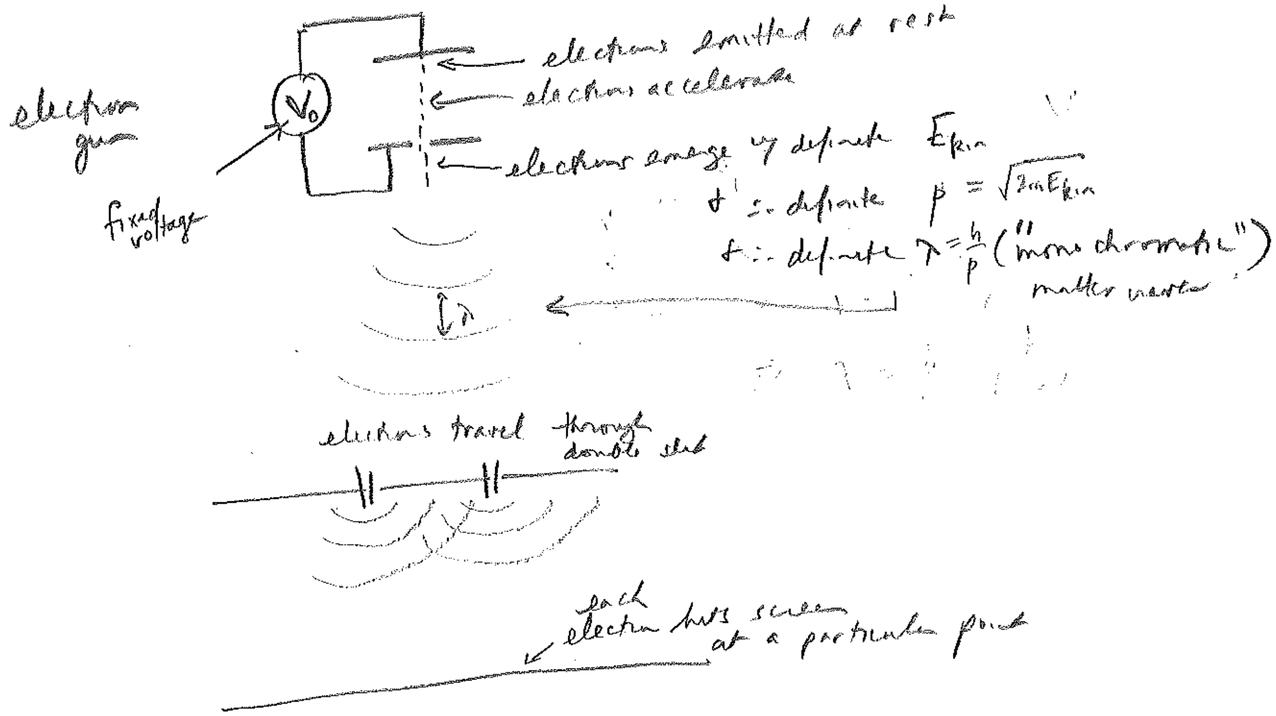
$\psi = A\cos(kx-\omega t) + i\,A\sin(kx-\omega t)$

Define probability density $\rho = |\psi|^2 \equiv \psi_R^2 + \psi_I^2$

$= A^2\cos^2(kx-\omega t) + A^2\sin^2(kx-\omega t)$

$= A^2$

Two slit experiment w/ electrons



Electrons, like classical particles, are emitted (eg from cathode) and detected (eg at screen) one at a time in specific locations but unlike classical particles, they do not follow a definite trajectory. Each electron (wave) goes through both slits. (see transparency)

