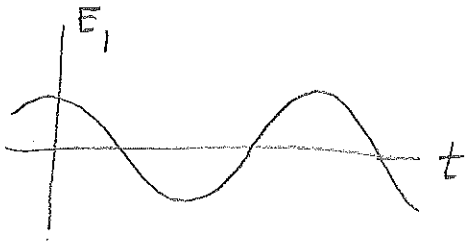
Interference caused by superposition of 2 or more waves

Consider an electromagnetic wave at a fixed point in space

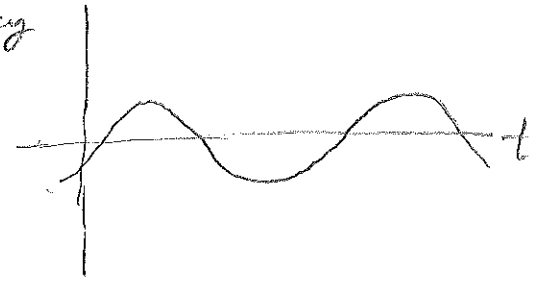
$E_1 = A_1 \sin(\omega t + \phi_1)$

ϕ_1 is called the phase of the wave



Consider a second wave w/ same frequency

$E_2 = A_2 \sin(\omega t + \phi_2)$



Superposition principle

$E_{tot} = E_1 + E_2$

[DEMO: show how to add waves using transparencies]

Interference depends on the difference of the phases.

Define phase difference $\Delta\phi = \phi_2 - \phi_1$

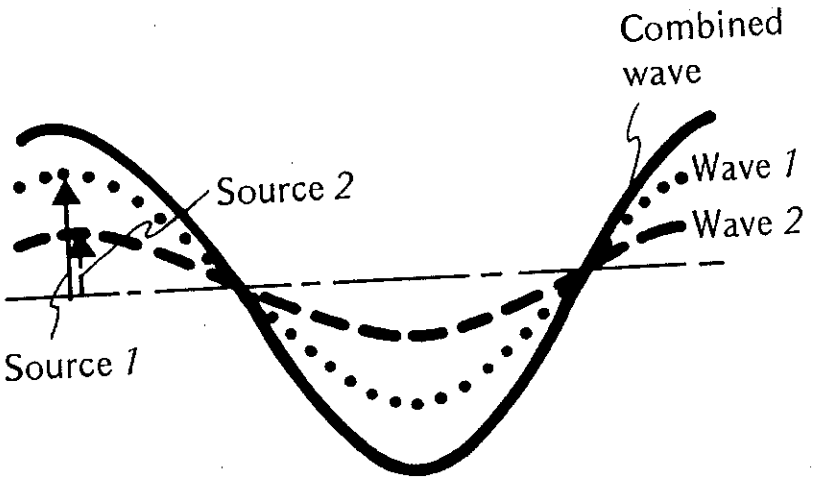
If $A_2 = A_1$ and $\phi_2 = \phi_1$, then $E_2 = E_1$ so $E_{tot} = 2E_1$

$\Delta\phi = 0$; waves are "in phase" and we have "constructive interference"

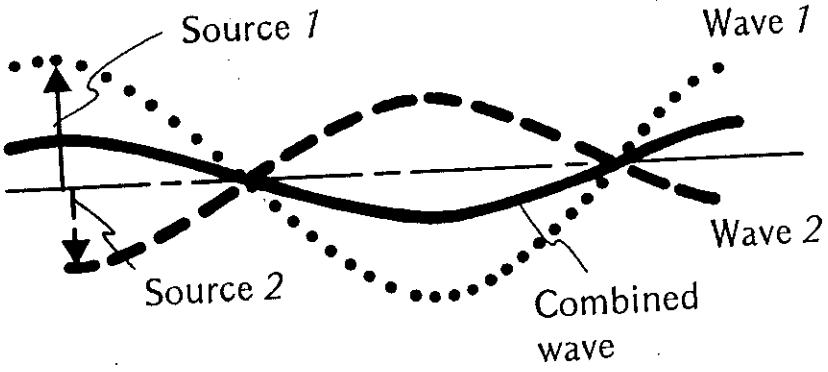
If $A_2 = A_1$ and $\phi_2 = \phi_1 + \pi$, then $E_2 = -E_1$ so $E_{tot} = 0$

