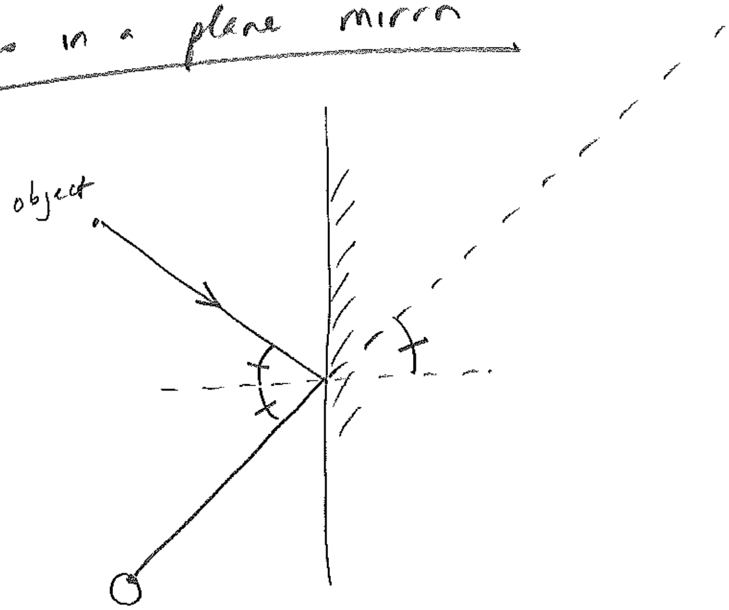
Images in a plane mirror



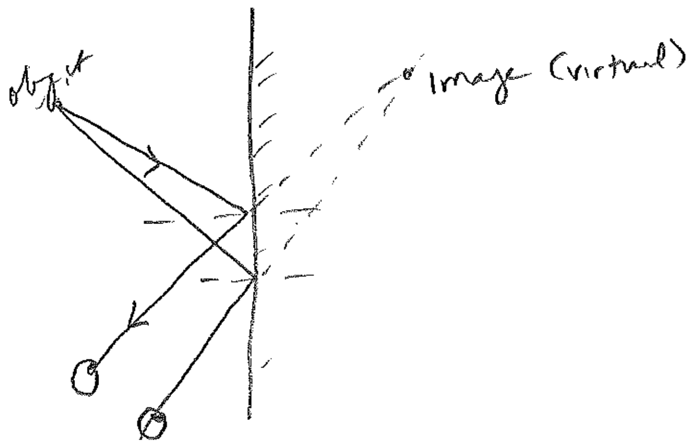
[where is image located?

object appears to be in direction of dashed line, but how far out?

ie how determine distance?

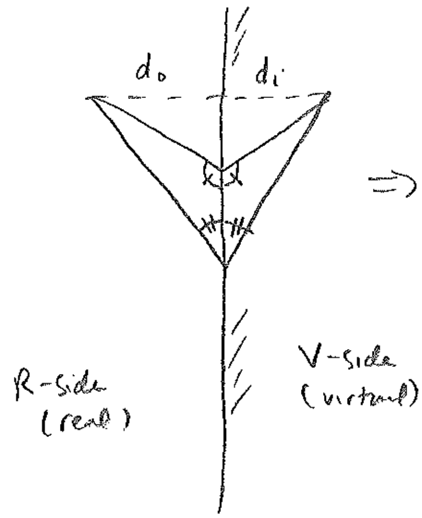
Binocular vision]

also parallax



The image is virtual because light rays appear to come from that point, but no rays actually pass through it.

[brick wall back there]



$\Rightarrow$ congruent triangles

$\Rightarrow |d_o| = |d_i|$

$|d_o|$ = distance from object to mirror surface

$|d_i|$ = distance from image to mirror surface

Sign conventions:

