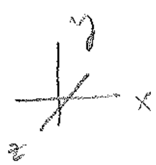
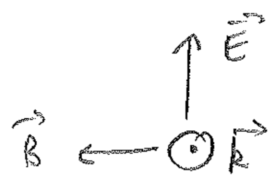
Polarized light

An electromagnetic wave in which the $\vec{E}$ field is confined to one plane and the $\vec{B}$ field to a perpendicular plane is called "plane polarized light"

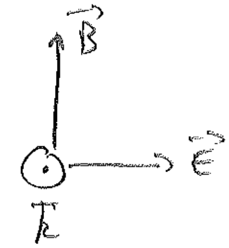
Plane of polarization = plane spanned by $\vec{E}$ and $\vec{k}$

[Consider light coming toward you]
+z direction

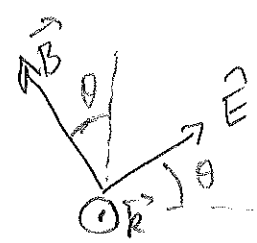




polarized in yz plane



polarized in xz plane



← also plane polarized

"ordinary" light is unpolarized
[random mixture of polarizations]

A polarizing filter (ideally) absorbs the component of $\vec{E}$ field in one direction and transmits the rest.

[DEMO: big filter against white screen:
It's grey because only lets through 50%

[HAND OUT POLARIZERS]

[Look through polarizer at the big one.]

[Look around room. Look at calculator.
If rotating makes a difference, then you are looking at partially polarized light.
E.g. glossy reflection from wooden beams.]

[CHI sign]

[HRK 5e, 38-6]

Energy carried by an EM wave is characterized by

$$\boxed{\text{Poynting vector} \quad \vec{S} \equiv \frac{1}{\mu_0} \vec{E} \times \vec{B}}$$

Since (wave speed)^2 is $\frac{1}{\mu_0 \epsilon_0} = c^2$, this becomes $\vec{S} = \epsilon_0 c^2 \vec{E} \times \vec{B}$

Direction of $\vec{S}$ is same as direction of wave travel $\vec{k}$.

Magnitude of S measures the power per unit area carried by the wave
where power = energy / time

Recall magnitude of cross product $|\vec{u} \times \vec{w}| = |\vec{u}||\vec{w}| \sin\theta$
θ = angle between $\vec{u}$ + $\vec{w}$

$|\vec{S}| = \epsilon_0 c^2 |\vec{E}| |\vec{B}| \sin(90°)$ (Since $\vec{E} \perp \vec{B}$)

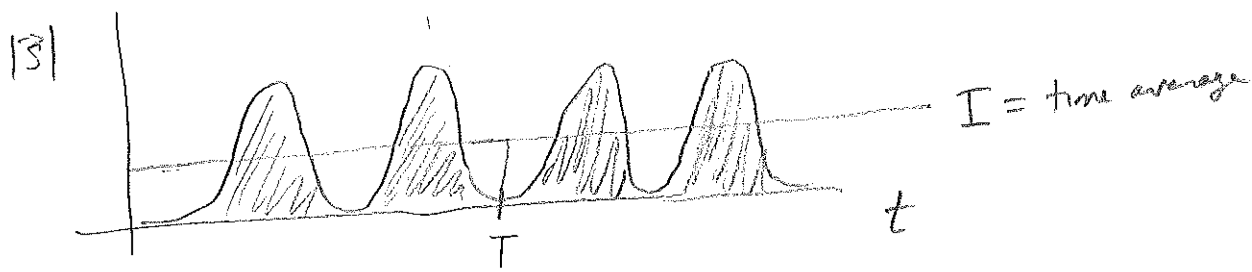
Maxwell's equations predict that an EM wave obeys $|\vec{B}| = \frac{|\vec{E}|}{c}$

$|\vec{S}| = \epsilon_0 c |\vec{E}|^2$

energy carried by EM wave is prop. to $|\vec{E}|^2$

Because $\vec{E}$ oscillates rapidly in time, so does $|\vec{S}|$.

A more useful quantity is the <u>intensity I</u> of the EM wave, defined as the <u>time average</u> of $|\vec{S}|$.



