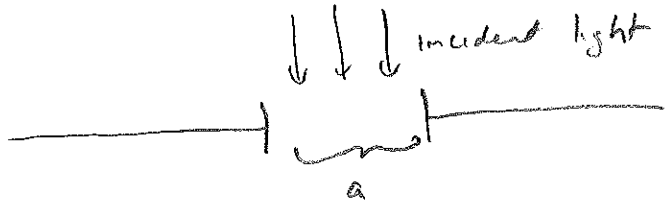
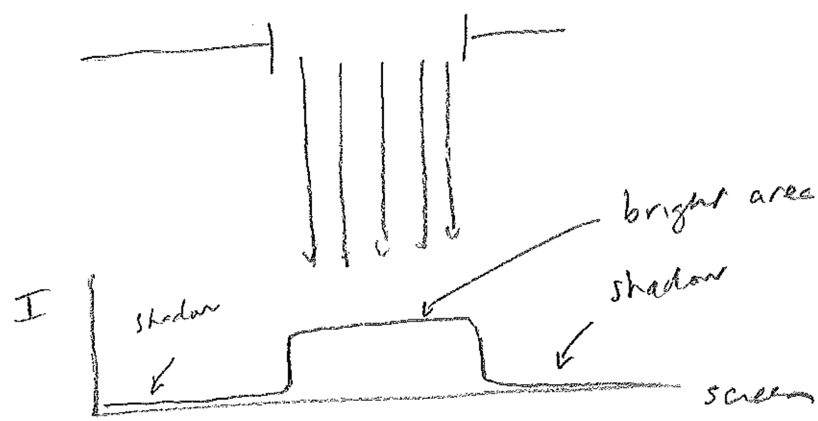
Single slit diffraction

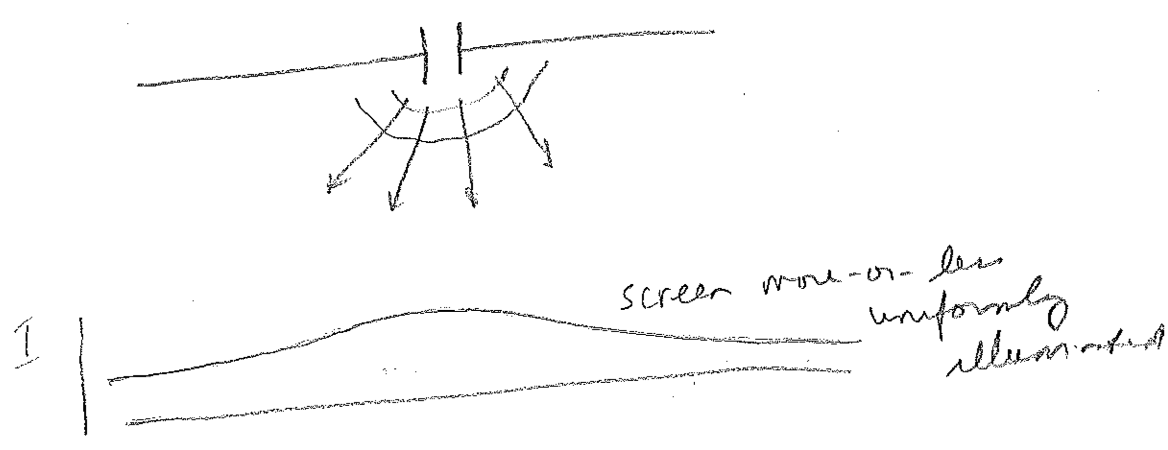
Consider an opaque screen w/ a single slit of width



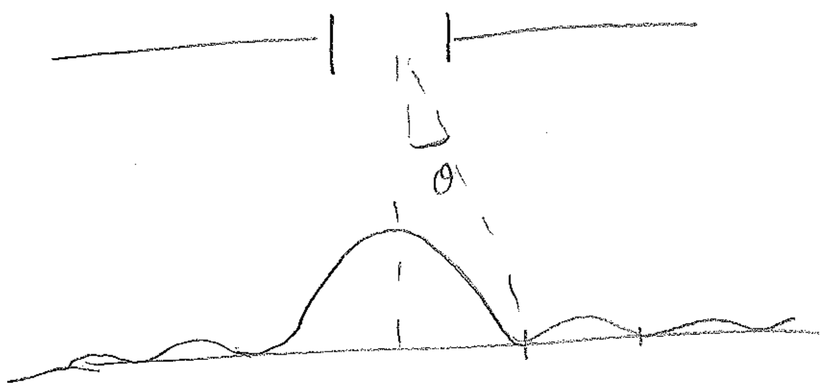
① if $a \gg \lambda$, use geometric optics (light goes in straight lines)



② if $a \ll \lambda$, diffraction



What if $a \gtrsim \lambda$? Single slit diffraction pattern w/ a large central peak