distances on R-side are positive, so $d_o > 0$

distances on V-side are negative, so $d_i < 0$

$$d_o = |d_o| = |d_i| = -d_i$$

$\boxed{d_o = -d_i}$ for a plane mirror

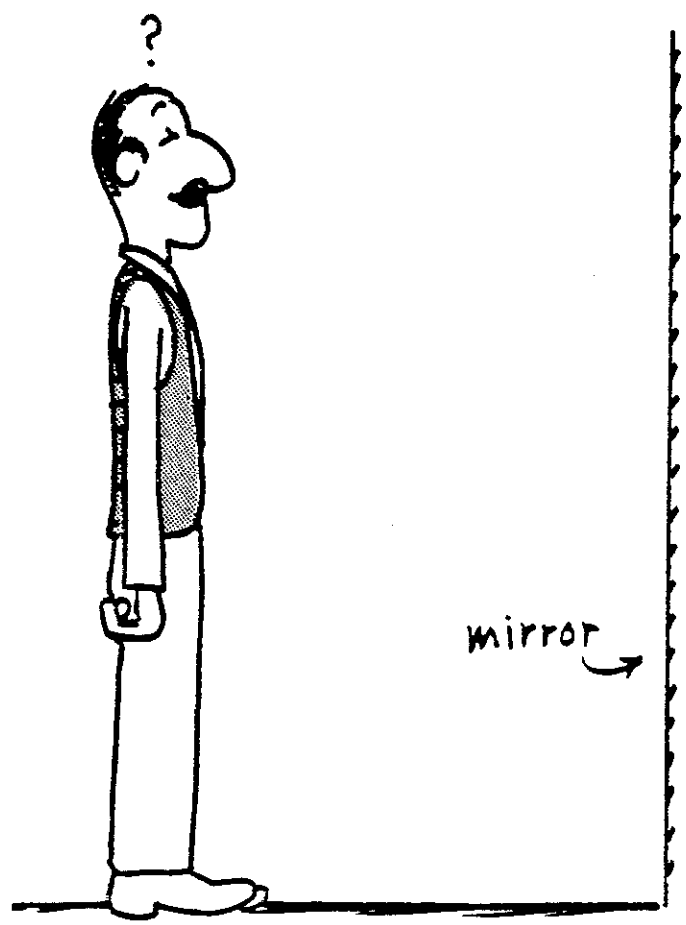
PLANE MIRROR

skipped in F'06 for time

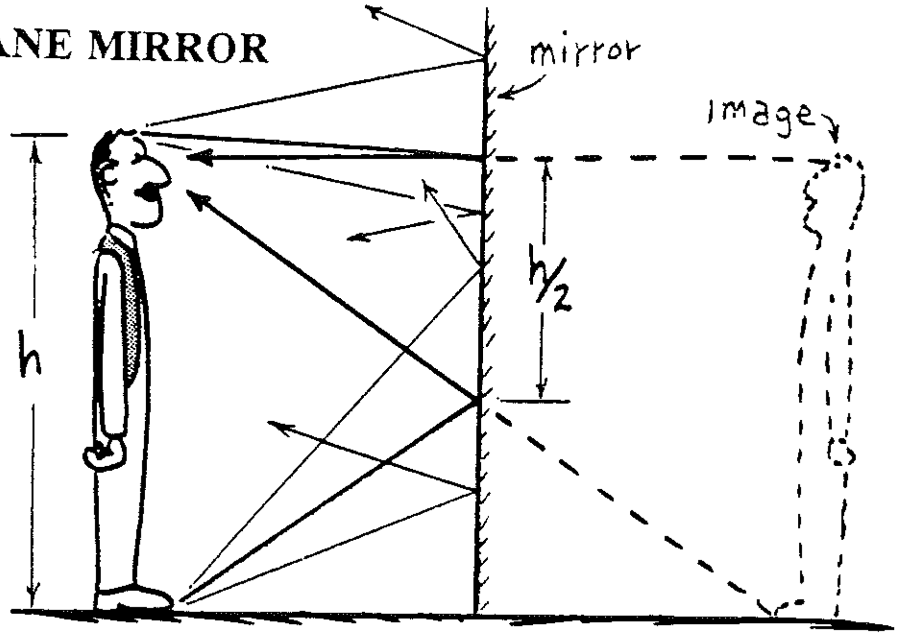
N2.5

What must be the minimum length of a plane mirror in order for you to see a full view of yourself?