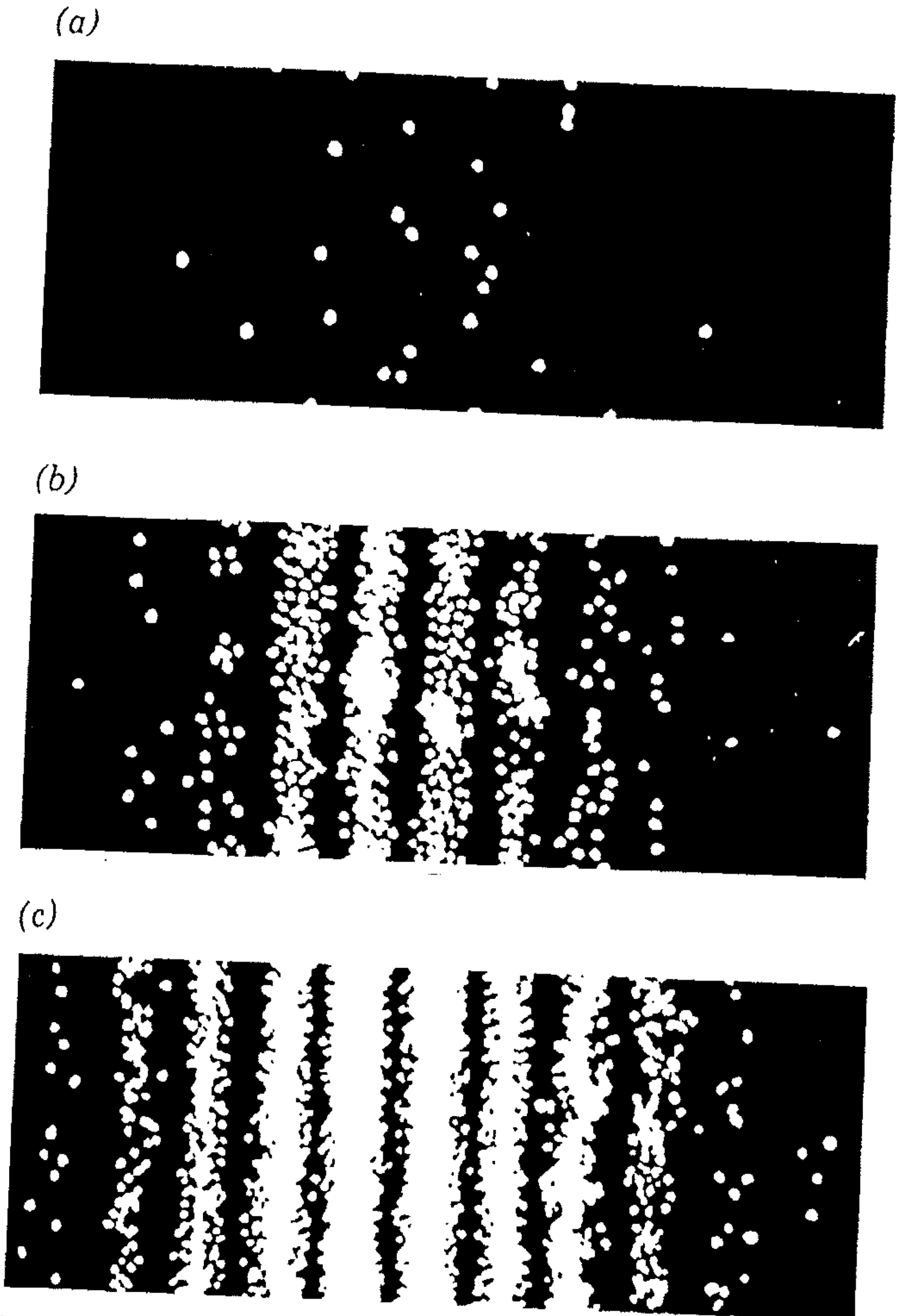
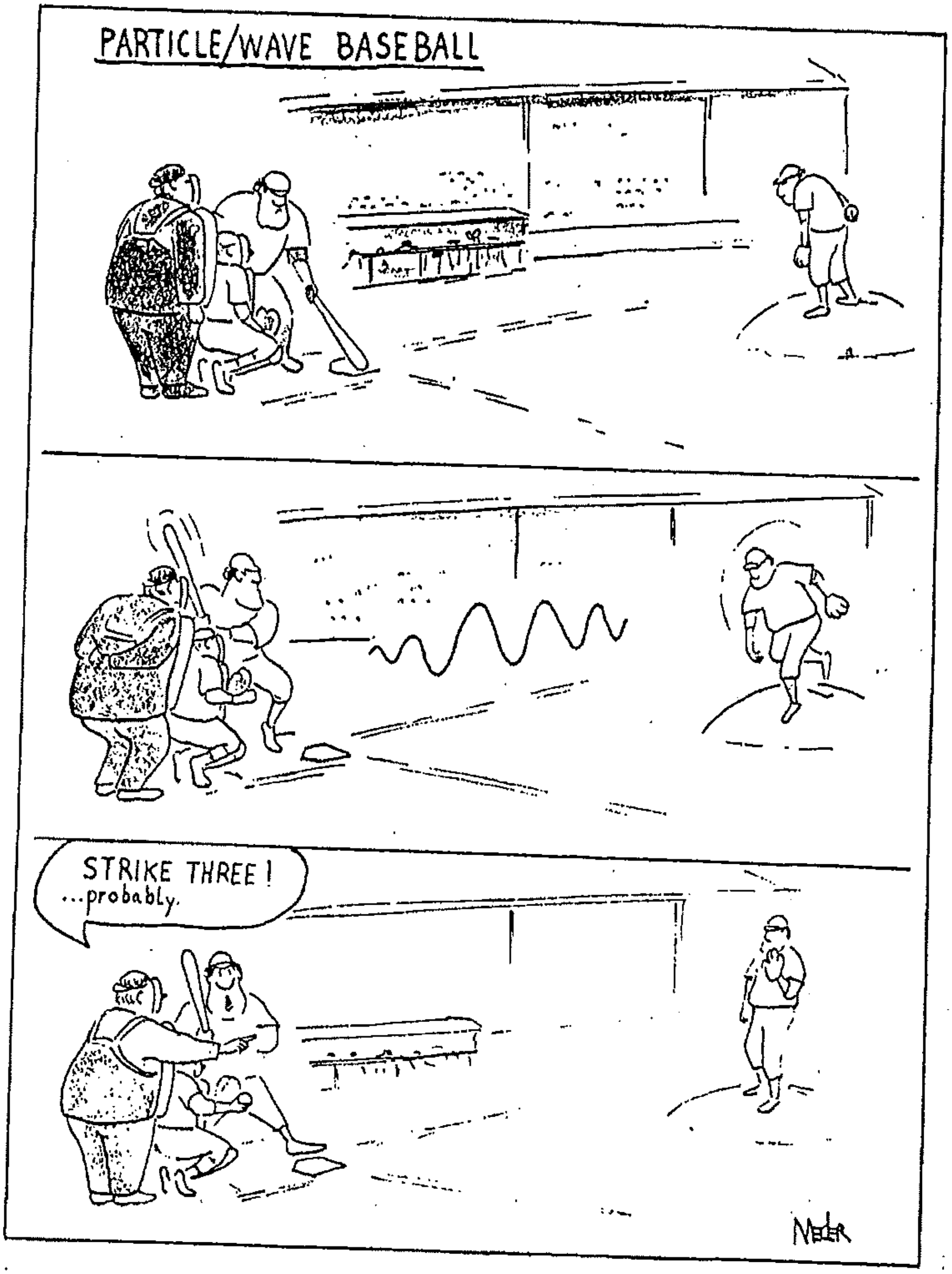
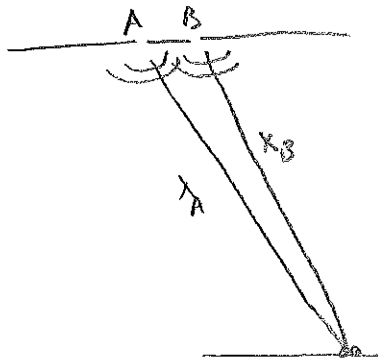


