Thermal radiation

Any object w/ $T > 0$ emits electromagnetic radiation

[if it's hot enough, that radiation will be visible]

Temperature is proportional to internal kinetic energy (due to random motion of atoms)

Roughly $E_{molecule} \sim k_B T$ (up to some numerical factor)

$\frac{3}{2}$ for atom

$\frac{5}{2}$ for diatomic

k_B = Boltzmann constant $= 1.38 \times 10^{-23} \frac{J}{K}$

$= 8.62 \times 10^{-5} \frac{eV}{K}$

At room temperature ($T \sim 300\,K$)

$k_B T \sim \frac{1}{40}\,eV$ (typical kinetic energy of each atom or molecule)

Random motion ⟹ random acceleration of charges (electrons)

⟹ emission of electromagnetic waves of all frequencies by

1) heated solids (eg W filaments)
2) heated liquids (eg molten steel)
3) high pressure gas (eg sun)

any substance w/ lots of interactions among atoms

[leads to loss of energy (radiant cooling)
— thermoses
— cloudless nights]

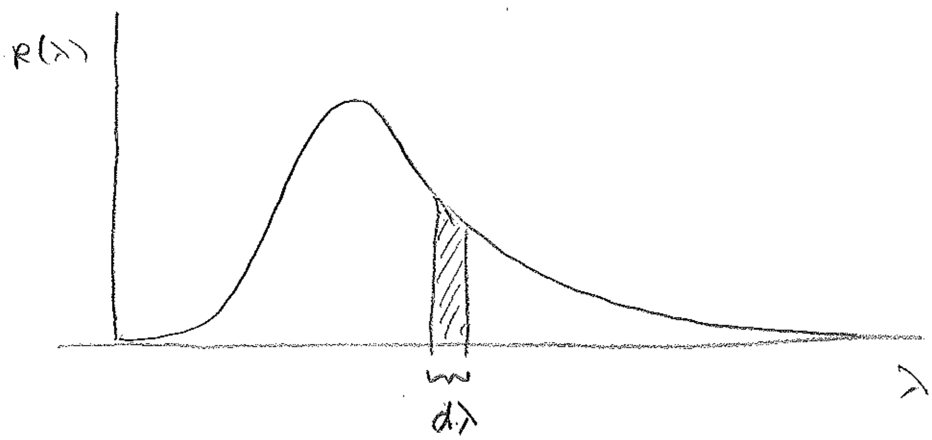
⟹ continuous spectrum

[How describe a continuous spectrum?] →

Intensity = (time average of) power/area $\left(\frac{J}{s \cdot m^2}\right)$

Radiancy $R(\lambda) = \dfrac{\text{intensity}}{\text{unit wavelength}}$ $\left(\frac{J}{s \cdot m^3}\right)$

Thermal radiation radiancy curve (a.k.a. "blackbody spectrum")

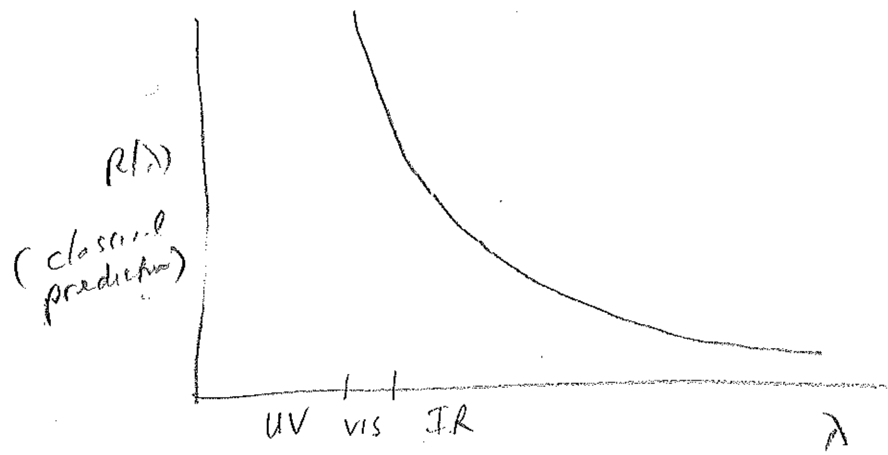


Area of strip = $R(\lambda)\,d\lambda$ = intensity emitted between λ and $\lambda + d\lambda$

Total intensity emitted I = area under curve = $\int_0^\infty R(\lambda)\,d\lambda$

The classical (ie nonquantum) theory of electromagnetism (19th c'y) predicts thermal radiation spectrum

$$R(\lambda) = \frac{2\pi c\, k_B T}{\lambda^4} \quad \text{(Rayleigh Jeans law)}$$