$\Delta\phi = \pi$; waves are "180° out of phase" and we have "destructive interference"

[DEMO: show overhead w/ $A_2 \neq A_1$]


Combined
wave
Wave 1
Wave 2
Source 2
Source 1

(a)

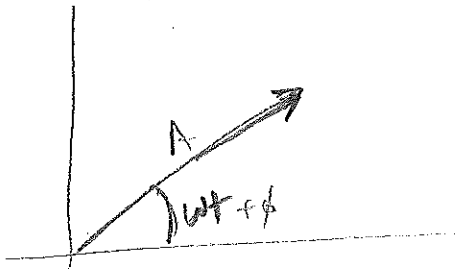

Source 1
Wave 1
Wave 2
Source 2
Combined
wave

(b)

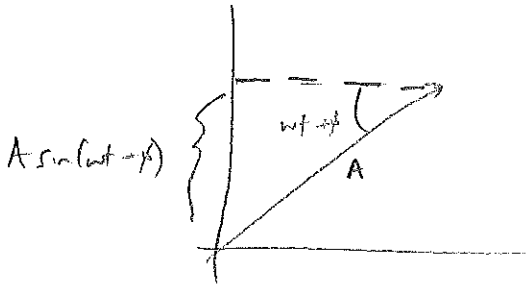
Phasor methods

Phasor: a graphical representation of an oscillating field

= a vector in a plane w/ magnitude A & angle $(\omega t + \phi)$



The projection of phasor onto the vertical axis gives the field

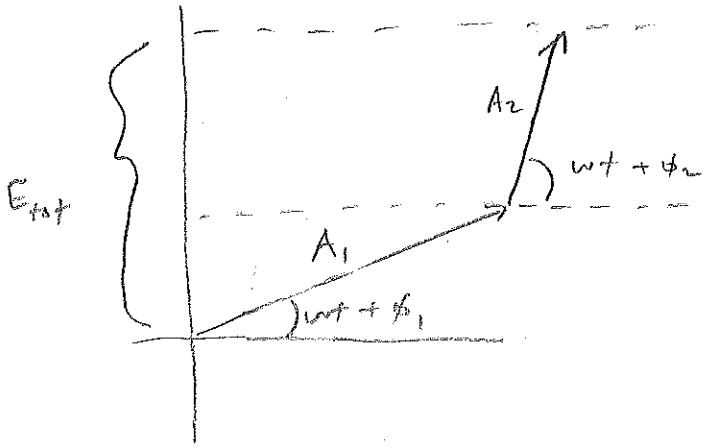


Phasor is rotating w/ const angular velocity ω

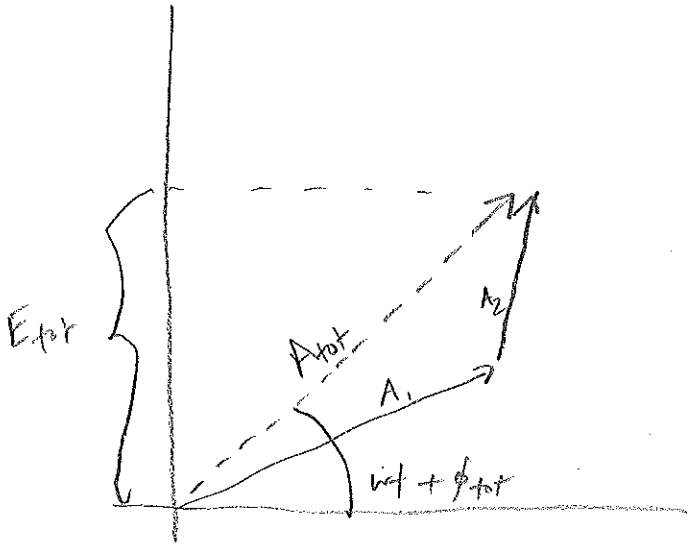
The superposition of two fields

$$E_{tot} = A_1 \sin(\omega t + \phi_1) + A_2 \sin(\omega t + \phi_2)$$

is represented by placing phasors head-to-tail



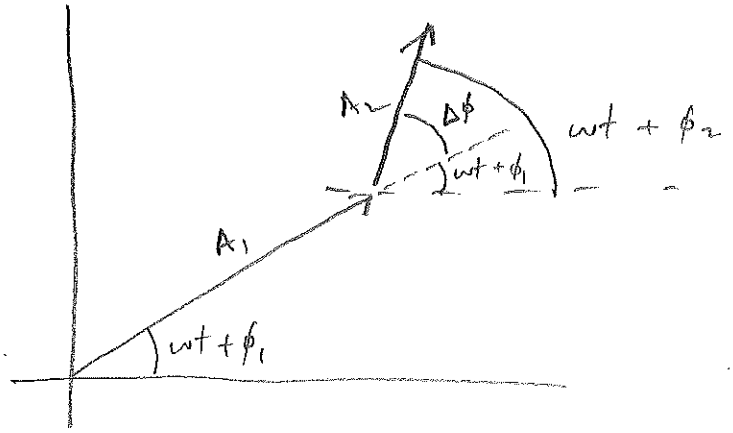
But this is the <u>same</u> as the field represented by the vector sum of the phasors



we need to find A_{tot} in terms of A_1, A_2, and $\Delta\phi$

Recall $\Delta\phi = \phi_2 - \phi_1$.