We will derive this pattern.
and show that first minimum occurs when

$$a \sin\theta = \lambda$$

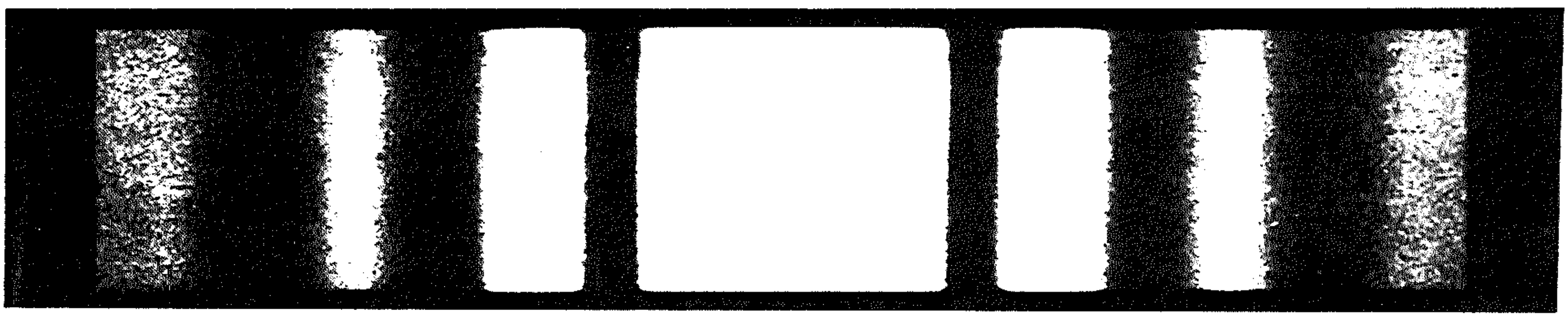
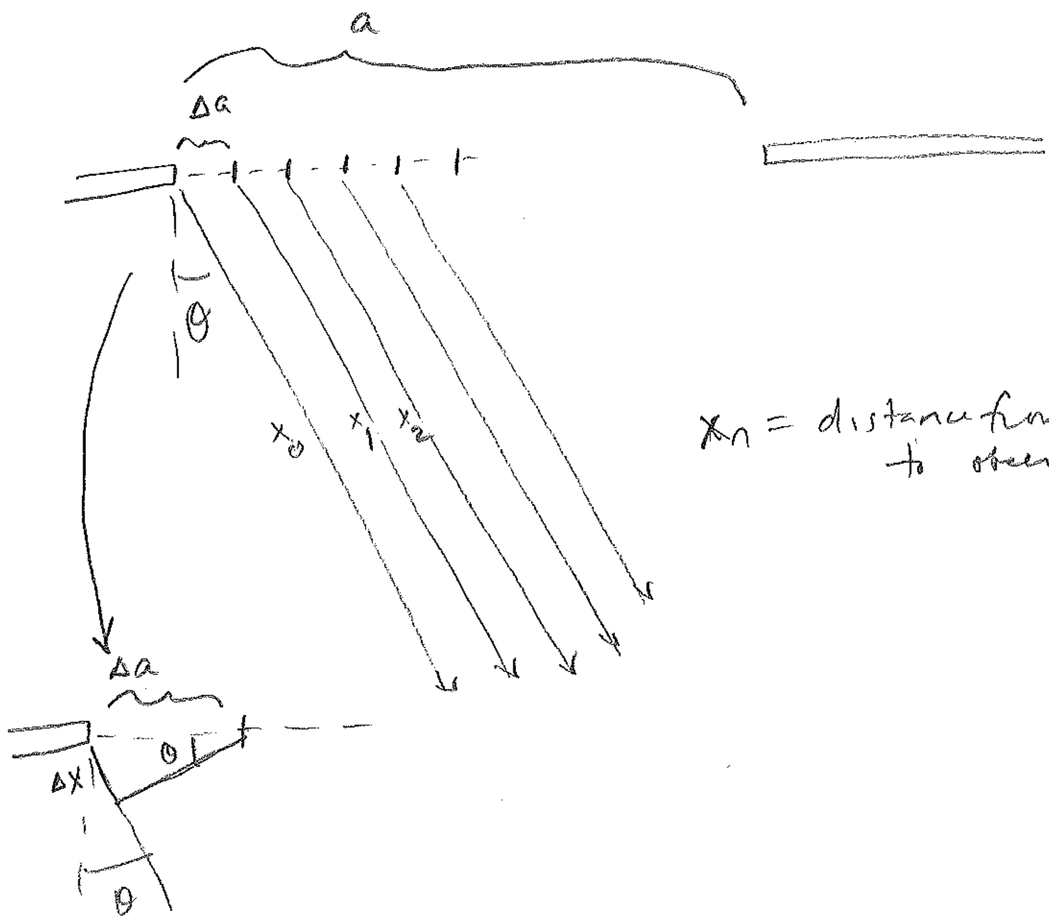


Figure 1 The diffraction pattern produced when light passes through a narrow slit.

HR
46, fig 1

Divide slit of width a into N strips of width $\Delta a = \frac{a}{N}$



x_n = distance from nth strip to observation point

$\Delta x = \Delta a \sin\theta$ = path difference between adjacent strips

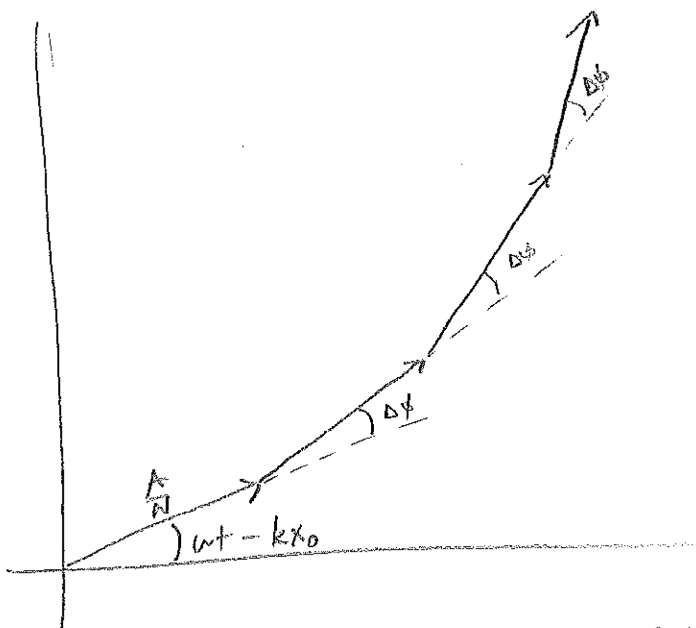
Each strip produces an EM wave $E_n = \frac{A}{N}\sin(\omega t - kx_n)$

$$x_1 = x_0 - \Delta x$$
$$x_2 = x_1 - \Delta x = x_0 - 2\Delta x$$
$$\vdots$$
$$x_n = x_0 - n\Delta x$$

$$\Rightarrow E_n = \frac{A}{N}\sin(\omega t - kx_0 + nk\Delta x)$$

$$E_{tot} = \sum_{n=0}^{N-1} E_n$$

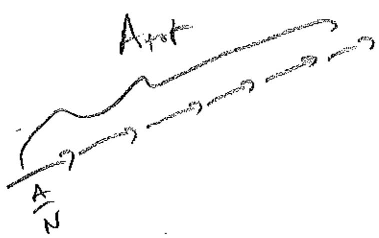
Represent waves by phasors



$\Delta\phi$ = phase difference between adjacent strips $= k\,\Delta x = \frac{2\pi}{\lambda}\,\Delta a \sin\theta$

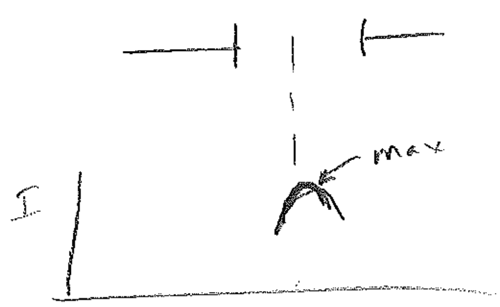
Fully constructive interference occurs if all phasors are parallel