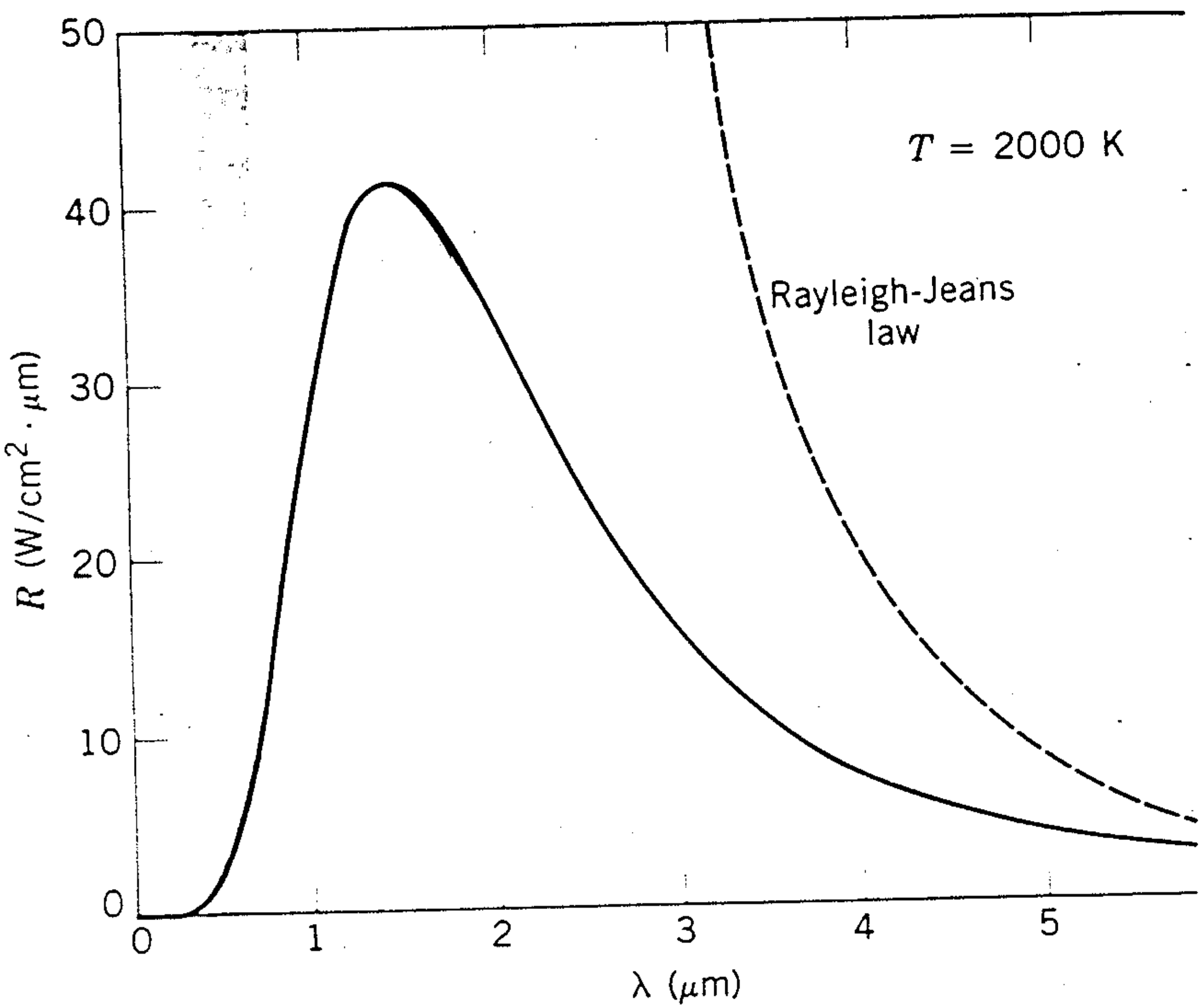


[predicts more vis than IR, more UV than vis, more X-rays than UV] lethal!

Total intensity $I = \int_0^\infty R(\lambda)\, d\lambda \sim \int_0^\infty \frac{d\lambda}{\lambda^4} \sim -\frac{1}{3\lambda^3}\Big|_0^\infty = \infty$

ultraviolet catastrophe!

classical theory fails!



50
40
30
20
10
0
R (W/cm² · μm)
0
1
2
3
4
5
λ (μm)
T = 2000 K
Rayleigh-Jeans law

Max Planck (1900) invoked the quantum hypothesis

$E_{photon} = hf$ to predict radiancy

$$R(\lambda) = \frac{2\pi c^2 h}{\lambda^5 \left(e^{\frac{hc}{\lambda k_B T}} - 1 \right)}$$

[agrees w/ Rayleigh-Jeans for large λ but not small λ]

$\frac{dR}{d\lambda} = 0$ at the maximum of R

which occurs at approximately:

$$\lambda_{peak} \approx \frac{1}{5} \frac{hc}{k_B T} \approx \frac{(2.9 \times 10^6 \text{ nm} \cdot \text{K})}{T}$$

"Wien's displacement law"

$\frac{dR}{d\lambda} = 0$ [HW: exact $= \frac{1}{4.96511}$]