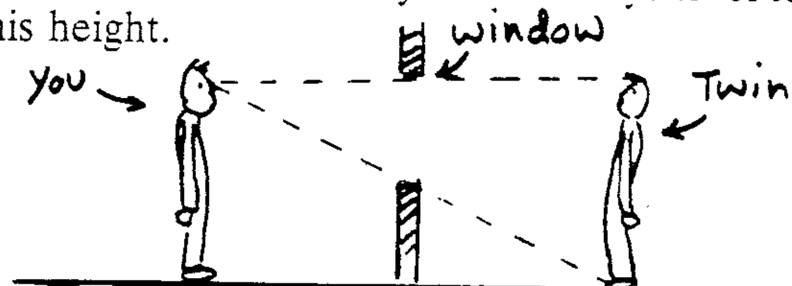
a) One-quarter your height
b) One-half your height
c) Three-quarters your height
d) Your full height
e) The answer depends on your distance.

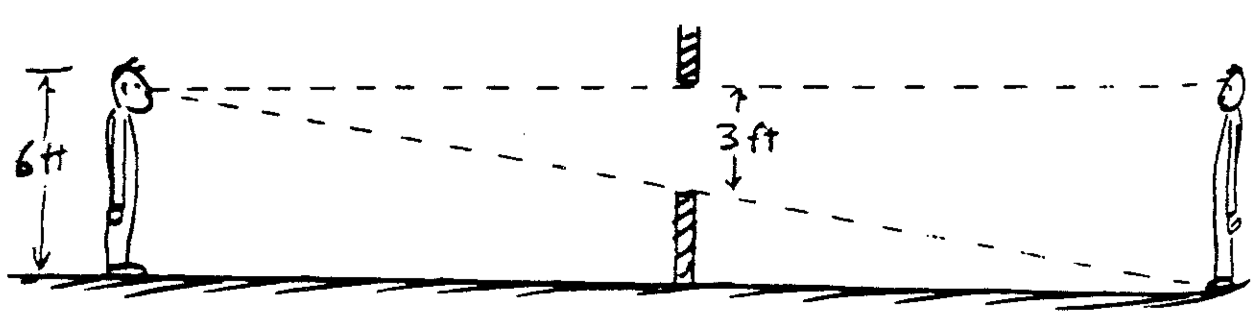


ANSWER: PLANE MIRROR



The answer is: b. Exactly one-half your height. Why? Because for reflection, the angle of incidence is equal to the angle of reflection. Consider a man standing in front of a very large mirror as shown in the sketch. The only rays of light from his shoes to encounter his eyes are those that are incident upon the mirror at a level halfway up from ground level to eye level. Rays of "shoe light" that are incident higher reflect above his eyes and rays incident lower reflect below his eyes. So the part of the mirror below the halfway level is not needed — it shows only a reflection of the floor in front of his feet. Similarly for the top part of the mirror. The only rays to reach his eyes from the top of his head are those that are incident upon the mirror halfway between the top of his head and his eye level. The part of the mirror above this is not needed. So the portion of the mirror useful for seeing his image lies halfway above his eye level to the top of his head, to halfway below his eye level to his toes — that's half his height.





Mirrors act like windows to a world behind them. Everything in mirrorland is the mirror image of this land. The sketch shows that to see your image in mirrorland the window need be only one half your height—regardless of how near or how far you stand from the window.

The next time you view yourself in a mirror, mark it where you see the top of your head and where you see the bottom of your chin. Note that the distance between the pair of marks is half your face size and that as you move closer to or farther from the mirror the image of your face still fills the space between the marks.

[HRK 5e, 40]