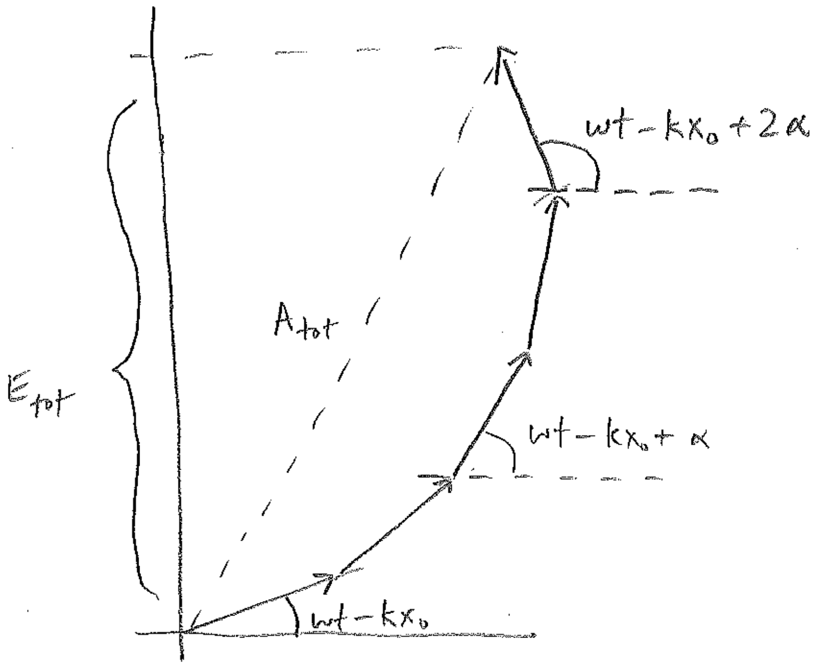
$$A_{tot} = N\left(\frac{A}{N}\right) = A$$



$\Delta\phi = 0$ which requires $\theta = 0$

Maximum of intensity occurs at $\theta = 0$, directly behind slit





Let 2α = phase difference between 1st + last phasors

Then α = phase difference between 1st + "middle" phasor

phasor representing E_{tot} is parallel to middle phasor

$$E_{tot} = A_{tot} \sin(\omega t - kx_0 + \alpha)$$

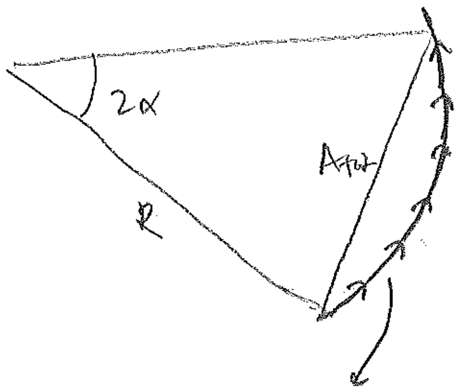
$$I = \epsilon_0 c A_{tot}^2 \langle \sin^2(\omega t - kx_0 + \alpha) \rangle = \frac{1}{2} \epsilon_0 c A_{tot}^2$$

A_{tot} is maximized when phasors are all parallel: $A_{tot} = A$

$$I_{max} = \frac{1}{2} \epsilon_0 c A^2$$

$$\frac{I}{I_{max}} = \frac{A_{tot}^2}{A^2}$$

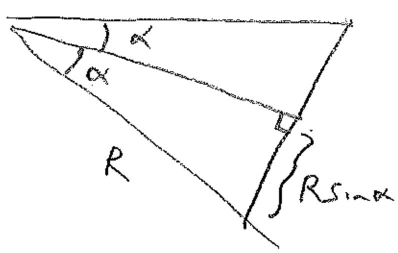
Now we need to find A_{tot} as a function of α



As $N \to \infty$, the phasors lie on an arc of a circle of radius R and angle 2α

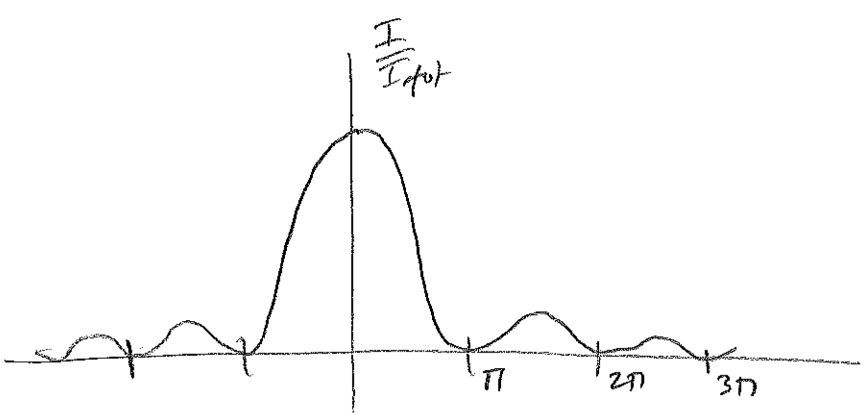
$$\text{arclength} = R \cdot 2\alpha = N \cdot \left(\frac{A}{N}\right) \Rightarrow R = \frac{A}{2\alpha}$$

A_{tot} is a chord on that circle



$$A_{tot} = 2R\sin\alpha = A\,\frac{\sin\alpha}{\alpha}$$

$$\Rightarrow \frac{I}{I_{tot}} = \left(\frac{A_{tot}}{A}\right)^2 = \left(\frac{\sin\alpha}{\alpha}\right)^2$$

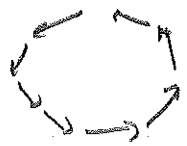


$I = 0$ if $\sin\alpha = 0 \Rightarrow \alpha = m\pi$ (m = nonzero integer)

If $\alpha = 0$ then $\frac{I}{I_{tot}} = \lim_{\alpha\to 0}\left(\frac{\sin\alpha}{\alpha}\right)^2 = 1$ (l'Hôpital)

[Recall: small $\alpha \Rightarrow \sin\alpha \approx \alpha$, $\frac{\sin\alpha}{\alpha} \approx 1$.]