As $T \uparrow$, $\lambda_{peak} \downarrow$

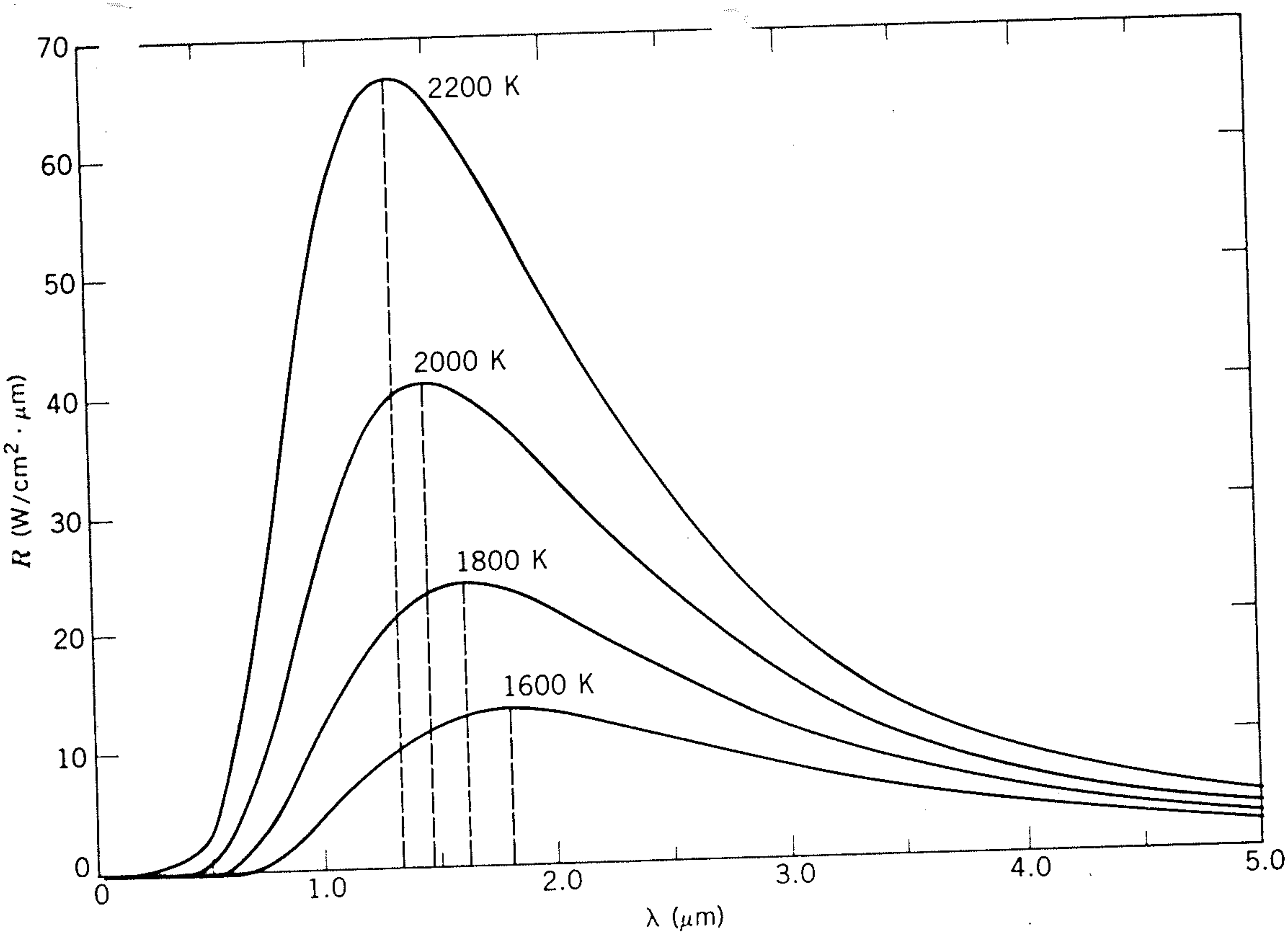


Figure 3 Spectral radiancy curves for cavity radiation at four selected temperatures. Note that as the temperature increases, the wavelength of the maximum spectral radiancy shifts to lower values.

[Rough explanation]

typical $E_{atom} \sim k_B T$

If the atom gives that energy to a photon

$$E_{photon} \sim E_{atom}$$

$$hf \sim k_B T$$

$$\frac{hc}{\lambda} \sim k_B T$$

$$\lambda \sim \frac{hc}{k_B T}$$

higher temp $\Rightarrow$ more energetic photons

$$\lambda_{peak} = \frac{2.9 \times 10^6 \text{ nm} \cdot \text{K}}{T}$$

[DEMO: hand out diffraction gratings]

$T = 300K$ (room temp) $\Rightarrow \lambda_{peak} = 10{,}000$ nm $= 10\ \mu m$ (IR)

[heat scopes]

$T = 1000$ K (hot stove element) $\Rightarrow \lambda_{peak} = 3{,}000$ nm

[dull red glow]

$T = 1750K$ (candle) $\Rightarrow \lambda_{peak} = 1{,}700$ nm [yellow]

[carbon particles in the flame not gas atoms]

[show transparency; light candle]

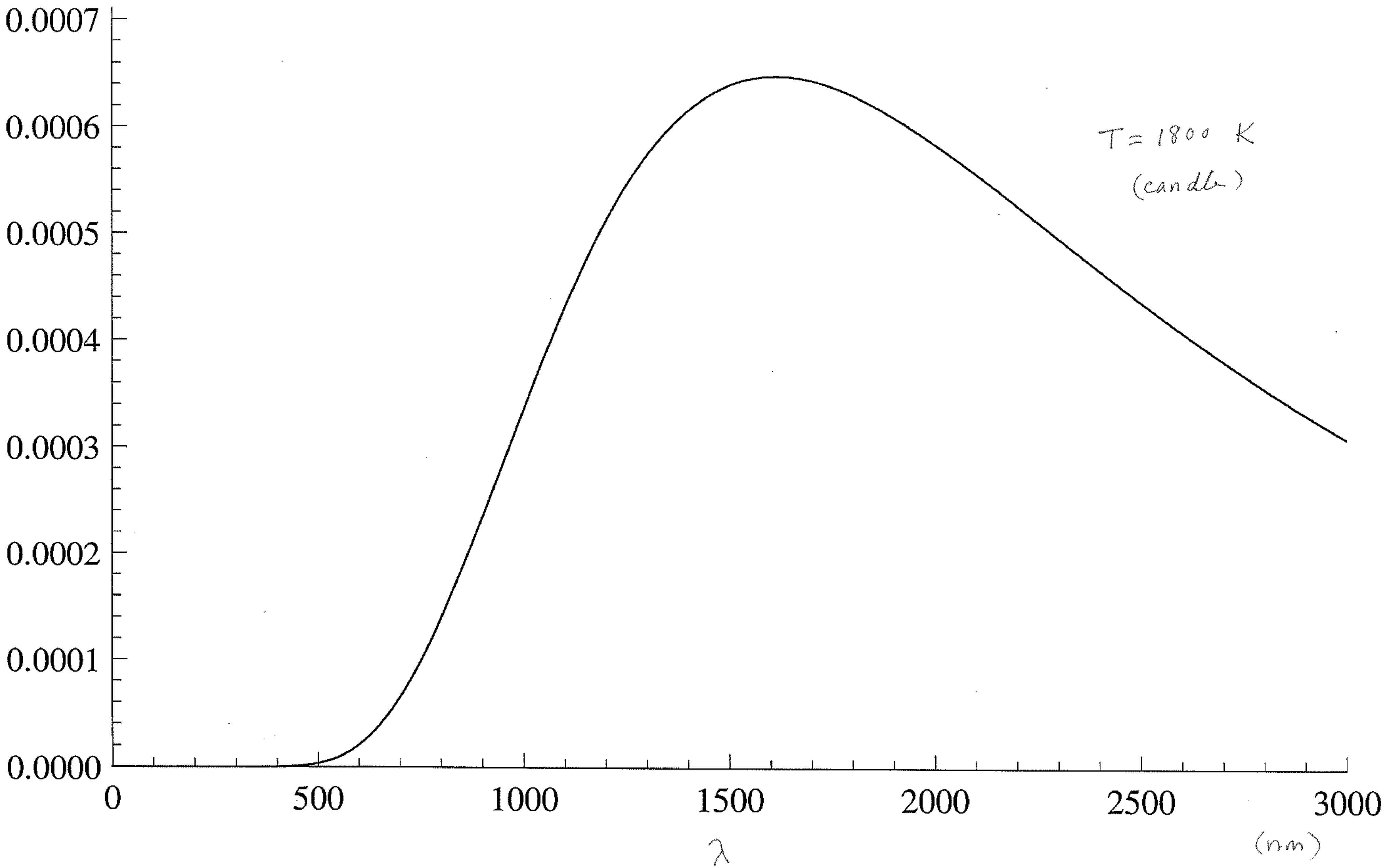
$T = 2800$ K (filament of incandescent bulb) $\lambda_{peak} = 1100$ nm