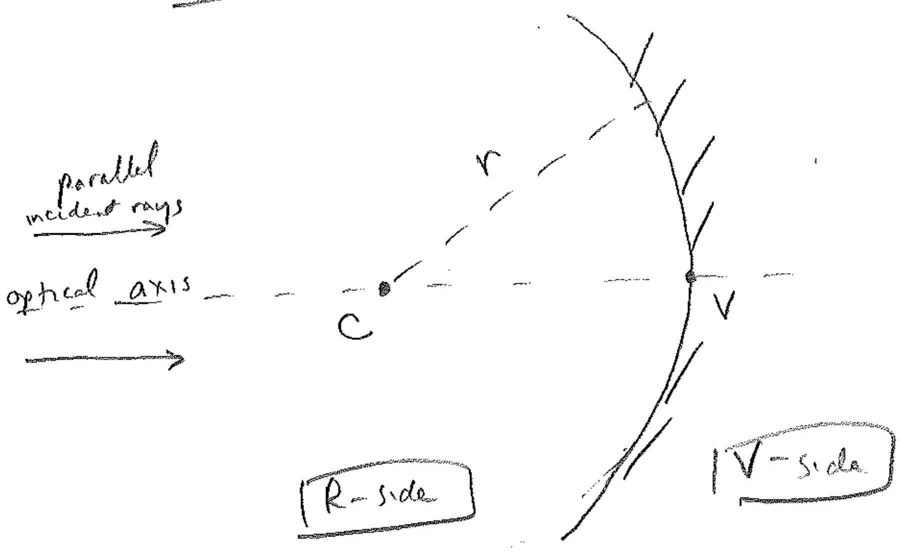
Spherical mirrors

part. of a mirror: show concave

sometimes a whole one: show convex

2 in 1: show soup spoon

Concave



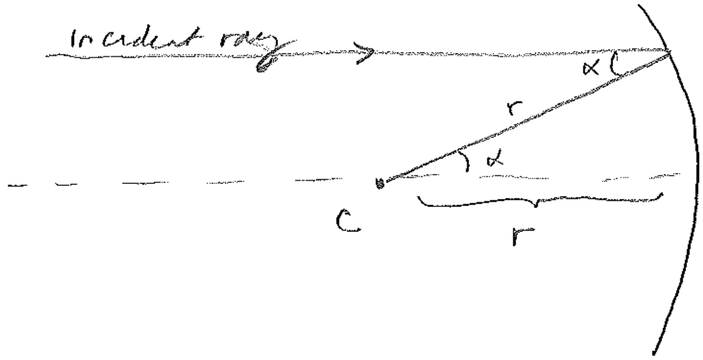
C = center of curvature

axis = line thru C parallel to incident rays

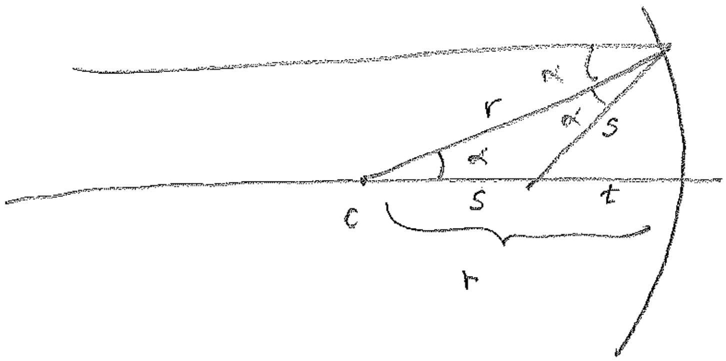
V = vertex = where axis intersects mirror

$r = CV$ = radius of curvature

$r > 0$ because C is on R-side



[alternate interior]



[law of reflection]

[isosceles triangle]

$t + s = r$

We define paraxial rays as parallel rays close to the optical axis

$\Rightarrow \alpha$ is a small angle $\Rightarrow$ $r \approx 2s$

$s \approx \frac{1}{2} r$

$t = r - s \approx \frac{1}{2} r$

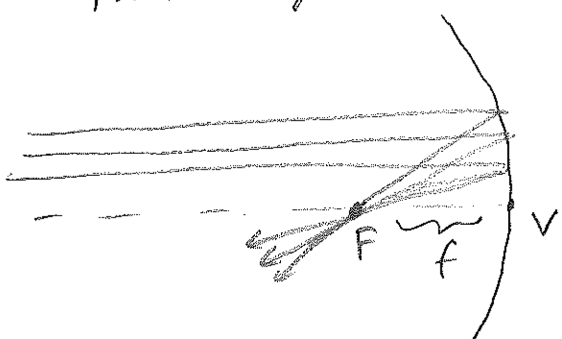
[Alternatively

$\alpha = \frac{mv}{CV} = \frac{mv}{r}$

$2\alpha = \frac{mv}{FV} = \frac{mv}{f}$

$f = \frac{r}{2}$]