Complete destructive interference occurs when phasors form a circle

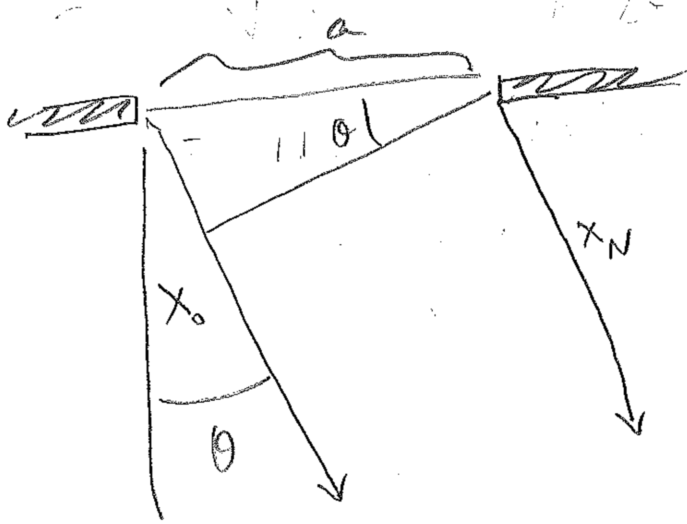


$m=1$

phase difference between last & first phasor is $2\pi m = 2\alpha \Rightarrow \alpha = m\pi$

$m=2$

How is α related to the angle of diffraction θ?

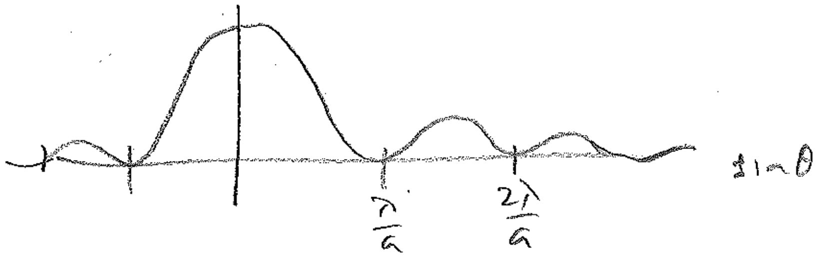


Phase difference between 1st & last phasor

$$2\alpha = k(x_0 - x_N) = \frac{2\pi}{\lambda} a \sin\theta \Rightarrow \alpha = \frac{\pi a \sin\theta}{\lambda}$$

Minima occur when $\alpha = m\pi \Rightarrow a \sin\theta = m\lambda$

m = nonzero integer



~~[optional] straight edge~~ ~~[see fig 211, 210 (Cornu) 213, 206 (razor blade) need monochromatic light]~~

circular aperture (diameter = a)

similar to single slit

but 1st minimum occurs at $\sin\theta = 1.22\frac{\lambda}{a}$

(not $\sin\theta = \frac{\lambda}{a}$)

area inside 1st minimum called

Airy disk

[OH: HR 46-13]

[optional] story of Fresnel + "Poisson spot"

[OH: 46-2]

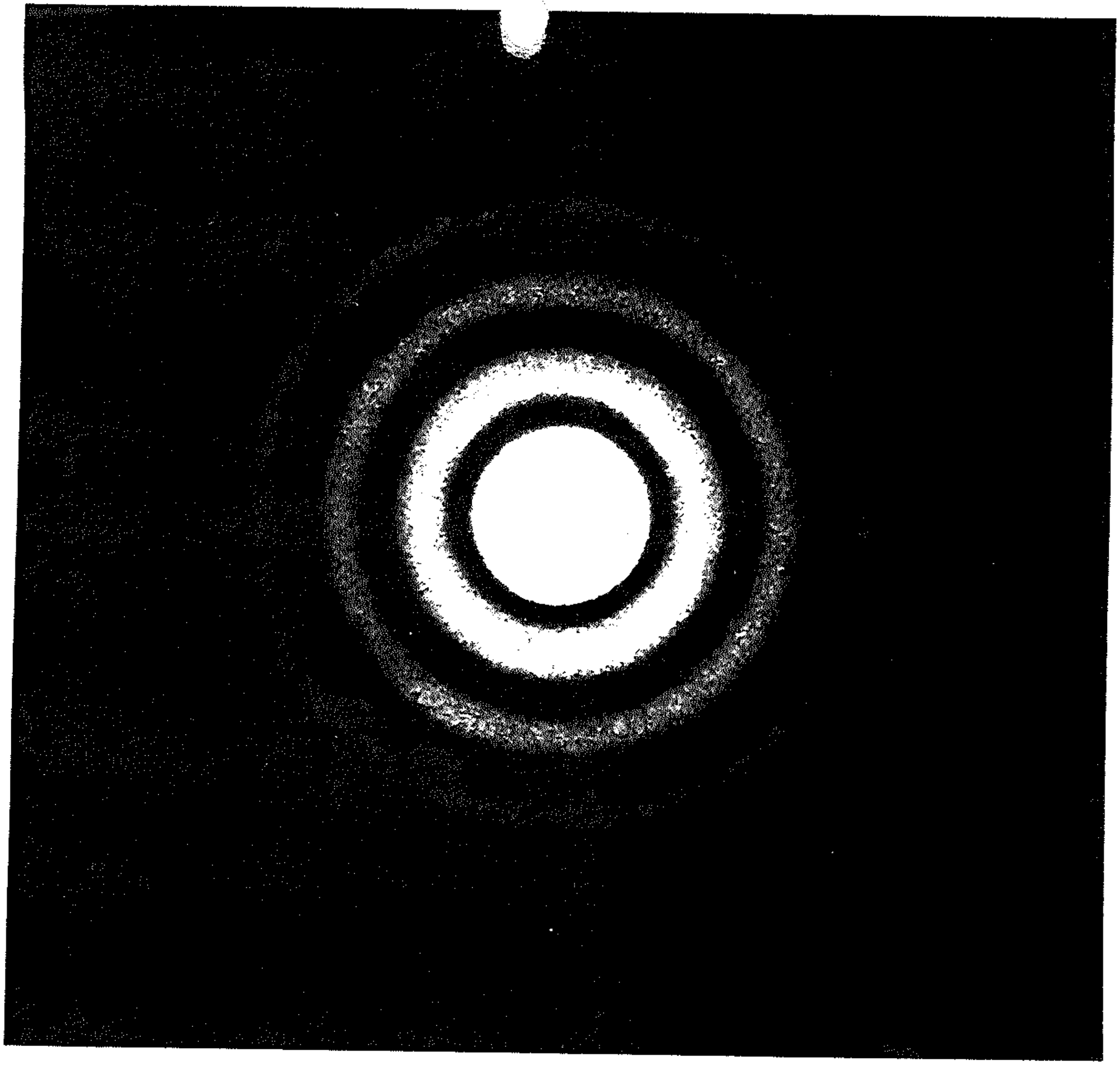


Figure 13 The diffraction pattern of a circular aperture. The central maximum is sometimes called the Airy disk (after Sir George Airy, who first solved the problem of diffraction by a circular aperture in 1835). Note the circular secondary maxima.