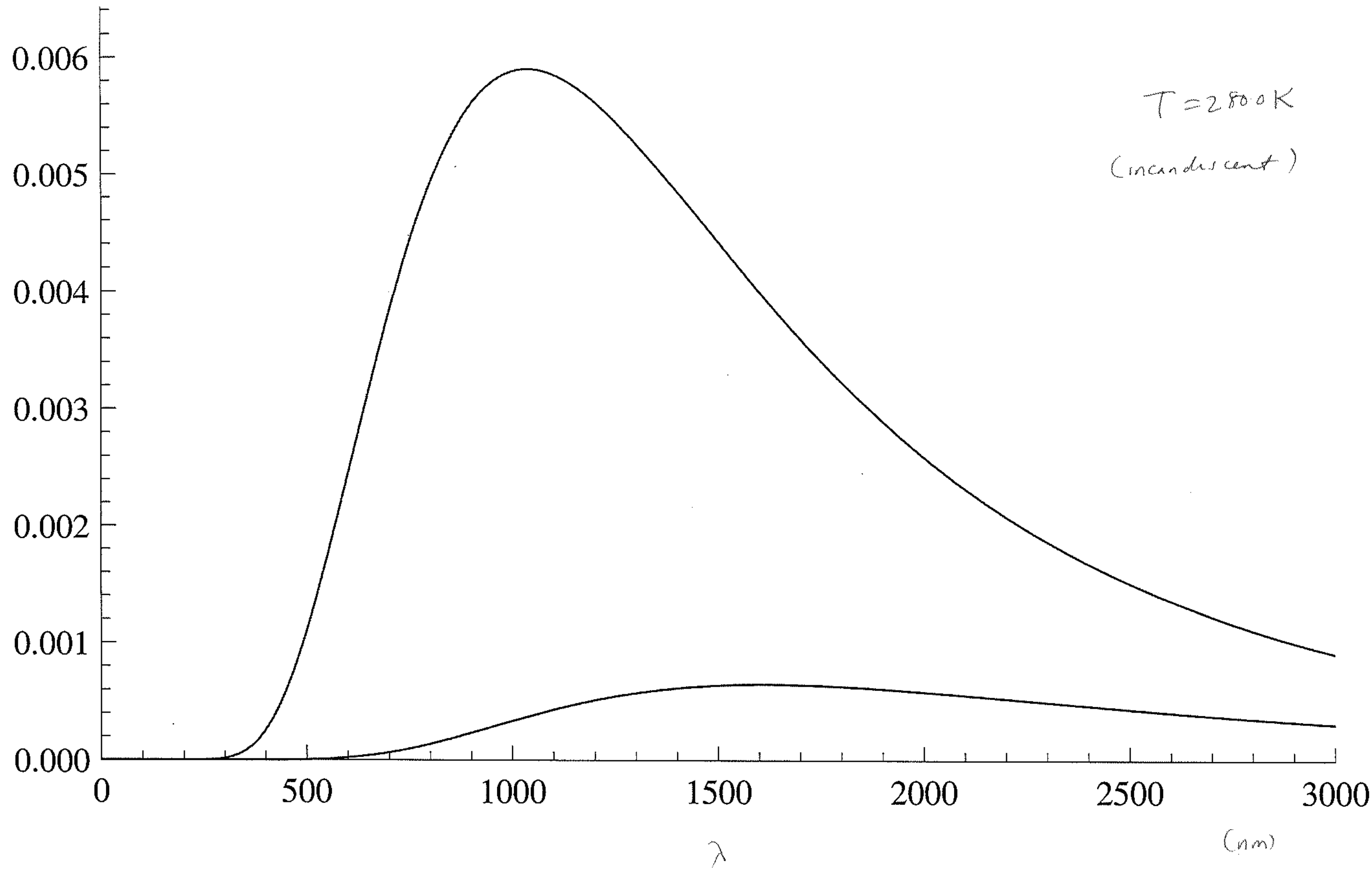
[show transparency; adjustable bulb]

$T = 5800K$ (Sun's surface) $\lambda_{peak} = 500$ nm [white]