Paraxial rays all reflect to (& through) a focal point F (a.k.a focus)

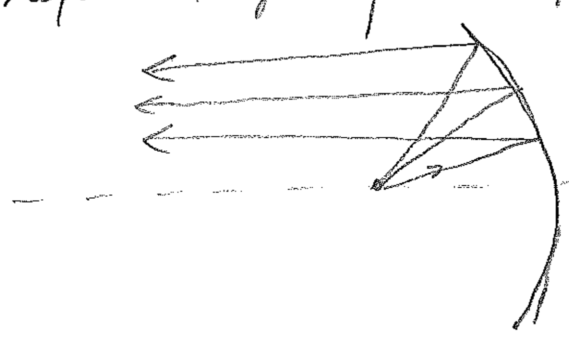


focal length $f = FV$

For a concave mirror

$f = \frac{1}{2} r$

Principle of reversibility: rays coming from (or through F) reflect to give parallel rays



[auto head lights]

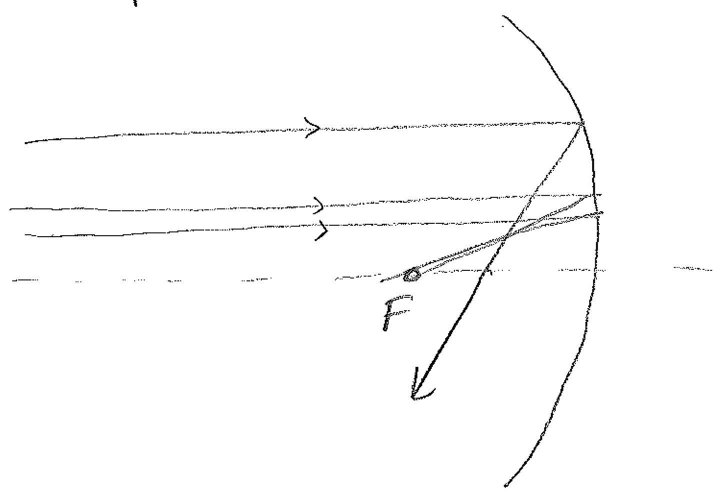
[What about rays not so close to axis?]

α not small $\Rightarrow$ $r < 2s$

$s > \frac{1}{2}r$

$t < \frac{1}{2}r$

Nonparaxial rays reflect through a point between F + V



Fact that not all rays reflect to F is called <u>spherical aberration</u>.

[How cure spherical aberration?
Don't use a sphere. Parabola reflects all rays to focus.

TV satellite dish.]

[Demo: auto reflector over candle, see light on ceiling]

↳ follow this up demo w/ mirror #38 causing parallel rays from candle to come to a focus; + then show formation of an image

[DEMO: Candle, concave mirror #38 ← don't use, too heavy!
large grid glass. ($r \approx 2.5$m, $f \approx 1.25$ m)

Have student stand w/ candle far away.
→ I find focus (≈ 1.25 m) in grid glass
Paraxial rays focus to a point.
Student moves in, image moves out
Begin to see image is inverted
Image gets larger & is eventually on the wall.

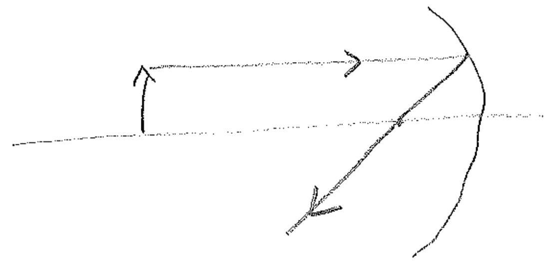
[Ask for conclusions]

- Real image formed by a concave mirror is inverted
- As $d_o \downarrow$, $d_i \uparrow$
- Size $\uparrow$ as $d_i \uparrow$

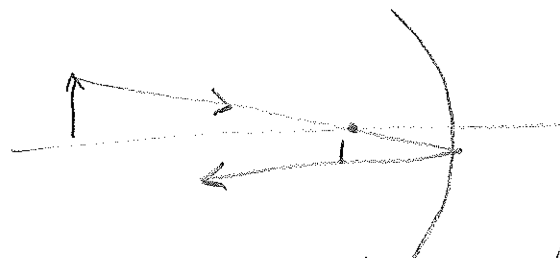
[Next time when they come to class, use the hidden bulb in a box]