To compute I, we integrate the area under the $|\vec{S}|$ curve over one period T and then divide by T

$$I = \frac{1}{T}\int_0^T |\vec{S}|\, dt$$

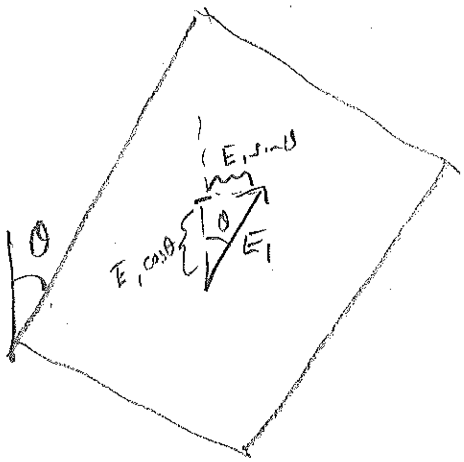
If $|E| = A\sin(kx - \omega t)$ then $I = \frac{\epsilon_0 c}{2} A^2$

H5

Suppose 2 polarizers are placed behind one another, w/a relative orientation of θ.

What fraction of light gets through the second one? (Light is travelling toward you.)

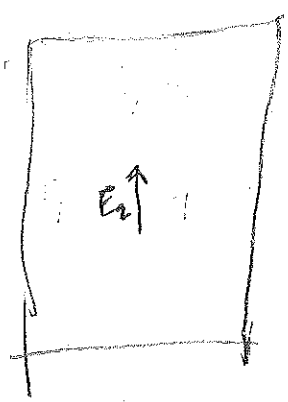
first filter



Let $E_1 = |\vec{E}_1|$ be the magnitude of the electric field of the wave emerging from the 1st filter

$$I_1 \propto |\vec{E}_1|^2 = E_1^2$$

second filter



Second filter absorbs the horizontal component and transmits the vertical.

$$E_2 = E_1 \cos\theta$$

$$I_2 \propto |\vec{E}_2|^2 = E_1^2 \cos^2\theta$$

ratio of intensities $\dfrac{I_2}{I_1} = \cos^2\theta$ (Malus's law)

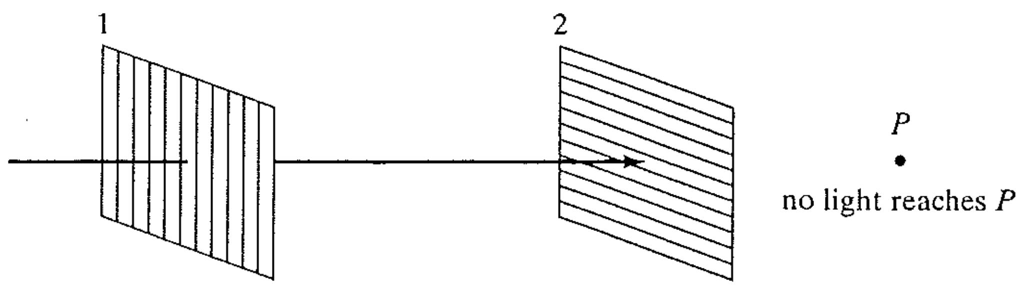
$\theta = 0 \Rightarrow \cos^2\theta = 1$ ✓

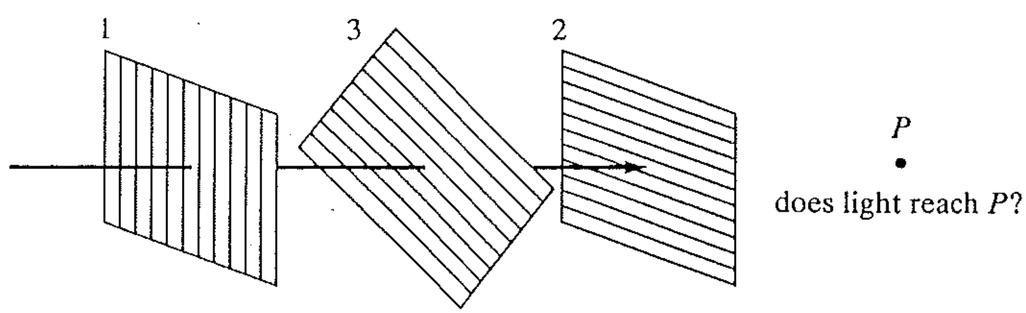
$\theta = 90 \Rightarrow \cos^2\theta = 0$ ✓

$\theta = 45 \Rightarrow \cos^2\theta = \frac{1}{2}$ ✓

H6

When a ray of light is incident on two polarizers with their polarization axes perpendicular, no light is transmitted. If a third polarizer is inserted between these two with its polarization axis at 45° to that of the other two, does any light get through to point P?