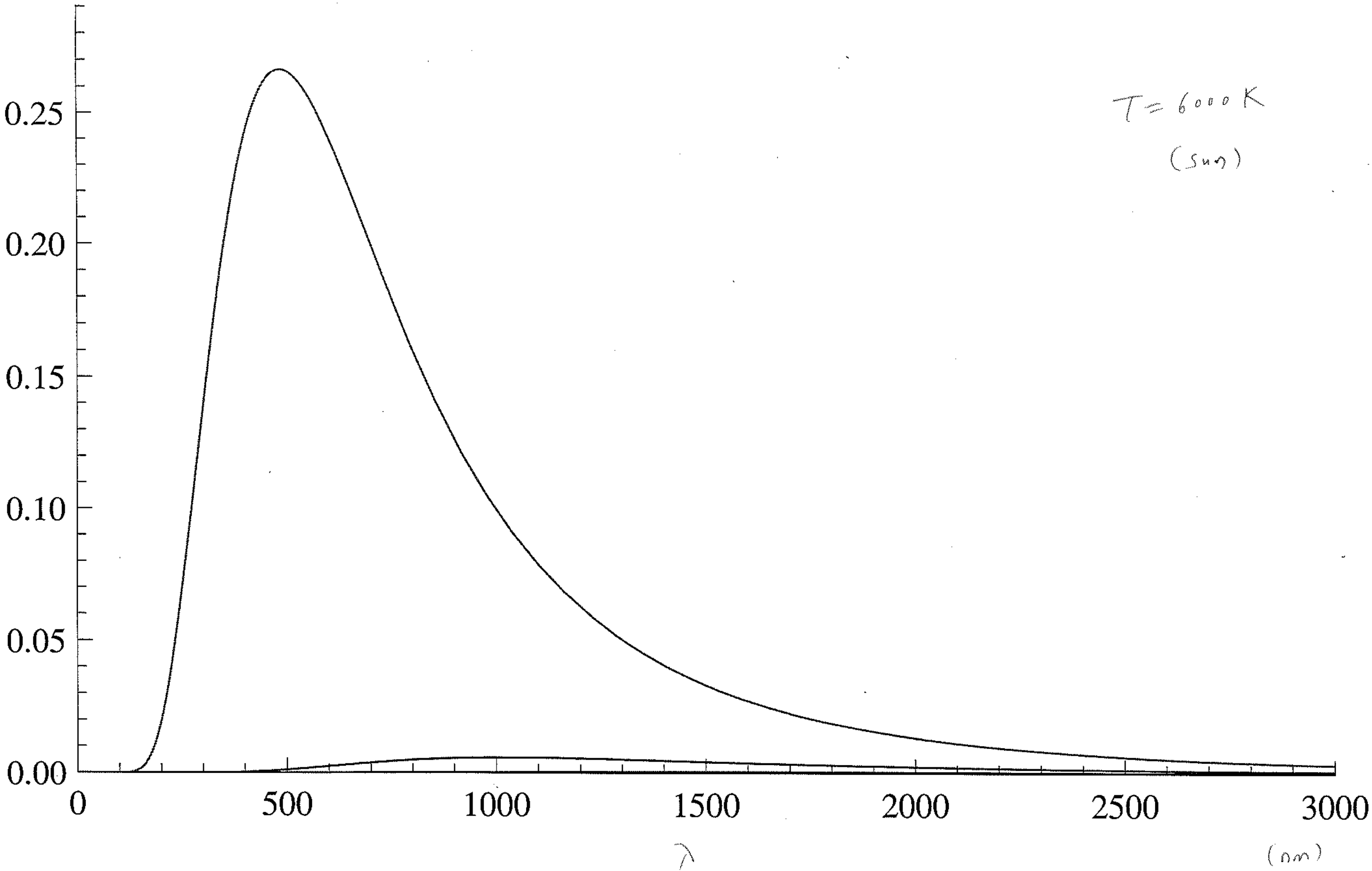
$T = 12{,}000K$ (hot stars) $\lambda_{peak} = 250$ nm [UV peak] [blue]