Rules of ray tracing

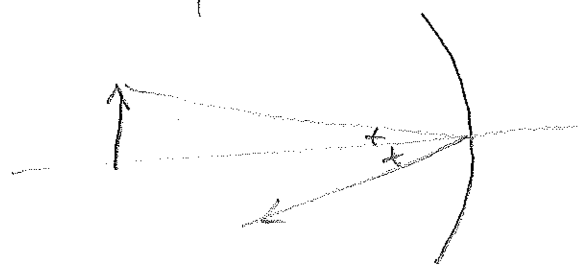
1) a ray parallel to axis is reflected to pass thru F



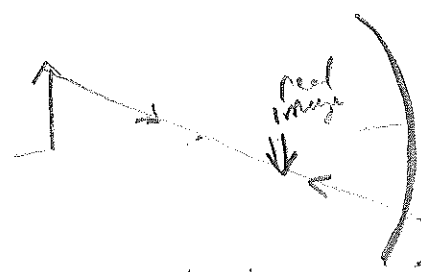
2) a ray passing thru F is reflected parallel to axis



3) a ray striking V is reflected at an equal angle across the axis

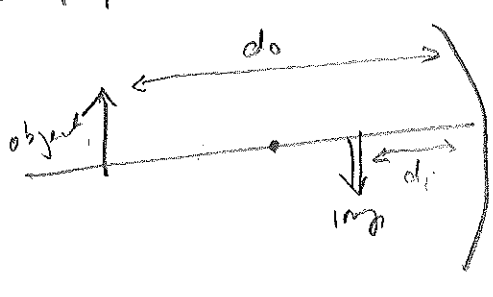


4) a ray passing thru C is reflected to return thru C



[These rays + all others pass thru a single pt. ⇒ image]

Real image: rays appear to come from the real image + ~~actually pass through it~~ [one can put one's hand thru it]



d_o = distance from object to mirror

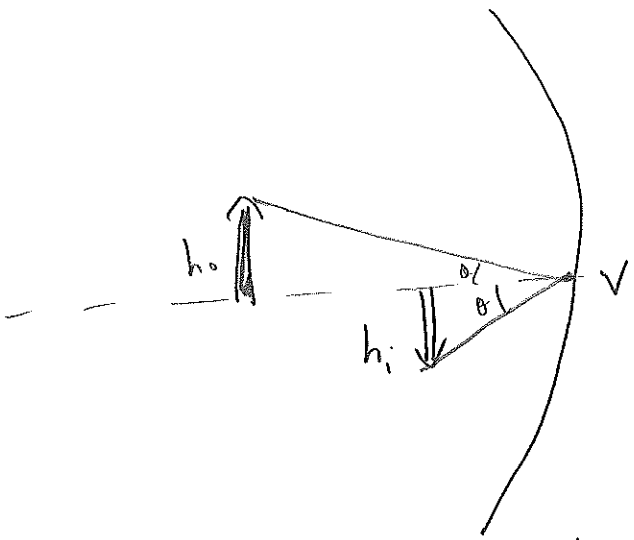
d_i = distance from image to mirror

$d_o, d_i > 0$ (Both on R-side)

Magnification

m = (lateral) magnification = $\dfrac{\text{transverse size of image}}{\text{transverse size of object}} = \pm \dfrac{h_i}{h_o}$