HR 46 - fig 13

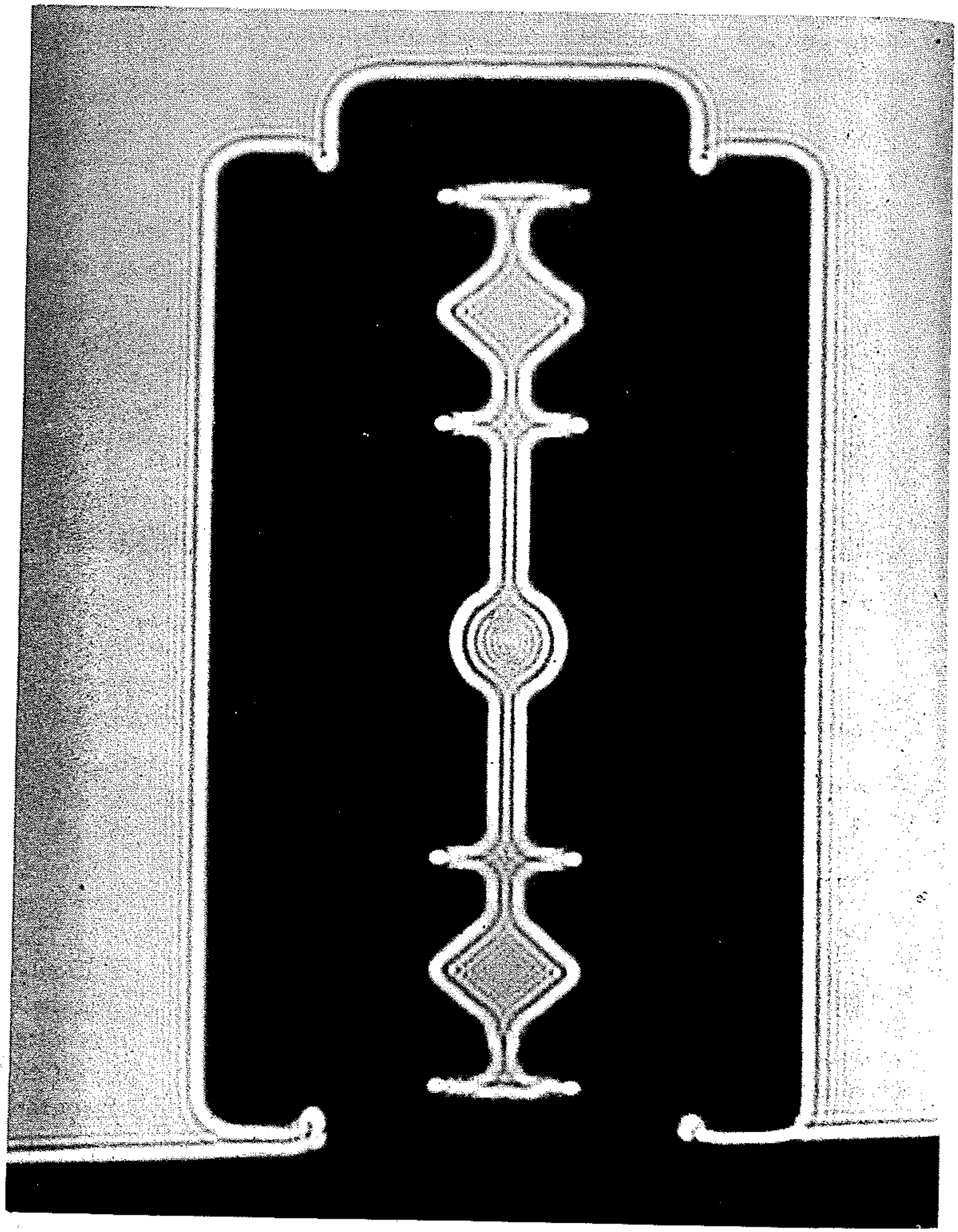


Figure 206. Diffraction pattern of a razor blade.