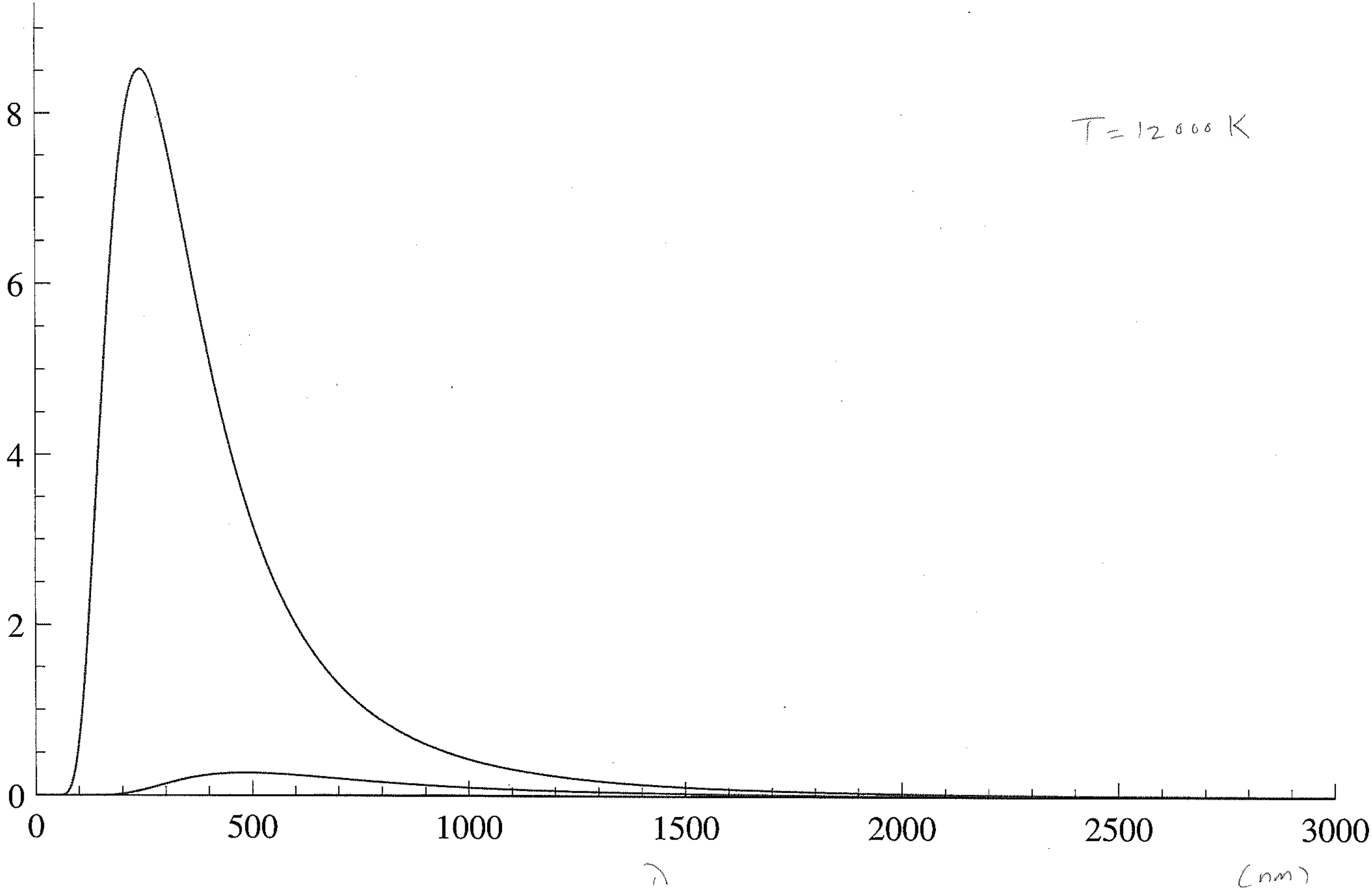
[20,000K: B-type stars]


0.0007
0.0006
0.0005
0.0004
0.0003
0.0002
0.0001
0.0000
0
500
1000
1500
2000
2500
3000
λ
(nm)
T = 1800 K
(candle)



0.006
0.005
0.004
0.003
0.002
0.001
0.000
0
500
1000
1500
2000
2500
3000
λ
(nm)
T = 2800 K
(incandescent)


0.25
0.20
0.15
0.10
0.05
0.00
0
500
1000
1500
2000
2500
3000
λ
(nm)
T = 6000 K
(sun)



8
6
4
2
0
0
500
1000
1500
2000
2500
3000
T = 12000 K
λ
(nm)

Total radiation emitted = area under curve

$$I = \int_0^\infty R(\lambda)\, d\lambda = 2\pi h c^2 \int_{\lambda=0}^{\lambda=\infty} \frac{d\lambda}{\lambda^5 \left(e^{\frac{hc}{\lambda k_B T}} - 1\right)}$$

[How to do an integral w/o doing an integral]

Change to dimensionless variable $x = \frac{hc}{kT} \frac{1}{\lambda}$

$$\lambda = \frac{hc}{k_B T} \frac{1}{x}$$

$$\frac{d\lambda}{dx} = -\frac{hc}{k_B T} \frac{1}{x^2}$$

$$d\lambda = -\frac{hc}{k_B T} \frac{dx}{x^2}$$

$$I = 2\pi h c^2 \int_{x=\infty}^{x=0} \frac{\left(-\frac{hc}{k_B T} \frac{dx}{x^2}\right)}{\left(\frac{hc}{k_B T} \frac{1}{x}\right)^5 (e^x - 1)}$$

$$= 2\pi h c^2 \left(\frac{k_B T}{hc}\right)^4 \int_\infty^0 \frac{(-x^3 dx)}{e^x - 1}$$

$$= \frac{k_B^4 T^4}{h^3 c^2} \, 2\pi \underbrace{\int_0^\infty \frac{x^3 dx}{e^x - 1}}_{\text{Tables} \Rightarrow \frac{\pi^4}{15}}$$

$$= \underbrace{\left(\frac{2\pi^5}{15} \frac{k_B^4}{h^3 c^2}\right)}_{\sigma_B \,=\, 5.67 \times 10^{-8} \frac{J}{s\, m^2 K^4}} T^4$$

= Stefan Boltzmann constant

$$\boxed{I = \sigma_B T^4}$$ Stefan Boltzmann law

Thermal radiation emitted by sun

$$I = \sigma_B T_s^4$$

T_s = temp of sun's surface $\approx 5800\,K$

$$I = \frac{\text{power}}{\text{area}}$$

P_s = (intensity at sun's surface)(surface area of sun)

$= (\sigma_B T_s^4)(4\pi R_s^2)$

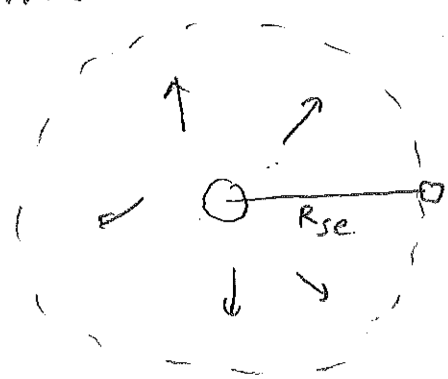
$R_s \approx 6.95 \times 10^5$ km

$\Rightarrow P_s = 3.9 \times 10^{26}$ W

[as found in prev. prob.]

[where does it come from?
chemical rxns? $< 10^5$ yrs
gravitational collapse?
nuclear fusion $\Rightarrow 10^{10}$ yrs]

Intensity of sunlight at earth's surface



$$I = \frac{P_s}{4\pi R_{se}^2}$$

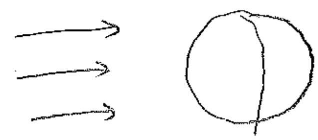
R_{se} = distance between earth + sun

$= 1.5 \times 10^8$ km

$$I = 1380 \,\frac{W}{m^2}$$

Solar energy intercepted by earth?