$m > 0$ if image is upright (same as object)
$m < 0$ if image is inverted

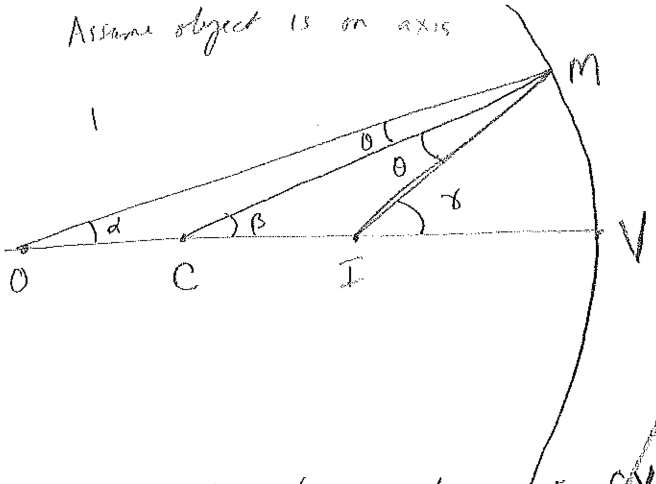


Similar triangles: $\dfrac{h_i}{h_o} = \dfrac{d_i}{d_o}$

Image inverted $\Rightarrow$ $m = -\dfrac{h_i}{h_o} = -\dfrac{d_i}{d_o}$

Plane mirror: $d_i = -d_o$ so $m = +1$ [same size, upright]

Assume object is on axis



N8 replaced by new N9

r = radius of curvature $= CV$
d_o = distance from object to mirror $= OV$
d_i = distance from image to mirror $= IV$

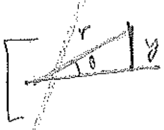
replaced

Both on R-side: $d_o > 0$
$d_i > 0$

Geometry $\Rightarrow$ $\alpha + \theta = \beta$
$\beta + \theta = \gamma$

Subtract: $\alpha - \beta = \beta - \gamma$
$\alpha + \gamma = 2\beta$

Recall  s = arclength $= r\theta \Rightarrow \theta = \frac{s}{r}$

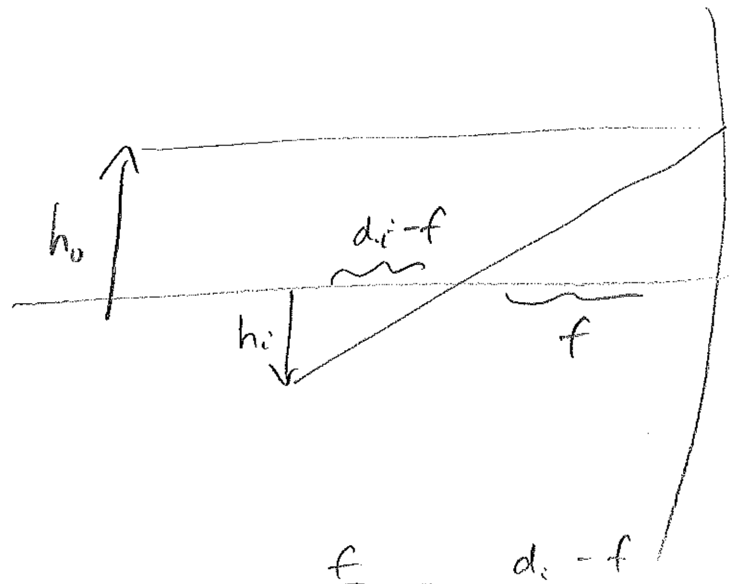
[ $\sin\theta = \frac{y}{r}$
For small θ, $y \approx s$, ie $\sin\theta \approx \theta$] ← skip until later

assume small angles
$$\beta = \frac{MV}{CV} = \frac{MV}{r} \quad \text{(if angles not small, spherical aberration)}$$
$$\alpha \approx \frac{MV}{OV} = \frac{MV}{d_o}$$
$$\gamma \approx \frac{MV}{IV} = \frac{MV}{d_i}$$

So $\alpha + \gamma = 2\beta \Rightarrow \frac{1}{d_o} + \frac{1}{d_i} = \frac{2}{r}$ (mirror eqn)

Since $f = \frac{r}{2} \Rightarrow \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$

Plane mirror: $r = \infty \Rightarrow \frac{1}{d_o} + \frac{1}{d_i} = 0 \Rightarrow d_i = -d_o \Rightarrow$ virtual because $d_i < 0$



Similar Δ: $\frac{f}{h_o} = \frac{d_i - f}{h_i}$

but $\frac{h_o}{d_o} = \frac{h_i}{d_i}$

Multiply $\frac{f}{d_o} = \frac{d_i - f}{d_i} = 1 - \frac{f}{d_i}$

$\frac{f}{d_o} + \frac{f}{d_i} = 1$

$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$

For mirrors $f = \frac{r}{2}$ so

$\frac{1}{d_o} + \frac{1}{d_i} = \frac{2}{r}$ (mirror eqn)