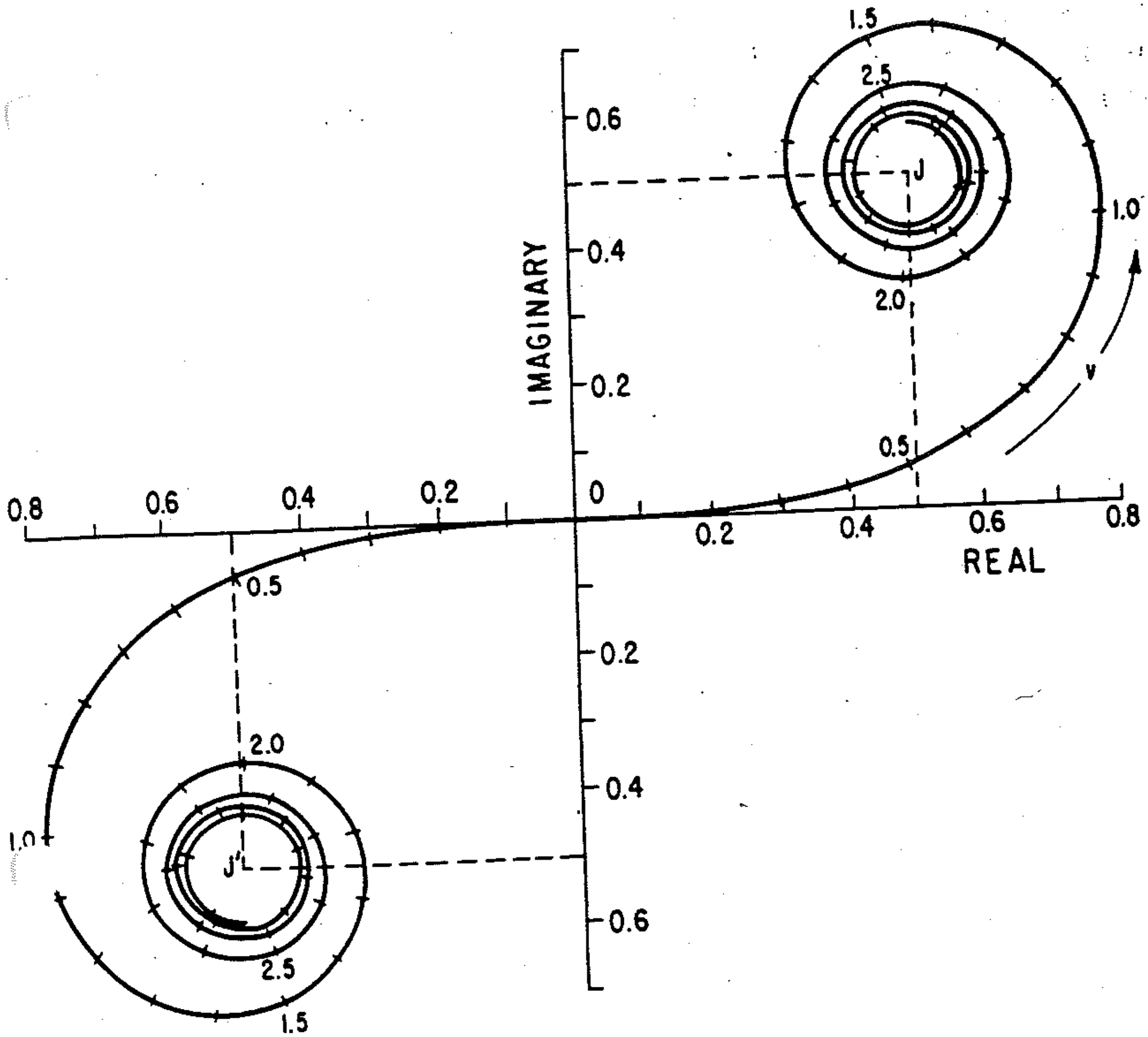


Figure 210. Cornu spiral.

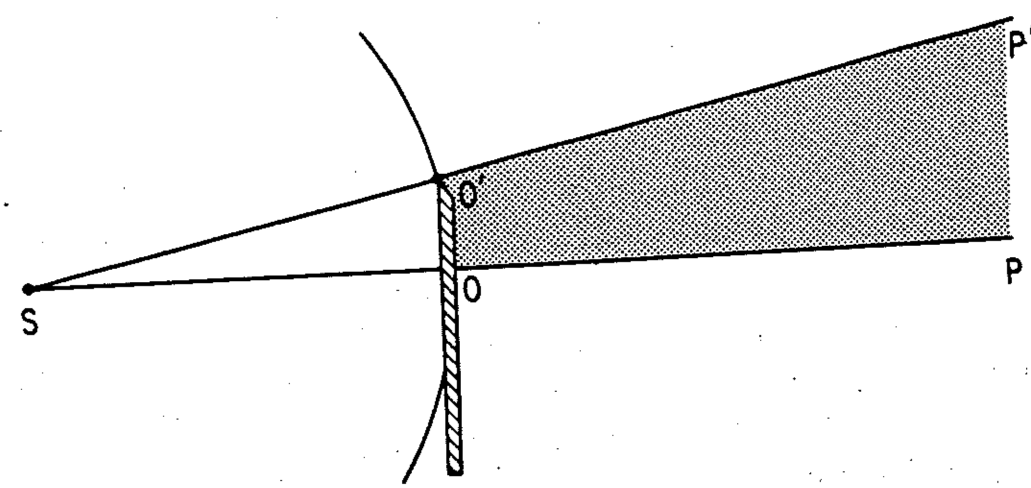


Figure 211. Determination of the diffraction pattern in the shadow of a straight edge.

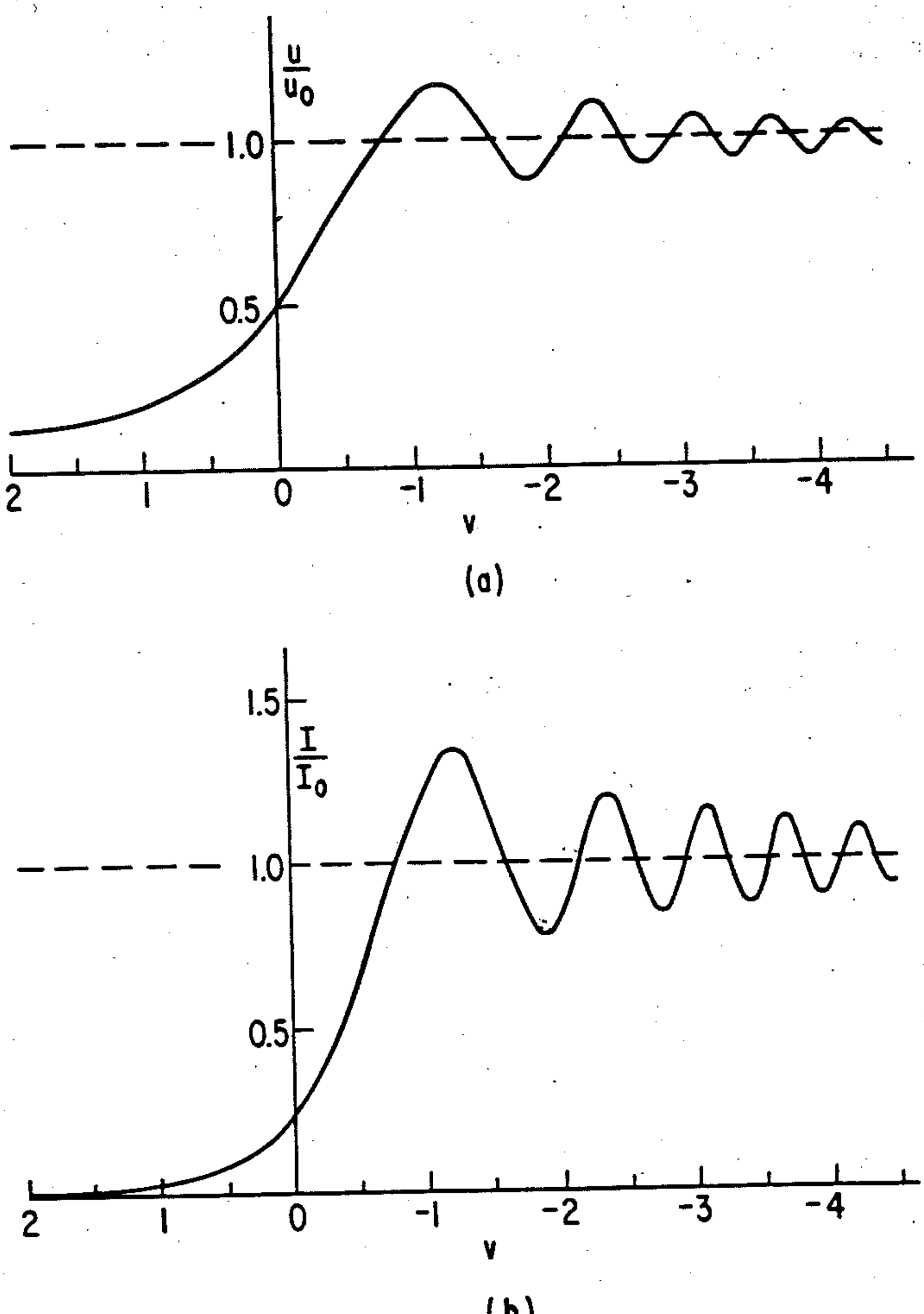


Figure 213. Plots of (a) amplitude and (b) intensity of the diffraction pattern of a straight edge in the far field.