Figure 26 The buildup of interference fringes as electrons fall on screen B of Fig. 25. In (*a*), about 30 electrons have landed on the screen, in (*b*) about 1000, and in (*c*) about 10,000. The probability density of the wave describing the electron determines where the electrons will land on the screen.


PARTICLE/WAVE BASEBALL
STRIKE THREE!
...probably.

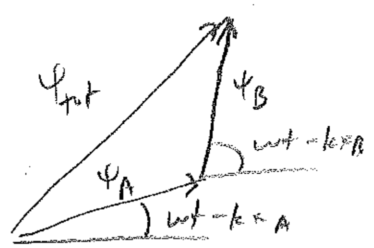
Two slit experiment



Superposition principle for matter waves

$$\psi_{tot} = \psi_A + \psi_B$$

Represent waves by phasors



Phase difference $\Delta\phi = kx_A - kx_B$

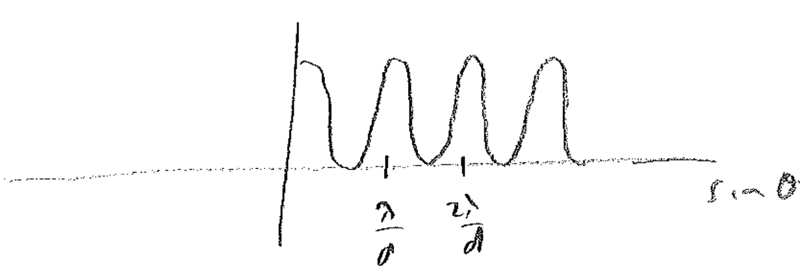
$$= \frac{2\pi}{\lambda}\Delta x$$

just as for light, but using de Broglie wavelength $\lambda = \frac{h}{p}$

For light, intensity $I \sim |A_{tot}|^2$

For electrons, probability density $P = |\psi_{tot}|^2$

$|\psi_{tot}|^2$ exhibits maxima (const. interf) when $d\sin\theta = m\lambda$



The interference pattern represents the probability density of electrons, i.e. the statistical distribution of electrons reaching the screen. Intensity is proportional to number of electrons or photons