Treat earth as a flat disk
w/all radiation perpendicular.



$$P_{in} = I \cdot (\pi R_e^2)$$

R_e = radius of earth = 6400 km

$$P_{in} = 1.8 \times 10^{17}\ W$$

Thermal power emitted by earth (mostly infrared)

$$P_{out} = (\sigma_B T_e^4)(4\pi R_e^2)$$

Assume thermal equilibrium

$$P_{out} = P_{in}$$

$$4\pi R_e^2 \sigma_B T_e^4 = I \pi R_e^2$$

$$T_e^4 = \frac{I}{4\sigma_B}$$

$$T_e = \left(\frac{I}{4\sigma_B}\right)^{\frac{1}{4}} \approx 280\ K$$

What happens to T_e if:

① R_s increases [$P_s \uparrow$ so $T_e \uparrow$]

② R_{se} increases [$I \downarrow$ so $T \downarrow$]

③ R_e increases [nothing]

④ T_s increases [$P_s \uparrow$ so $T_e \uparrow$]

[optional (TWB) Don't do as makes problems too easy

$$T_e^4 = \frac{I}{4\sigma_B} = \frac{P_s}{4\sigma_B (4\pi R_{se}^2)} = \frac{\sigma_B T_s^4 (4\pi R_s^2)}{4\sigma_B (4\pi R_{se}^2)} = T_s^4 \frac{R_s^2}{4R_{se}^2}$$

$$T_e = T_s \left(\frac{R_s}{2R_{se}}\right)^{\frac{1}{2}}$$
]

P_{in} reduced by albedo [earth not a perfect absorber]

P_{out} reduced by greenhouse effect [IR radiation trapped + reflected]

What would happen to the temperature of the earth if

1. R_{sun} doubled $T \uparrow \sqrt{2}$

2. $D_{earth-sun}$ doubled $T \downarrow \frac{1}{\sqrt{2}}$

3. R_{earth} doubled (T same)

What would happen to the temperature of the Earth if:

1. R_{sun} doubled
2. R_{earth} doubled
3. $D_{earth\text{-}sun}$ doubled

Choose from:

A. $T_{new} = \frac{1}{4} T_{old}$
B. $T_{new} = \frac{1}{2} T_{old}$
C. $T_{new} = 1/\sqrt{2}\, T_{old}$
D. $T_{new} = T_{old}$
E. $T_{new} = \sqrt{2}\, T_{old}$
F. $T_{new} = 2\, T_{old}$
G. $T_{new} = 4\, T_{old}$