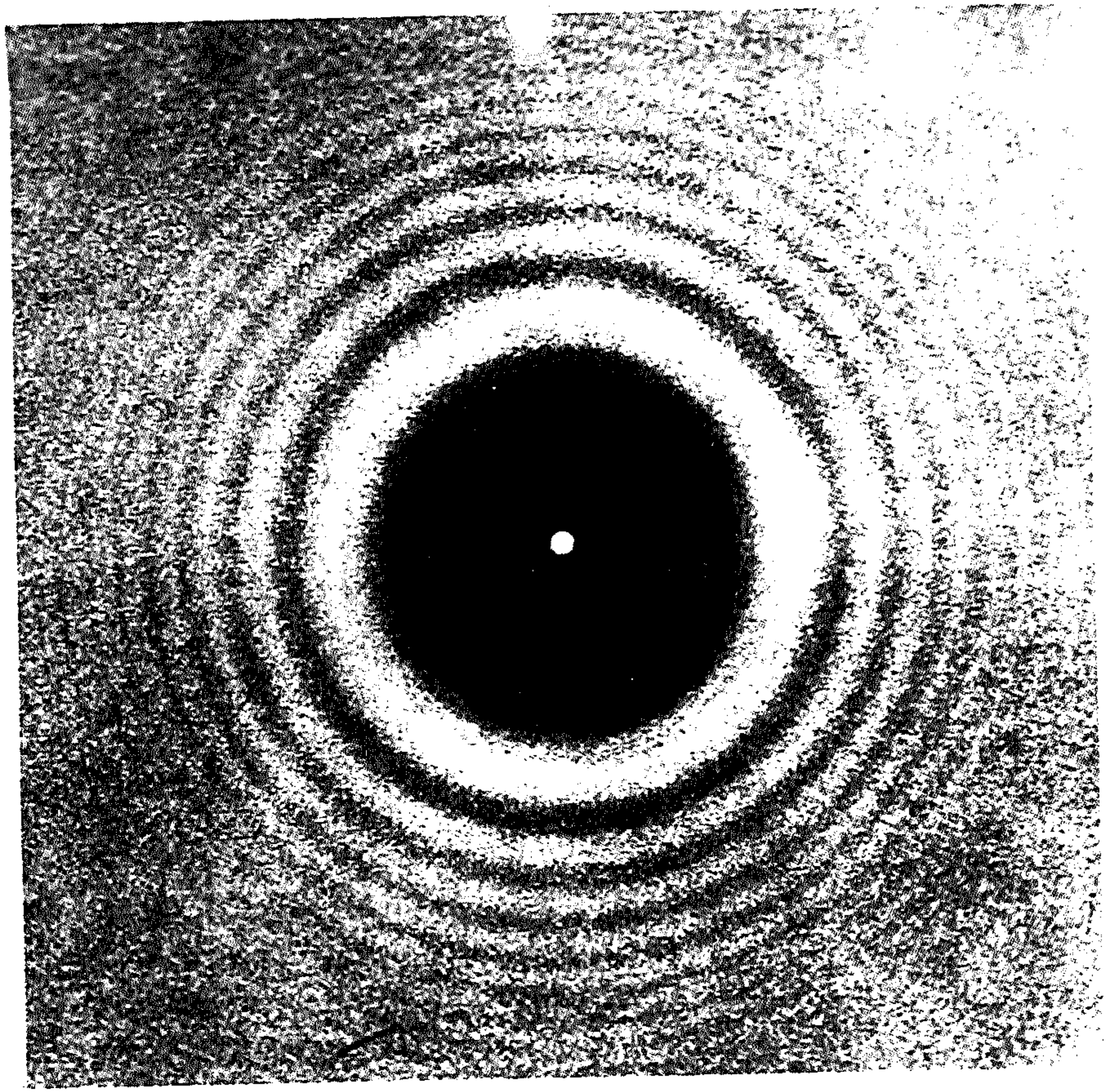
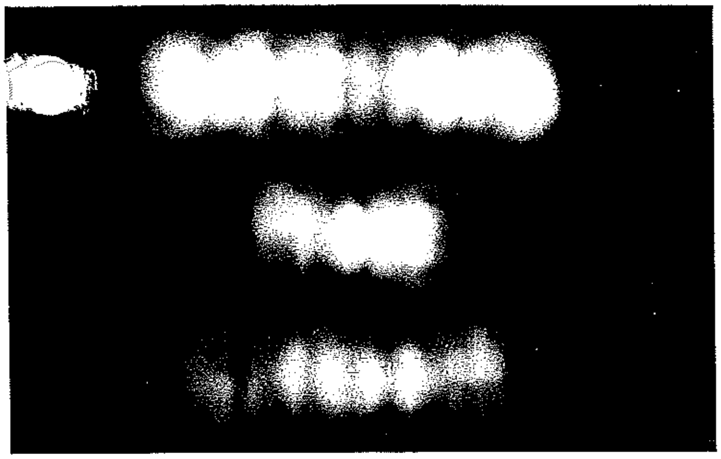
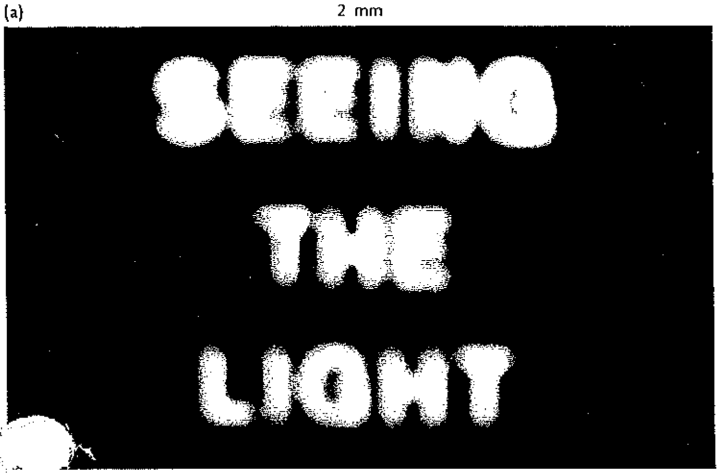


Figure 2 The diffraction pattern of a disk. Note the bright Poisson spot at the center of the pattern.