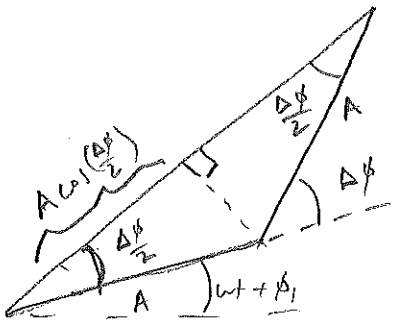
where is $\Delta\phi$ on the phasor diagram?



$\Delta\phi$ is the difference in angles between the phasors.

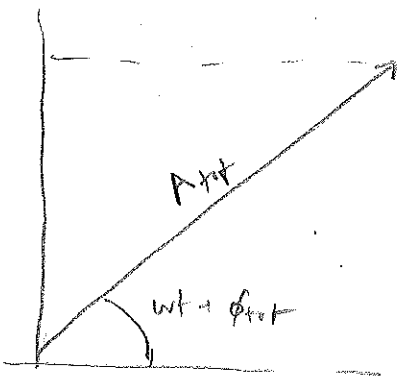
What are A_{tot} and ϕ_{tot}?

For simplicity, let $A_2 = A_1$ $\Rightarrow$ phasors form an isosceles triangle



$$A_{tot} = 2A\cos\left(\frac{\Delta\phi}{2}\right)$$

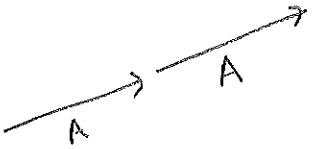
Also $\phi_{tot} = \phi_1 + \frac{\Delta\phi}{2}$



$$E_{tot} = A_{tot}\sin(\omega t + \phi_{tot})$$

$$A_{tot} = 2A \cos\left(\frac{\Delta\phi}{2}\right)$$

Constructive interference occurs when phasors are parallel

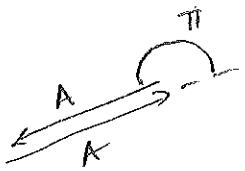


$\Delta\phi = 0$ (or a multiple of 2π)

$\Delta\phi = 2\pi m$ where m = integer

$A_{tot} = 2A$

Complete destructive interference occurs when phasors add up to zero



$\Delta\phi = \pi$ ($\pi + 2\pi m$)

$\Delta\phi = 2\pi\left(m + \frac{1}{2}\right)$

$A_{tot} = 2A\cos\left(\frac{\pi}{2}\right) = 0$

Partial interference occurs between $\Delta\phi = 0$ and π