Plane mirror: $r = \infty \Rightarrow \frac{1}{d_i} + \frac{1}{d_o} = 0$

$\Rightarrow d_i = -d_o$ virtual because $d_i < 0$

$$\frac{1}{d_i}+\frac{1}{d_o}=\frac{1}{f}$$

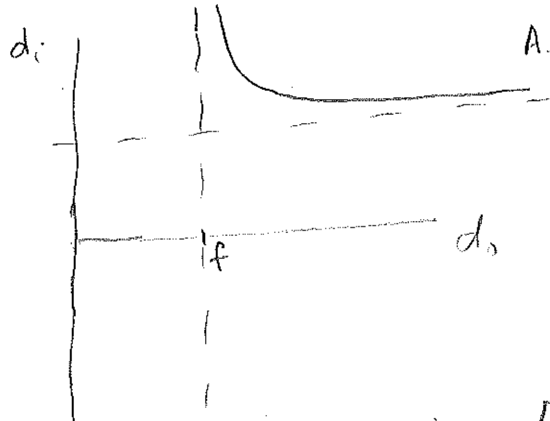
$$\frac{1}{d_i}=\frac{1}{f}-\frac{1}{d_o}=\frac{d_o-f}{f d_o}$$

$$d_i=\frac{d_o f}{d_o-f}$$

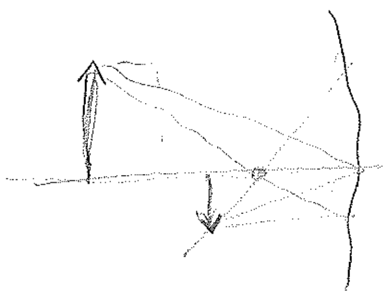
⇒ if $d_o=\infty$ (parallel rays), $d_i=f$

if $d_o=f$ (at focus), $d_i=\infty$

As $d_o \downarrow$, $d_i \uparrow$



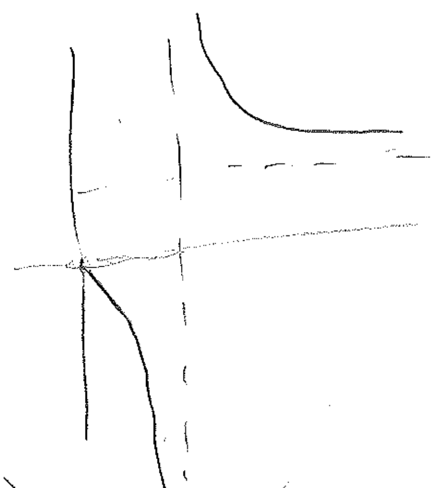
(agrees w/ observation)



R side

[what about $d_o<f$?]

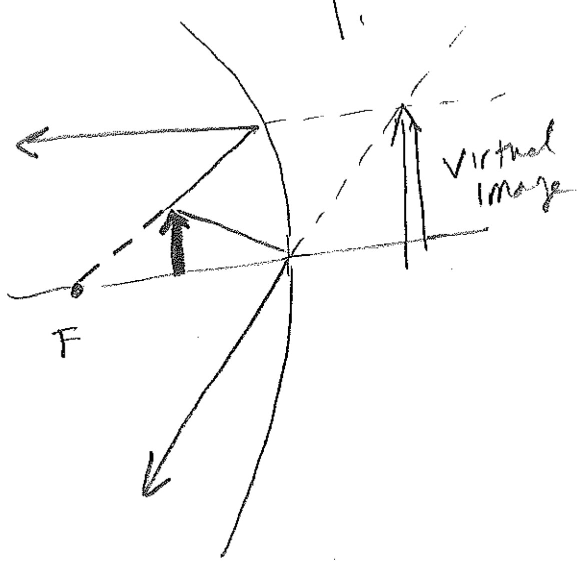
$d_o=0 \Rightarrow d_i=0$



If $0<d_o<f$

then $d_i<0$

ie Image is on V-side



$m=-\frac{d_i}{d_o}>0$

upright image

[NB convex mirror done in problem set]