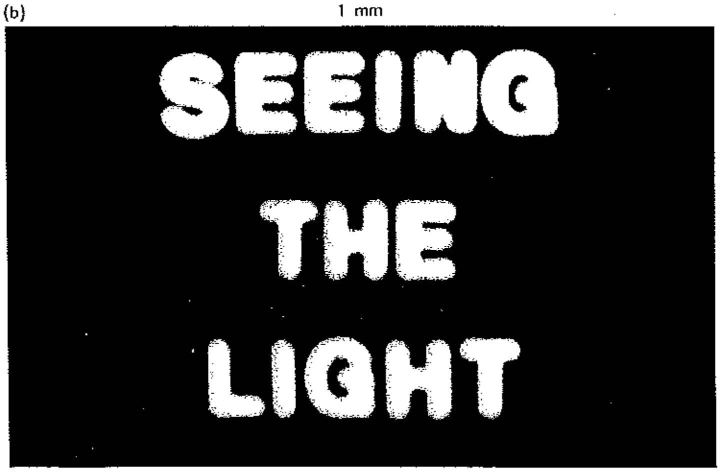
HR 46, fig 2



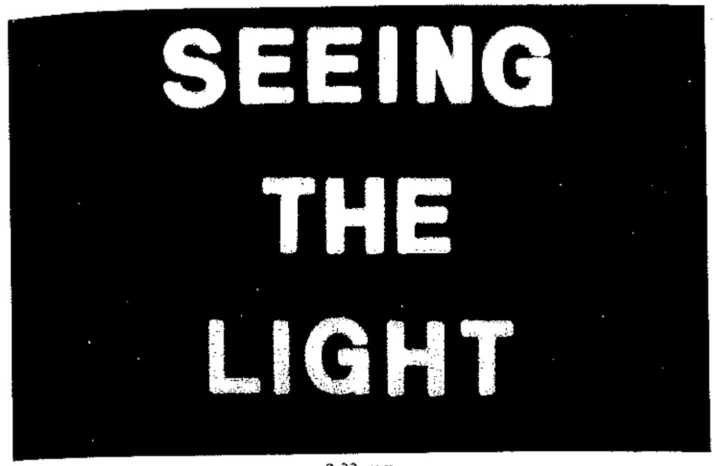
(a) 2 mm



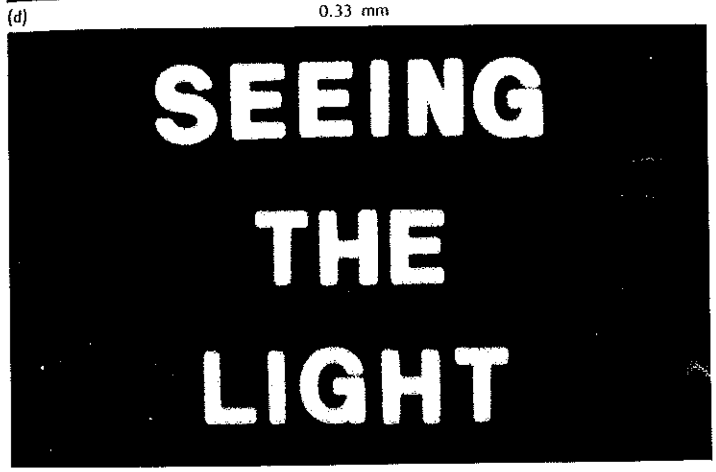
(b) 1 mm



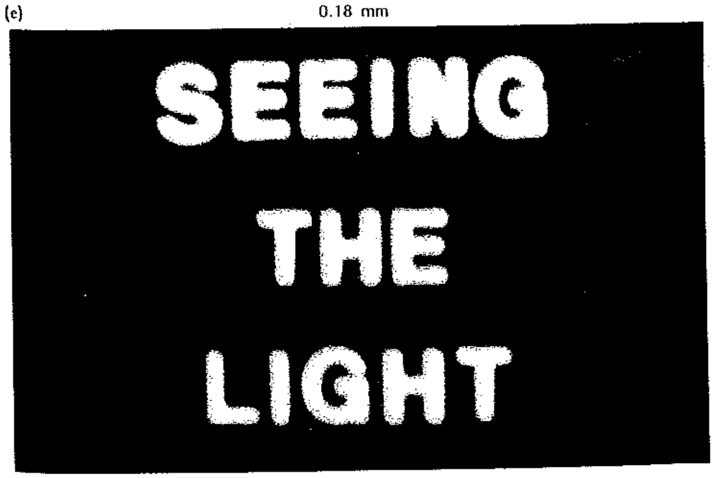
(c) 0.65 mm



(d) 0.33 mm



(e) 0.18 mm



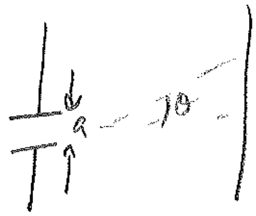
(f) 0.10 mm

QE9

(optional)

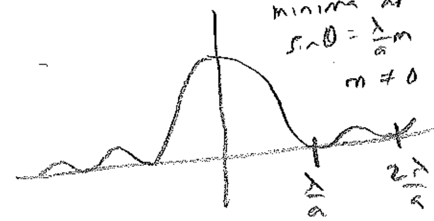
Cr 4.5

Single slit

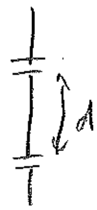


$$I = I_{max}\left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$\alpha = \frac{\pi a \sin\theta}{\lambda}$$

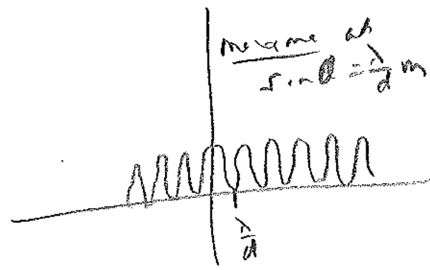


double slit

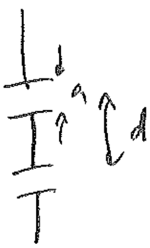


$$I = I_{max}(\cos\beta)^2$$

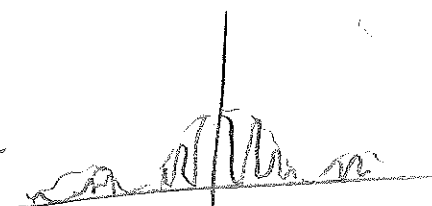
$$\beta = \frac{\pi d \sin\theta}{\lambda}$$



finite size double slit



$$I = I_{max}\left(\frac{\sin\alpha}{\alpha}\right)^2(\cos\beta)^2$$



(double slit pattern modulated by single slit pattern)