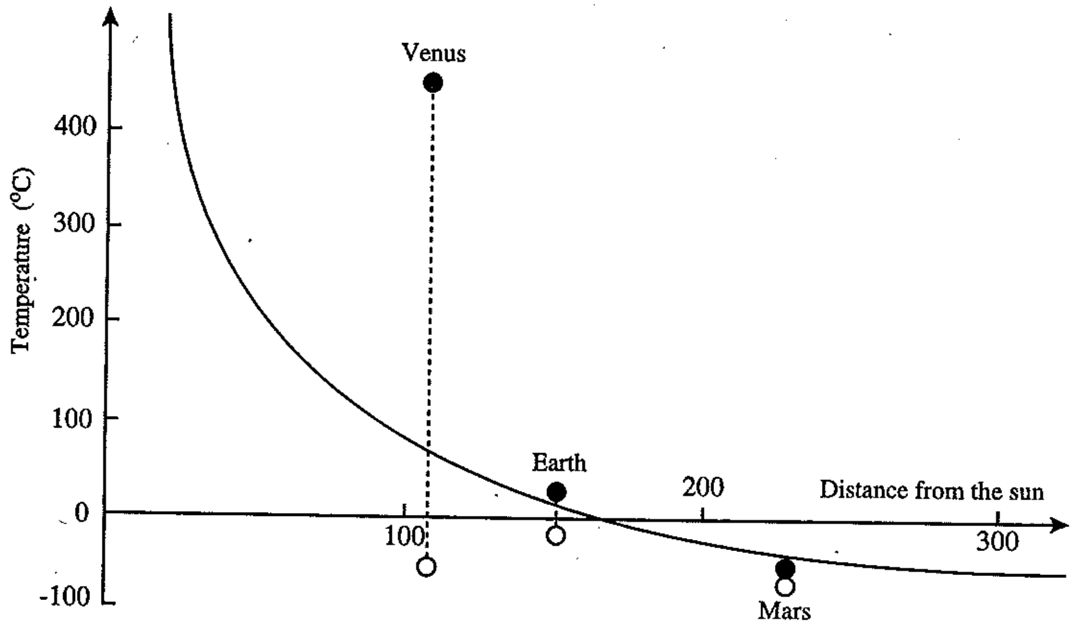


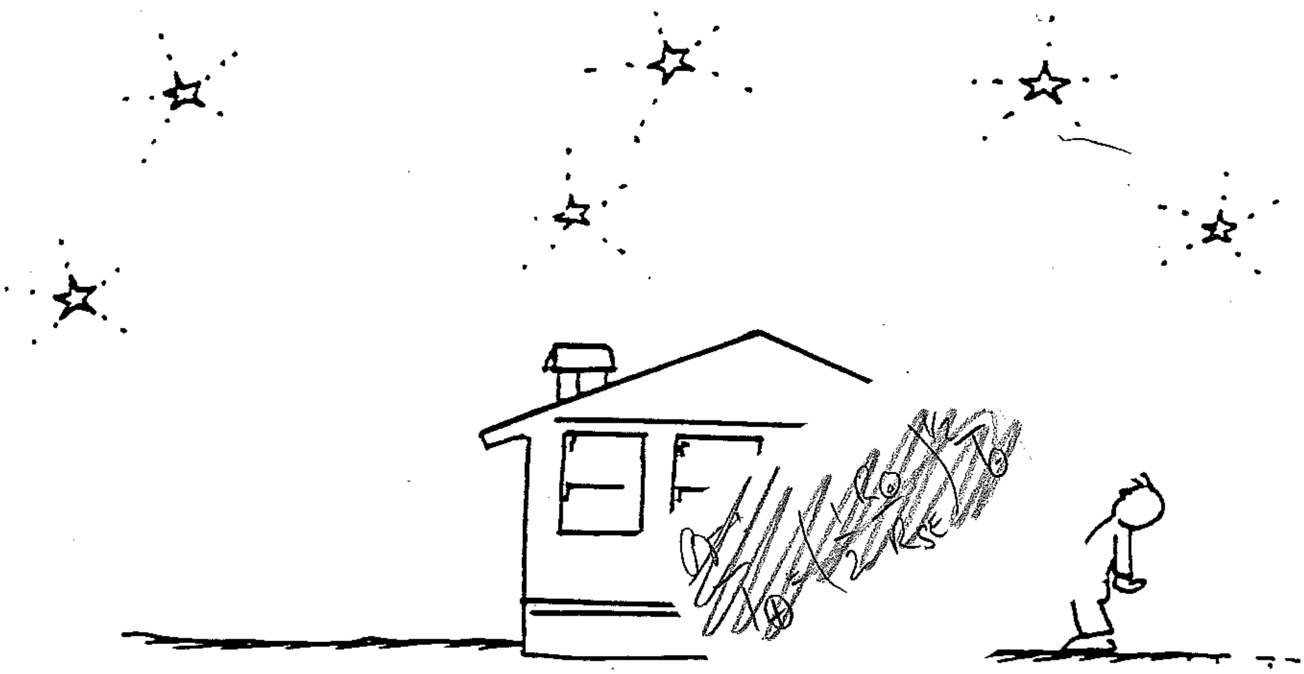
Figure 3.1 The curve shows the decrease in temperature, with increasing distance from the Sun (in units of 10^6 km), for planets that absorb all the incident sunlight and that have neither internal sources of energy nor atmospheres. The open circles take into account that each planet reflects some sunlight. The solid circles correspond to the actual temperatures at the surfaces of the planets. The length of each dashed line is a measure of the greenhouse effect.

← good

HOT STAR

Can you tell, just by looking, which are the hottest stars in the sky?

a) Yes, you can tell
b) No, you cannot tell

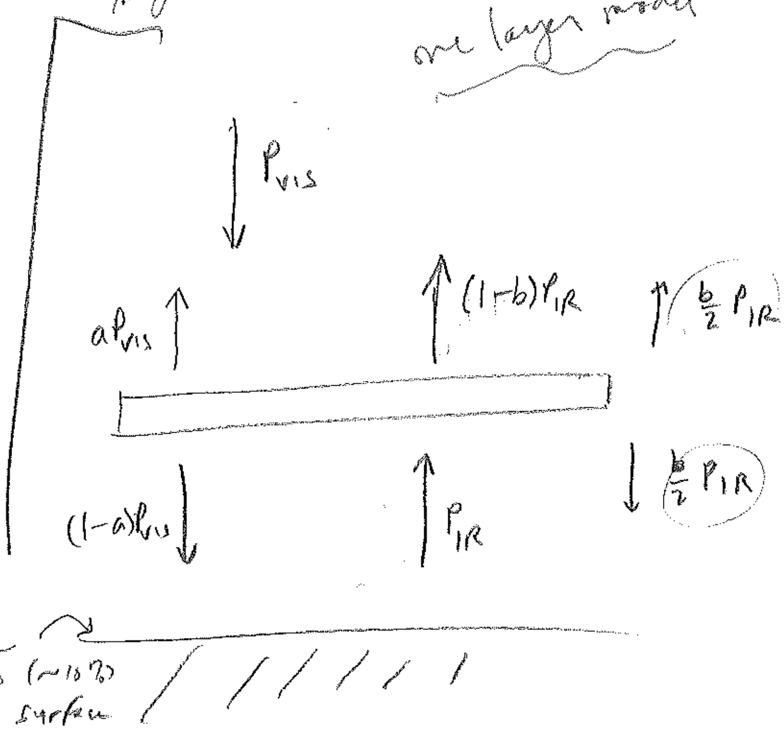


The hottest stars in the sky are [illegible] brightest stars in the sky.

a) True
b) False

Just for my interest

one layer model



$$P_{IR} = \sigma T_\oplus^4 \, 4\pi R_\oplus^2$$

$$P_{VIS} = \pi R_\oplus^2 \, I_{sun}$$

$1368 \frac{W}{m^2}$

$$T_\oplus = \sqrt[4]{\frac{I_{sun}}{4\sigma}} = 279\,K$$

(45°F)

$$(1-a)P_{VIS} = \left(1-\frac{b}{2}\right)P_{IR}$$

$$P_{IR} = \frac{(1-a)}{\left(1-\frac{b}{2}\right)} P_{VIS}$$

$$T_{earth} \sim \sqrt[4]{\frac{(1-a)}{\left(1-\frac{b}{2}\right)}}\,(279K)$$

albedo $a = 0.31$ $\quad \sqrt[4]{1-a} = 0.9114 \Rightarrow 254\,K$ (~0°F)

greenhouse $b = 0.8$ $\quad \sqrt[4]{\frac{1}{1-0.4}} = 1.14 \Rightarrow 290\,K$ (63°F)

(In fact 288)

300K ⇒ 80°
290 → 63°
280 = 45°