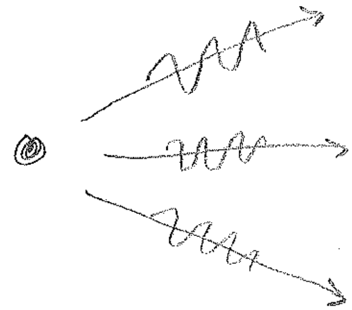


[DEMO: 3 sheet trick]

2 crossed polarizers
Add 3rd above them (nothing)
Add 3rd between (AHA!)

Explain (qualitatively).
HW: do quantitative

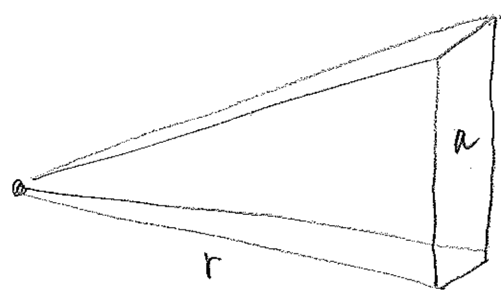
Consider EM waves travelling radially outward from a source



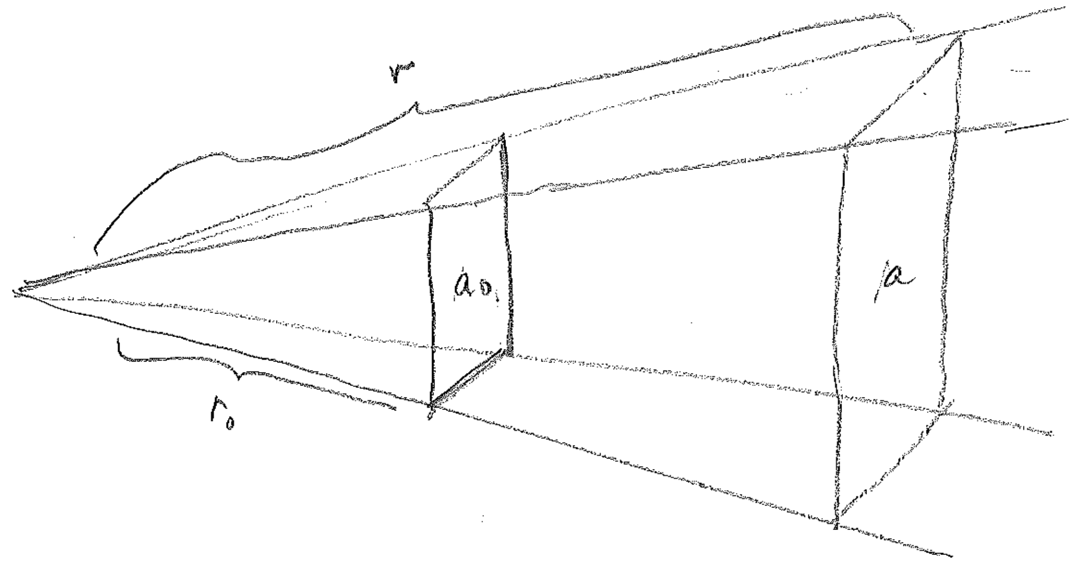
[Recall]

Intensity I = time avg of $|\vec{S}| = \frac{\text{energy}}{\text{area} \cdot \text{time}} = \frac{\text{power}}{\text{area}}$

e.g. sunlight at earth's surface has $I = 1380 \frac{W}{m^2} = 1380 \frac{J}{m^2 s}$



The power transmitted through area a is $P = I a$



By energy conservation, power thru a = power thru a_0

$$P = P_0$$

$$I a = I_0 a_0$$

$$I = I_0 \frac{a_0}{a}$$

But $\frac{a_0}{a} = \frac{r_0^2}{r^2}$ so

$$I = I_0 \left(\frac{r_0}{r}\right)^2$$

Intensity diminishes by inverse square of distance

Since $I \propto |\vec{E}|^2 \Rightarrow$ $|\vec{E}|$ of a travelling wave diminishes by $\frac{1}{r}$.